Computer algebra independent integration tests

4-Trig-functions/4.3-Tangent/4.3.1.3-d-sin-^m-a+b-tan-^n

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [91]. This is test number [102].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (91)	% 0. (0)
Mathematica	% 98.9 (90)	% 1.1 (1)
Maple	% 91.21 (83)	% 8.79 (8)
Maxima	% 81.32 (74)	% 18.68 (17)
Fricas	% 91.21 (83)	% 8.79 (8)
Sympy	% 5.49 (5)	% 94.51 (86)
Giac	% 83.52 (76)	% 16.48 (15)

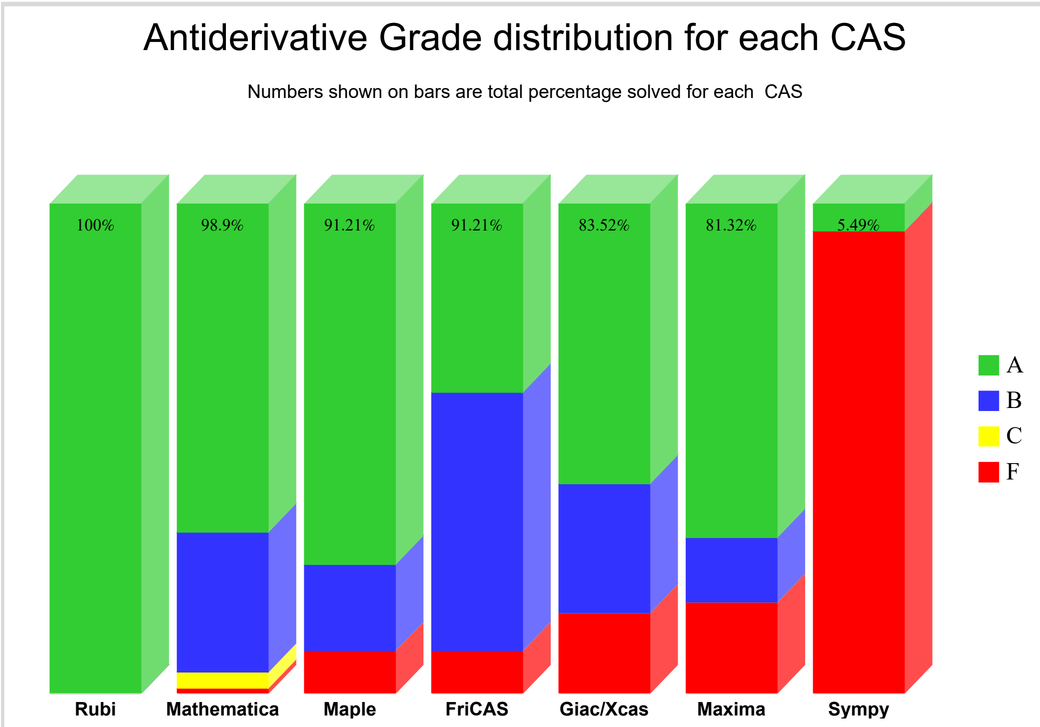
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

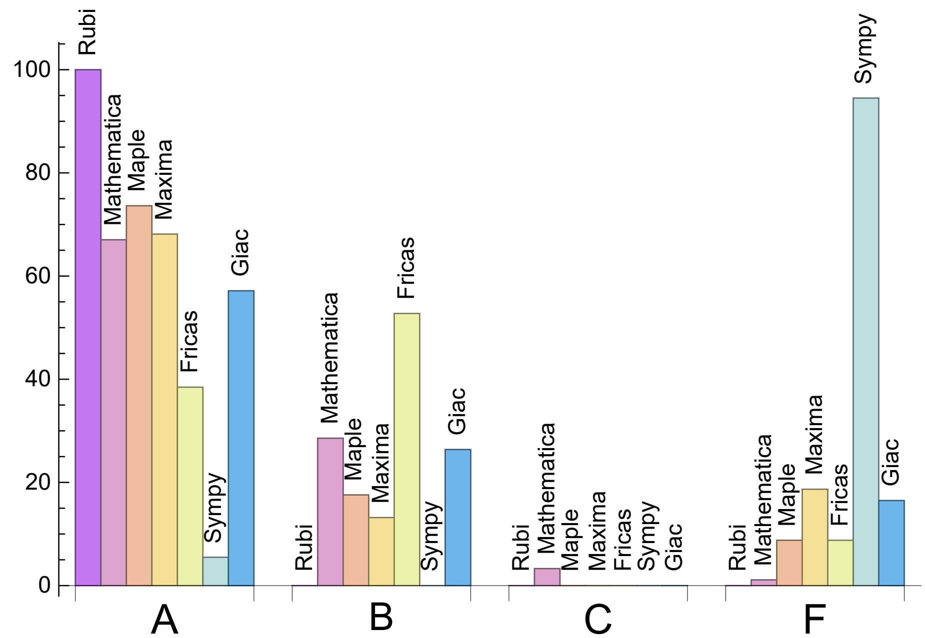
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	67.03	28.57	3.3	1.1
Maple	73.63	17.58	0.	8.79
Maxima	68.13	13.19	0.	18.68
Fricas	38.46	52.75	0.	8.79
Sympy	5.49	0.	0.	94.51
Giac	57.14	26.37	0.	16.48

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.32	136.4	0.95	109.	1.
Mathematica	2.34	277.16	1.88	162.	1.63
Maple	0.09	222.58	1.48	141.	1.32
Maxima	1.23	239.86	1.61	151.5	1.43
Fricas	2.35	672.3	5.02	444.	4.75
Sympy	0.26	27.2	0.69	31.	0.65
Giac	1.7	685.49	6.92	243.5	2.15

1.4 list of integrals that has no closed form antiderivative

$\{83, 88, 89, 90, 91\}$

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{30, 39, 48\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

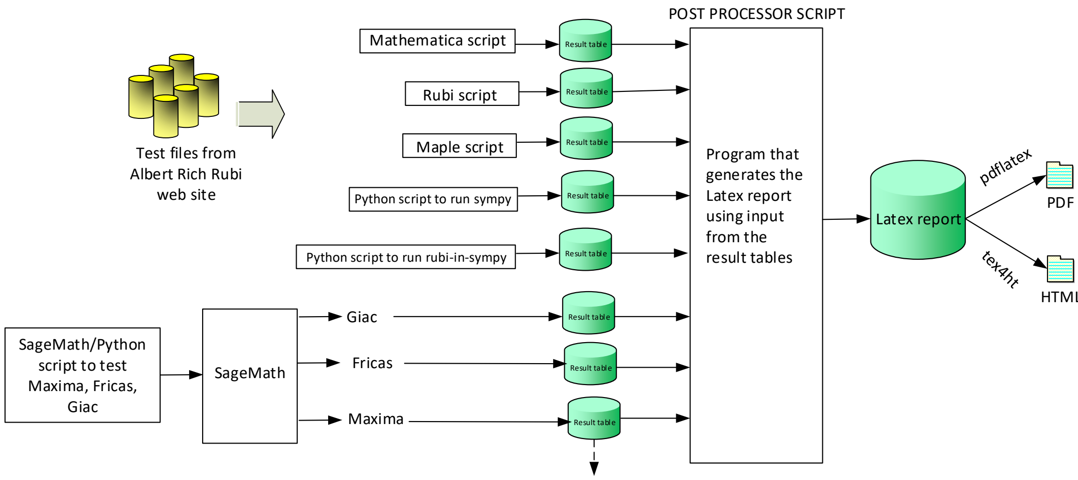
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 19, 21, 23, 25, 29, 31, 33, 36, 38, 42, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 72, 73, 74, 79, 80, 81, 83, 85, 86, 87, 88, 89, 90, 91 }

B grade: { 7, 9, 22, 24, 26, 27, 28, 30, 32, 34, 35, 37, 39, 40, 41, 43, 44, 46, 48, 66, 70, 71, 75, 76, 77, 84 }

C grade: { 18, 20, 78 }

F grade: { 82}

2.1.3 Maple

A grade: { 1, 3, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 55, 56, 57, 58, 59, 60, 64, 65, 66, 70, 71, 72, 75, 76, 77, 78, 83, 88, 89, 90, 91 }

B grade: { 2, 4, 7, 24, 33, 42, 52, 54, 61, 62, 63, 67, 68, 69, 73, 74 }

C grade: { }

F grade: { 79, 80, 81, 82, 84, 85, 86, 87 }

2.1.4 Maxima

A grade: { 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 54, 57, 59, 60, 64, 65, 66, 70, 71, 72, 75, 76, 77, 83, 88, 89, 90, 91

B grade: { 5, 7, 9, 61, 62, 63, 67, 68, 69, 73, 74, 78 }

C grade: { }

F grade: { 1, 2, 3, 4, 51, 53, 55, 56, 58, 79, 80, 81, 82, 84, 85, 86, 87 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 6, 11, 12, 13, 14, 15, 22, 23, 24, 25, 32, 33, 34, 35, 41, 42, 43, 44, 45, 47, 50, 51, 52, 53, 54, 57, 83, 88, 89, 90, 91 }

B grade: { 5, 7, 8, 9, 10, 16, 17, 18, 19, 20, 21, 26, 27, 28, 29, 30, 31, 36, 37, 38, 39, 40, 46, 48, 49, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

C grade: { }

F grade: { 79, 80, 81, 82, 84, 85, 86, 87 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 89 }

B grade: { }

C grade: { }

F grade: { 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91 }

2.1.7 Giac

A grade: { 1, 3, 5, 6, 8, 10, 16, 17, 19, 20, 21, 26, 27, 28, 29, 30, 31, 35, 36, 38, 39, 40, 44, 45, 46, 47, 48, 49, 50, 51, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 70, 71, 72, 75, 76, 77, 78, 88, 89, 90, 91 }

B grade: { 2, 4, 7, 9, 12, 13, 14, 15, 18, 22, 24, 25, 33, 37, 42, 52, 54, 61, 62, 67, 68, 69, 73, 74 }

C grade: { }

F grade: { 11, 23, 32, 34, 41, 43, 79, 80, 81, 82, 83, 84, 85, 86, 87 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	67	66	0	136	51	72
normalized size	1	1.	0.86	0.85	0.	1.74	0.65	0.92
time (sec)	N/A	0.066	0.08	0.075	0.	2.065	0.395	1.306

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	51	81	0	101	37	96
normalized size	1	1.	1.76	2.79	0.	3.48	1.28	3.31
time (sec)	N/A	0.137	0.019	0.051	0.	2.061	0.289	1.298

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	39	47	0	89	31	55
normalized size	1	1.	0.78	0.94	0.	1.78	0.62	1.1
time (sec)	N/A	0.054	0.091	0.041	0.	2.125	0.425	1.341

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	33	47	0	47	17	45
normalized size	1	1.	1.74	2.47	0.	2.47	0.89	2.37
time (sec)	N/A	0.089	0.012	0.035	0.	1.994	0.174	1.34

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	31	21	38	73	0	30
normalized size	1	1.	1.94	1.31	2.38	4.56	0.	1.88
time (sec)	N/A	0.086	0.016	0.033	1.478	2.205	0.	1.333

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	15	20	23	78	0	24
normalized size	1	1.	0.83	1.11	1.28	4.33	0.	1.33
time (sec)	N/A	0.033	0.019	0.033	1.404	2.02	0.	1.32

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	75	42	80	232	0	63
normalized size	1	1.	3.12	1.75	3.33	9.67	0.	2.62
time (sec)	N/A	0.132	0.022	0.039	1.246	2.133	0.	1.385

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	29	15	16	109	0	16
normalized size	1	1.	1.53	0.79	0.84	5.74	0.	0.84
time (sec)	N/A	0.039	0.018	0.038	1.366	1.996	0.	1.417

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	139	58	112	424	0	85
normalized size	1	1.	3.48	1.45	2.8	10.6	0.	2.12
time (sec)	N/A	0.151	0.023	0.043	1.301	2.139	0.	1.402

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	41	28	32	176	0	32
normalized size	1	1.	1.11	0.76	0.86	4.76	0.	0.86
time (sec)	N/A	0.047	0.019	0.041	1.363	1.987	0.	1.388

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	103	113	123	267	0	0
normalized size	1	1.	1.02	1.12	1.22	2.64	0.	0.
time (sec)	N/A	0.076	0.032	0.041	1.389	2.277	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	82	92	117	189	0	1439
normalized size	1	1.	0.99	1.11	1.41	2.28	0.	17.34
time (sec)	N/A	0.168	0.079	0.033	2.159	2.359	0.	1.7

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	71	79	95	197	0	7223
normalized size	1	1.	1.03	1.14	1.38	2.86	0.	104.68
time (sec)	N/A	0.068	0.027	0.035	1.361	2.303	0.	2.784

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	56	58	70	120	0	558
normalized size	1	1.	1.14	1.18	1.43	2.45	0.	11.39
time (sec)	N/A	0.084	0.048	0.035	2.511	2.255	0.	1.437

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	48	45	62	134	0	1669
normalized size	1	1.	1.3	1.22	1.68	3.62	0.	45.11
time (sec)	N/A	0.036	0.02	0.027	1.521	2.3	0.	1.658

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	52	42	62	170	0	66
normalized size	1	1.	2.	1.62	2.38	6.54	0.	2.54
time (sec)	N/A	0.029	0.024	0.036	1.535	2.351	0.	1.413

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	36	26	34	171	0	47
normalized size	1	1.	1.44	1.04	1.36	6.84	0.	1.88
time (sec)	N/A	0.078	0.059	0.077	1.388	2.495	0.	1.364

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	107	75	112	367	0	159
normalized size	1	1.	1.78	1.25	1.87	6.12	0.	2.65
time (sec)	N/A	0.068	0.027	0.082	1.142	2.89	0.	1.512

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	72	60	68	316	0	84
normalized size	1	1.	1.26	1.05	1.19	5.54	0.	1.47
time (sec)	N/A	0.088	0.245	0.081	1.128	2.795	0.	1.362

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	151	109	166	585	0	239
normalized size	1	1.	1.54	1.11	1.69	5.97	0.	2.44
time (sec)	N/A	0.092	0.029	0.084	1.26	2.723	0.	1.427

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	104	94	97	473	0	113
normalized size	1	1.	1.2	1.08	1.11	5.44	0.	1.3
time (sec)	N/A	0.108	0.545	0.085	1.132	2.1	0.	1.384

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	240	204	173	333	0	7808
normalized size	1	1.	2.12	1.81	1.53	2.95	0.	69.1
time (sec)	N/A	0.186	3.197	0.04	1.477	2.063	0.	6.669

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	152	167	140	321	0	0
normalized size	1	1.	1.25	1.37	1.15	2.63	0.	0.
time (sec)	N/A	0.131	1.023	0.041	0.974	1.993	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	162	145	111	240	0	1432
normalized size	1	1.	2.13	1.91	1.46	3.16	0.	18.84
time (sec)	N/A	0.113	2.362	0.039	1.476	2.016	0.	1.879

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	111	108	90	230	0	3830
normalized size	1	1.	1.63	1.59	1.32	3.38	0.	56.32
time (sec)	N/A	0.074	0.439	0.036	1.119	2.322	0.	4.538

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	97	61	81	288	0	100
normalized size	1	1.	2.26	1.42	1.88	6.7	0.	2.33
time (sec)	N/A	0.056	0.233	0.044	1.069	2.434	0.	1.572

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	91	43	53	246	0	69
normalized size	1	1.	2.17	1.02	1.26	5.86	0.	1.64
time (sec)	N/A	0.047	0.544	0.042	1.107	2.282	0.	1.539

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	250	120	165	581	0	232
normalized size	1	1.	2.63	1.26	1.74	6.12	0.	2.44
time (sec)	N/A	0.118	1.951	0.051	1.101	2.778	0.	1.636

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	127	104	93	446	0	123
normalized size	1	1.	1.61	1.32	1.18	5.65	0.	1.56
time (sec)	N/A	0.07	1.409	0.053	1.086	2.394	0.	1.619

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	994	183	252	856	0	363
normalized size	1	1.	6.02	1.11	1.53	5.19	0.	2.2
time (sec)	N/A	0.16	6.2	0.054	1.075	2.974	0.	1.652

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	114	166	140	628	0	177
normalized size	1	1.	0.93	1.36	1.15	5.15	0.	1.45
time (sec)	N/A	0.101	1.51	0.055	1.04	2.428	0.	1.408

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	771	271	234	455	0	0
normalized size	1	1.	3.76	1.32	1.14	2.22	0.	0.
time (sec)	N/A	0.189	6.266	0.053	1.037	2.396	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	203	226	153	342	0	3497
normalized size	1	1.	1.97	2.19	1.49	3.32	0.	33.95
time (sec)	N/A	0.145	4.227	0.052	1.515	2.168	0.	3.227

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	637	193	173	346	0	0
normalized size	1	1.	4.79	1.45	1.3	2.6	0.	0.
time (sec)	N/A	0.116	6.156	0.043	1.118	2.124	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	241	125	150	383	0	194
normalized size	1	1.	2.8	1.45	1.74	4.45	0.	2.26
time (sec)	N/A	0.086	2.26	0.061	1.085	2.497	0.	2.06

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	126	63	76	327	0	95
normalized size	1	1.	1.97	0.98	1.19	5.11	0.	1.48
time (sec)	N/A	0.053	0.976	0.056	1.101	2.047	0.	2.067

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	897	170	231	720	0	410
normalized size	1	1.	6.36	1.21	1.64	5.11	0.	2.91
time (sec)	N/A	0.135	6.183	0.065	1.152	2.74	0.	1.985

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	212	141	132	575	0	180
normalized size	1	1.	1.88	1.25	1.17	5.09	0.	1.59
time (sec)	N/A	0.086	2.207	0.068	1.064	2.018	0.	2.048

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	1229	254	338	1060	0	504
normalized size	1	1.	5.37	1.11	1.48	4.63	0.	2.2
time (sec)	N/A	0.208	6.2	0.068	1.106	2.955	0.	2.15

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	515	230	192	834	0	255
normalized size	1	1.	3.08	1.38	1.15	4.99	0.	1.53
time (sec)	N/A	0.134	1.758	0.069	1.118	2.284	0.	2.032

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	1017	412	294	528	0	0
normalized size	1	1.	3.7	1.5	1.07	1.92	0.	0.
time (sec)	N/A	0.248	6.275	0.059	1.148	2.133	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	263	368	208	431	0	5307
normalized size	1	1.	1.89	2.65	1.5	3.1	0.	38.18
time (sec)	N/A	0.175	6.266	0.057	1.709	2.248	0.	7.192

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	383	309	224	414	0	0
normalized size	1	1.	2.13	1.72	1.24	2.3	0.	0.
time (sec)	N/A	0.157	5.459	0.049	1.093	2.066	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	352	214	188	444	0	261
normalized size	1	1.	2.98	1.81	1.59	3.76	0.	2.21
time (sec)	N/A	0.114	5.132	0.069	1.108	2.565	0.	2.71

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	162	90	97	389	0	116
normalized size	1	1.	1.95	1.08	1.17	4.69	0.	1.4
time (sec)	N/A	0.058	1.16	0.063	1.12	2.049	0.	2.648

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	1128	192	254	818	0	405
normalized size	1	1.	7.01	1.19	1.58	5.08	0.	2.52
time (sec)	N/A	0.157	6.193	0.075	1.174	3.035	0.	2.77

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	188	184	162	633	0	217
normalized size	1	1.	1.37	1.34	1.18	4.62	0.	1.58
time (sec)	N/A	0.104	3.677	0.077	1.07	2.199	0.	2.645

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	274	274	1491	317	410	1289	0	647
normalized size	1	1.	5.44	1.16	1.5	4.7	0.	2.36
time (sec)	N/A	0.241	6.287	0.082	1.173	3.983	0.	2.684

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	233	301	231	922	0	317
normalized size	1	1.	1.2	1.55	1.19	4.75	0.	1.63
time (sec)	N/A	0.157	3.919	0.082	1.069	2.634	0.	2.653

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	660	442	522	1661	0	873
normalized size	1	1.	1.64	1.1	1.3	4.13	0.	2.17
time (sec)	N/A	0.306	6.259	0.127	1.154	4.568	0.	2.8

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	289	361	0	840	0	626
normalized size	1	1.	1.05	1.32	0.	3.07	0.	2.28
time (sec)	N/A	0.354	3.124	0.079	0.	2.327	0.	1.342

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	249	565	378	481	0	451
normalized size	1	1.	1.58	3.58	2.39	3.04	0.	2.85
time (sec)	N/A	0.339	2.592	0.068	1.655	2.289	0.	1.196

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	139	205	0	597	0	325
normalized size	1	1.	0.83	1.22	0.	3.55	0.	1.93
time (sec)	N/A	0.223	1.375	0.065	0.	2.278	0.	1.386

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	170	238	194	278	0	248
normalized size	1	1.	1.81	2.53	2.06	2.96	0.	2.64
time (sec)	N/A	0.163	0.723	0.059	1.572	2.123	0.	1.148

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	79	100	0	435	0	159
normalized size	1	1.	0.88	1.11	0.	4.83	0.	1.77
time (sec)	N/A	0.106	0.336	0.05	0.	2.046	0.	1.334

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	75	65	0	450	0	127
normalized size	1	1.	1.14	0.98	0.	6.82	0.	1.92
time (sec)	N/A	0.129	0.105	0.056	0.	2.377	0.	1.436

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	47	53	63	246	0	81
normalized size	1	1.	0.94	1.06	1.26	4.92	0.	1.62
time (sec)	N/A	0.06	0.131	0.062	1.068	2.055	0.	1.208

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	179	162	0	644	0	282
normalized size	1	1.	1.47	1.33	0.	5.28	0.	2.31
time (sec)	N/A	0.307	0.827	0.069	0.	2.683	0.	1.351

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	95	144	131	500	0	194
normalized size	1	1.	0.88	1.33	1.21	4.63	0.	1.8
time (sec)	N/A	0.104	0.456	0.076	1.064	2.154	0.	1.181

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	150	273	227	902	0	339
normalized size	1	1.	0.89	1.62	1.34	5.34	0.	2.01
time (sec)	N/A	0.151	2.164	0.083	1.095	2.412	0.	1.244

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	526	1211	1079	1388	0	992
normalized size	1	1.	1.77	4.08	3.63	4.67	0.	3.34
time (sec)	N/A	0.914	6.48	0.103	1.796	2.896	0.	1.233

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	373	724	684	986	0	693
normalized size	1	1.	1.72	3.34	3.15	4.54	0.	3.19
time (sec)	N/A	0.562	3.807	0.099	1.53	2.632	0.	1.216

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	246	352	396	640	0	355
normalized size	1	1.	1.66	2.38	2.68	4.32	0.	2.4
time (sec)	N/A	0.295	3.26	0.089	1.556	2.249	0.	1.177

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	109	75	100	663	0	100
normalized size	1	1.	1.51	1.04	1.39	9.21	0.	1.39
time (sec)	N/A	0.065	0.373	0.092	0.987	2.199	0.	1.219

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	244	189	194	1004	0	274
normalized size	1	1.	1.74	1.35	1.39	7.17	0.	1.96
time (sec)	N/A	0.122	2.506	0.153	1.092	2.374	0.	1.298

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	589	343	304	1770	0	448
normalized size	1	1.	2.69	1.57	1.39	8.08	0.	2.05
time (sec)	N/A	0.198	6.231	0.122	1.225	2.59	0.	1.239

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	683	1449	1469	2137	0	1246
normalized size	1	1.	1.79	3.79	3.85	5.59	0.	3.26
time (sec)	N/A	1.432	6.604	0.118	1.933	3.59	0.	1.286

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	501	882	1004	1569	0	794
normalized size	1	1.	1.76	3.09	3.52	5.51	0.	2.79
time (sec)	N/A	0.85	6.438	0.109	1.811	3.095	0.	1.27

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	316	542	625	1152	0	651
normalized size	1	1.	1.53	2.63	3.03	5.59	0.	3.16
time (sec)	N/A	0.396	3.959	0.115	1.849	2.483	0.	1.237

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	241	96	146	1219	0	153
normalized size	1	1.	2.54	1.01	1.54	12.83	0.	1.61
time (sec)	N/A	0.077	2.589	0.119	1.146	2.46	0.	1.27

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	456	234	259	1818	0	320
normalized size	1	1.	2.56	1.31	1.46	10.21	0.	1.8
time (sec)	N/A	0.153	3.227	0.13	1.202	2.599	0.	1.346

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	494	410	379	2344	0	516
normalized size	1	1.	1.86	1.55	1.43	8.85	0.	1.95
time (sec)	N/A	0.239	4.675	0.137	1.219	3.015	0.	1.238

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	564	1215	1346	2412	0	1218
normalized size	1	1.	1.54	3.32	3.68	6.59	0.	3.33
time (sec)	N/A	1.37	5.419	0.123	1.896	4.005	0.	1.35

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	395	668	894	1789	0	867
normalized size	1	1.	1.5	2.53	3.39	6.78	0.	3.28
time (sec)	N/A	0.571	3.594	0.12	1.712	3.002	0.	1.324

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	259	117	189	1901	0	174
normalized size	1	1.	2.23	1.01	1.63	16.39	0.	1.5
time (sec)	N/A	0.088	2.064	0.131	1.14	2.846	0.	1.225

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	528	278	308	2732	0	300
normalized size	1	1.	2.58	1.36	1.5	13.33	0.	1.46
time (sec)	N/A	0.173	2.034	0.198	1.123	3.255	0.	1.334

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	673	476	439	3507	0	578
normalized size	1	1.	2.24	1.59	1.46	11.69	0.	1.93
time (sec)	N/A	0.271	1.634	0.165	1.24	4.431	0.	1.361

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	41	26	68	203	0	59
normalized size	1	1.	1.58	1.	2.62	7.81	0.	2.27
time (sec)	N/A	0.074	0.041	0.025	1.85	2.354	0.	1.296

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	205	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.45	2.52	0.392	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	166	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.268	1.194	0.248	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	109	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.145	0.257	0.488	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	765	765	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	4.173	12.792	0.164	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.463	3.184	0.644	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	435	910	0	0	0	0	0
normalized size	1	1.	2.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.803	6.579	0.556	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	270	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.367	1.088	0.473	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.875	0.239	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	78	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.13	1.244	0.229	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.979	3.069	0.542	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.855	2.185	0.041	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.487	1.52	0.23	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.7	2.852	0.189	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [5] had the largest ratio of [0.6364]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.	13	0.308
2	A	9	7	1.	13	0.538
3	A	5	4	1.	13	0.308
4	A	8	6	1.	11	0.546
5	A	8	7	1.	11	0.636
6	A	3	2	1.	13	0.154
7	A	8	7	1.	13	0.538
8	A	4	3	1.	13	0.231
9	A	9	8	1.	13	0.615
10	A	4	3	1.	13	0.231
11	A	8	5	1.	19	0.263
12	A	6	4	1.	19	0.21
13	A	8	5	1.	19	0.263
14	A	5	4	1.	19	0.21
15	A	6	5	1.	17	0.294
16	A	4	2	1.	17	0.118
17	A	3	1	1.	19	0.053
18	A	7	6	1.	19	0.316
19	A	3	1	1.	19	0.053
20	A	9	6	1.	19	0.316
21	A	3	1	1.	19	0.053
22	A	8	6	1.	21	0.286
23	A	11	7	1.	21	0.333
24	A	6	6	1.	21	0.286
25	A	9	7	1.	19	0.368
26	A	6	4	1.	19	0.21
27	A	3	2	1.	21	0.095
28	A	10	7	1.	21	0.333
29	A	3	2	1.	21	0.095
30	A	13	9	1.	21	0.429
31	A	3	2	1.	21	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
32	A	16	8	1.	21	0.381
33	A	7	6	1.	21	0.286
34	A	13	8	1.	19	0.421
35	A	8	5	1.	19	0.263
36	A	3	2	1.	21	0.095
37	A	12	7	1.	21	0.333
38	A	3	2	1.	21	0.095
39	A	17	9	1.	21	0.429
40	A	3	2	1.	21	0.095
41	A	19	8	1.	21	0.381
42	A	7	6	1.	21	0.286
43	A	16	9	1.	19	0.474
44	A	10	5	1.	19	0.263
45	A	3	2	1.	21	0.095
46	A	14	9	1.	21	0.429
47	A	3	2	1.	21	0.095
48	A	21	9	1.	21	0.429
49	A	3	2	1.	21	0.095
50	A	25	9	1.	21	0.429
51	A	13	9	1.	21	0.429
52	A	8	6	1.	21	0.286
53	A	10	9	1.	21	0.429
54	A	7	6	1.	21	0.286
55	A	6	6	1.	19	0.316
56	A	6	5	1.	19	0.263
57	A	3	2	1.	21	0.095
58	A	15	11	1.	21	0.524
59	A	3	2	1.	21	0.095
60	A	3	2	1.	21	0.095
61	A	9	6	1.	21	0.286
62	A	8	6	1.	21	0.286
63	A	7	6	1.	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	3	2	1.	21	0.095
65	A	3	2	1.	21	0.095
66	A	3	2	1.	21	0.095
67	A	9	6	1.	21	0.286
68	A	8	6	1.	21	0.286
69	A	7	6	1.	21	0.286
70	A	3	2	1.	21	0.095
71	A	3	2	1.	21	0.095
72	A	3	2	1.	21	0.095
73	A	8	6	1.	21	0.286
74	A	7	6	1.	21	0.286
75	A	3	2	1.	21	0.095
76	A	3	2	1.	21	0.095
77	A	3	2	1.	21	0.095
78	A	6	5	1.	9	0.556
79	A	8	5	1.	21	0.238
80	A	6	5	1.	21	0.238
81	A	5	4	1.	19	0.21
82	A	14	6	1.	21	0.286
83	A	0	0	0.	0	0.
84	A	7	4	1.	21	0.19
85	A	6	4	1.	21	0.19
86	A	2	2	1.	21	0.095
87	A	4	4	1.	21	0.19
88	A	0	0	0.	0	0.
89	A	0	0	0.	0	0.
90	A	0	0	0.	0	0.
91	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int \frac{\sin^4(x)}{i+\tan(x)}\,dx$

Optimal. Leaf size=78

$$-\frac{ix}{16}-\frac{i}{8(-\tan(x)+i)}-\frac{3i}{16(\tan(x)+i)}-\frac{1}{32(-\tan(x)+i)^2}-\frac{5}{32(\tan(x)+i)^2}+\frac{i}{24(\tan(x)+i)^3}$$

```
[Out] (-I/16)*x - 1/(32*(I - Tan[x])^2) - (I/8)/(I - Tan[x]) + (I/24)/(I + Tan[x]
)^3 - 5/(32*(I + Tan[x])^2) - ((3*I)/16)/(I + Tan[x])
```

Rubi [A] time = 0.0659966, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3516, 848, 88, 203}

$$-\frac{ix}{16}-\frac{i}{8(-\tan(x)+i)}-\frac{3i}{16(\tan(x)+i)}-\frac{1}{32(-\tan(x)+i)^2}-\frac{5}{32(\tan(x)+i)^2}+\frac{i}{24(\tan(x)+i)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(I + Tan[x]),x]
```

```
[Out] (-I/16)*x - 1/(32*(I - Tan[x])^2) - (I/8)/(I - Tan[x]) + (I/24)/(I + Tan[x]
)^3 - 5/(32*(I + Tan[x])^2) - ((3*I)/16)/(I + Tan[x])
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(x)}{i+\tan(x)}\,dx &= \text{Subst}\left(\int \frac{x^4}{(i+x)\left(1+x^2\right)^3}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \frac{x^4}{(-i+x)^3(i+x)^4}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{16(-i+x)^3}-\frac{i}{8(-i+x)^2}-\frac{i}{8(i+x)^4}+\frac{5}{16(i+x)^3}+\frac{3i}{16(i+x)^2}-\frac{i}{16\left(1+x^2\right)}\right)dx, x, \tan(\right.\\
&= -\frac{1}{32(i-\tan(x))^2}-\frac{i}{8(i-\tan(x))}+\frac{i}{24(i+\tan(x))^3}-\frac{5}{32(i+\tan(x))^2}-\frac{3i}{16(i+\tan(x))}-\frac{1}{16}i\,\text{Subst}\\
&= -\frac{ix}{16}-\frac{1}{32(i-\tan(x))^2}-\frac{i}{8(i-\tan(x))}+\frac{i}{24(i+\tan(x))^3}-\frac{5}{32(i+\tan(x))^2}-\frac{3i}{16(i+\tan(x))}
\end{aligned}$$

Mathematica [A] time = 0.0800587, size = 67, normalized size = 0.86

$$\frac{\sec(x)\left(-32\sin(x)-27\sin(3x)+5\sin(5x)-56i\cos(x)-9i\cos(3x)+i\cos(5x)+24\tan^{-1}(\tan(x))(\cos(x)-i\sin(x))\right)}{384(\tan(x)+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(I + Tan[x]),x]
```

```
[Out] (Sec[x]*((-56*I)*Cos[x] - (9*I)*Cos[3*x] + I*Cos[5*x] + 24*ArcTan[Tan[x]]*(
Cos[x] - I*Sin[x]) - 32*Sin[x] - 27*Sin[3*x] + 5*Sin[5*x]))/(384*(I + Tan[x
]))
```

Maple [A] time = 0.075, size = 66, normalized size = 0.9

$$\frac{\frac{i}{8}}{\tan(x)-i}-\frac{1}{32\,(\tan(x)-i)^2}-\frac{\ln(\tan(x)-i)}{32}+\frac{\frac{i}{24}}{(i+\tan(x))^3}-\frac{\frac{3\,i}{16}}{i+\tan(x)}-\frac{5}{32\,(i+\tan(x))^2}+\frac{\ln(i+\tan(x))}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(I+tan(x)),x)
```

```
[Out] 1/8*I/(tan(x)-I)-1/32/(tan(x)-I)^2-1/32*ln(tan(x)-I)+1/24*I/(I+tan(x))^3-3/
16*I/(I+tan(x))-5/32/(I+tan(x))^2+1/32*ln(I+tan(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.06533, size = 136, normalized size = 1.74

$$\frac{1}{384}\left(-24i\,xe^{(4i\,x)}-2\,e^{(10i\,x)}+9\,e^{(8i\,x)}-12\,e^{(6i\,x)}-18\,e^{(2i\,x)}+3\right)e^{(-4i\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] 1/384*(-24*I*x*e^(4*I*x) - 2*e^(10*I*x) + 9*e^(8*I*x) - 12*e^(6*I*x) - 18*e
^(2*I*x) + 3)*e^(-4*I*x)
```

Sympy [A] time = 0.395444, size = 51, normalized size = 0.65

$$-\frac{ix}{16} - \frac{e^{6ix}}{192} + \frac{3e^{4ix}}{128} - \frac{e^{2ix}}{32} - \frac{3e^{-2ix}}{64} + \frac{e^{-4ix}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(I+tan(x)),x)
```

```
[Out] -I*x/16 - exp(6*I*x)/192 + 3*exp(4*I*x)/128 - exp(2*I*x)/32 - 3*exp(-2*I*x)
/64 + exp(-4*I*x)/128
```

Giac [A] time = 1.3062, size = 72, normalized size = 0.92

$$-\frac{3i\tan(x)^4 + 21\tan(x)^3 + 13i\tan(x)^2 + 11\tan(x) + 8i}{48(\tan(x)+i)^3(\tan(x)-i)^2} + \frac{1}{32}\log(\tan(x)+i) - \frac{1}{32}\log(\tan(x)-i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/48*(3*I*tan(x)^4 + 21*tan(x)^3 + 13*I*tan(x)^2 + 11*tan(x) + 8*I)/((tan(
x) + I)^3*(tan(x) - I)^2) + 1/32*log(tan(x) + I) - 1/32*log(tan(x) - I)
```

3.2 $\int \frac{\sin^3(x)}{i+\tan(x)} \, dx$

Optimal. Leaf size=29

$$\frac{\sin^5(x)}{5} - \frac{1}{5} i \cos^5(x) + \frac{1}{3} i \cos^3(x)$$

```
[Out] (I/3)*Cos[x]^3 - (I/5)*Cos[x]^5 + Sin[x]^5/5
```

Rubi [A] time = 0.136883, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3518, 3108, 3107, 2565, 14, 2564, 30}

$$\frac{\sin^5(x)}{5} - \frac{1}{5} i \cos^5(x) + \frac{1}{3} i \cos^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(I + Tan[x]),x]
```

```
[Out] (I/3)*Cos[x]^3 - (I/5)*Cos[x]^5 + Sin[x]^5/5
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
```

```
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3(x)}{i+\tan(x)}\,dx &= \int \frac{\cos(x)\sin^3(x)}{i\cos(x)+\sin(x)}\,dx \\
&= -\left(i\int \cos(x)(\cos(x)+i\sin(x))\sin^3(x)\,dx\right) \\
&= -\left(i\int \left(\cos^2(x)\sin^3(x)+i\cos(x)\sin^4(x)\right)\,dx\right) \\
&= -\left(i\int \cos^2(x)\sin^3(x)\,dx\right)+\int \cos(x)\sin^4(x)\,dx \\
&= i\,\mathrm{Subst}\left(\int x^2\left(1-x^2\right)\,dx, x, \cos(x)\right)+\mathrm{Subst}\left(\int x^4\,dx, x, \sin(x)\right) \\
&= \frac{\sin^5(x)}{5}+i\,\mathrm{Subst}\left(\int \left(x^2-x^4\right)\,dx, x, \cos(x)\right) \\
&= \frac{1}{3}i\cos^3(x)-\frac{1}{5}i\cos^5(x)+\frac{\sin^5(x)}{5}
\end{aligned}
$$

Mathematica [A] time = 0.019013, size = 51, normalized size = 1.76

$$
\frac{\sin(x)}{8}-\frac{1}{16}\sin(3x)+\frac{1}{80}\sin(5x)+\frac{1}{8}i\cos(x)+\frac{1}{48}i\cos(3x)-\frac{1}{80}i\cos(5x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(I + Tan[x]),x]
```

```
[Out] (I/8)*Cos[x] + (I/48)*Cos[3*x] - (I/80)*Cos[5*x] + Sin[x]/8 - Sin[3*x]/16 +
 Sin[5*x]/80
```

Maple [B] time = 0.051, size = 81, normalized size = 2.8

$$
-\frac{i}{4}\left(\tan\left(\frac{x}{2}\right)-i\right)^{-2}+\frac{1}{6}\left(\tan\left(\frac{x}{2}\right)-i\right)^{-3}+\frac{1}{8}\left(\tan\left(\frac{x}{2}\right)-i\right)^{-1}+i\left(\tan\left(\frac{x}{2}\right)+i\right)^{-4}+\frac{2}{5}\left(\tan\left(\frac{x}{2}\right)+i\right)^{-5}-\frac{2}{3}\left(\tan\left(\frac{x}{2}\right)+i
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(I+tan(x)),x)
```

```
[Out] -1/4*I/(tan(1/2*x)-I)^2+1/6/(tan(1/2*x)-I)^3+1/8/(tan(1/2*x)-I)+I/(tan(1/2*
x)+I)^4+2/5/(tan(1/2*x)+I)^5-2/3/(tan(1/2*x)+I)^3-1/8/(tan(1/2*x)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.06113, size = 101, normalized size = 3.48

$$\frac{1}{240}\left(-3i\,e^{(8i\,x)}+10i\,e^{(6i\,x)}+30i\,e^{(2i\,x)}-5i\right)e^{(-3i\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] 1/240*(-3*I*e^(8*I*x) + 10*I*e^(6*I*x) + 30*I*e^(2*I*x) - 5*I)*e^(-3*I*x)
```

Sympy [A] time = 0.289148, size = 37, normalized size = 1.28

$$-\frac{ie^{5ix}}{80}+\frac{ie^{3ix}}{24}+\frac{ie^{-ix}}{8}-\frac{ie^{-3ix}}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(I+tan(x)),x)
```

```
[Out] -I*exp(5*I*x)/80 + I*exp(3*I*x)/24 + I*exp(-I*x)/8 - I*exp(-3*I*x)/48
```

Giac [B] time = 1.29783, size = 96, normalized size = 3.31

$$-\frac{-3i\,\tan\left(\frac{1}{2}\,x\right)^{2}-12\,\tan\left(\frac{1}{2}\,x\right)+5i}{24\left(-i\,\tan\left(\frac{1}{2}\,x\right)-1\right)^{3}}-\frac{15\,\tan\left(\frac{1}{2}\,x\right)^{4}+60i\,\tan\left(\frac{1}{2}\,x\right)^{3}-10\,\tan\left(\frac{1}{2}\,x\right)^{2}-20i\,\tan\left(\frac{1}{2}\,x\right)+7}{120\left(\tan\left(\frac{1}{2}\,x\right)+i\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/24*(-3*I*tan(1/2*x)^2 - 12*tan(1/2*x) + 5*I)/(-I*tan(1/2*x) - 1)^3 - 1/1
20*(15*tan(1/2*x)^4 + 60*I*tan(1/2*x)^3 - 10*tan(1/2*x)^2 - 20*I*tan(1/2*x)
 + 7)/(tan(1/2*x) + I)^5
```

3.3 $\int \frac{\sin^2(x)}{i+\tan(x)} \, dx$

Optimal. Leaf size=50

$$-\frac{ix}{8}-\frac{i}{8(-\tan(x)+i)}-\frac{i}{4(\tan(x)+i)}-\frac{1}{8(\tan(x)+i)^2}$$

```
[Out] (-I/8)*x - (I/8)/(I - Tan[x]) - 1/(8*(I + Tan[x])^2) - (I/4)/(I + Tan[x])
```

Rubi [A] time = 0.0539709, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3516, 848, 88, 203}

$$-\frac{ix}{8}-\frac{i}{8(-\tan(x)+i)}-\frac{i}{4(\tan(x)+i)}-\frac{1}{8(\tan(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(I + Tan[x]),x]
```

```
[Out] (-I/8)*x - (I/8)/(I - Tan[x]) - 1/(8*(I + Tan[x])^2) - (I/4)/(I + Tan[x])
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(x)}{i+\tan(x)}\,dx &= \text{Subst}\left(\int \frac{x^2}{(i+x)\left(1+x^2\right)^2}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \frac{x^2}{(-i+x)^2(i+x)^3}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{i}{8(-i+x)^2}+\frac{1}{4(i+x)^3}+\frac{i}{4(i+x)^2}-\frac{i}{8\left(1+x^2\right)}\right)dx, x, \tan(x)\right)\\
&= -\frac{i}{8(i-\tan(x))}-\frac{1}{8(i+\tan(x))^2}-\frac{i}{4(i+\tan(x))}-\frac{1}{8}i\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)\\
&= -\frac{ix}{8}-\frac{i}{8(i-\tan(x))}-\frac{1}{8(i+\tan(x))^2}-\frac{i}{4(i+\tan(x))}
\end{aligned}
$$

Mathematica [A] time = 0.091249, size = 39, normalized size = 0.78

$$-\frac{i\left(-3i\sin(2x)+\cos(2x)+2\tan^{-1}(\tan(x))(\tan(x)+i)+3\right)}{16(\tan(x)+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(I + Tan[x]),x]

[Out] ((-I/16)*(3 + Cos[2*x] - (3*I)*Sin[2*x] + 2*ArcTan[Tan[x]]*(I + Tan[x])))/(
I + Tan[x])
```

Maple [A] time = 0.041, size = 47, normalized size = 0.9

$$\frac{\frac{i}{8}}{\tan(x)-i}-\frac{\ln(\tan(x)-i)}{16}-\frac{\frac{i}{4}}{i+\tan(x)}-\frac{1}{8\,(i+\tan(x))^2}+\frac{\ln(i+\tan(x))}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(I+tan(x)),x)
```

```
[Out] 1/8*I/(tan(x)-I)-1/16*ln(tan(x)-I)-1/4*I/(I+tan(x))-1/8/(I+tan(x))^2+1/16*l
n(I+tan(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.12543, size = 89, normalized size = 1.78

$$\frac{1}{32}\left(-4i\,xe^{(2i\,x)} + e^{(6i\,x)} - 2\,e^{(4i\,x)} - 2\right)e^{(-2i\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] 1/32*(-4*I*x*e^(2*I*x) + e^(6*I*x) - 2*e^(4*I*x) - 2)*e^(-2*I*x)
```

Sympy [A] time = 0.425476, size = 31, normalized size = 0.62

$$-\frac{ix}{8} + \frac{e^{4ix}}{32} - \frac{e^{2ix}}{16} - \frac{e^{-2ix}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(I+tan(x)),x)
```

```
[Out] -I*x/8 + exp(4*I*x)/32 - exp(2*I*x)/16 - exp(-2*I*x)/16
```

Giac [A] time = 1.3412, size = 55, normalized size = 1.1

$$-\frac{i\,\tan\left(x\right)^{2}+3\,\tan\left(x\right)+2i}{8\left(\tan\left(x\right)+i\right)^{2}\left(\tan\left(x\right)-i\right)}+\frac{1}{16}\,\log\left(\tan\left(x\right)+i\right)-\frac{1}{16}\,\log\left(\tan\left(x\right)-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/8*(I*tan(x)^2 + 3*tan(x) + 2*I)/((tan(x) + I)^2*(tan(x) - I)) + 1/16*log
(tan(x) + I) - 1/16*log(tan(x) - I)
```

3.4 $\int \frac{\sin(x)}{i+\tan(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\sin^3(x)}{3}+\frac{1}{3}i\cos^3(x)$$

```
[Out] (I/3)*Cos[x]^3 + Sin[x]^3/3
```

Rubi [A] time = 0.0893613, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {3518, 3108, 3107, 2565, 30, 2564}

$$\frac{\sin^3(x)}{3}+\frac{1}{3}i\cos^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(I + Tan[x]),x]
```

```
[Out] (I/3)*Cos[x]^3 + Sin[x]^3/3
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
```

```
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(x)}{i+\tan(x)}\,dx &= \int \frac{\cos(x)\sin(x)}{i\cos(x)+\sin(x)}\,dx\\
&= -(i\int \cos(x)(\cos(x)+i\sin(x))\sin(x)\,dx)\\
&= -\left(i\int \left(\cos^2(x)\sin(x)+i\cos(x)\sin^2(x)\right)\,dx\right)\\
&= -\left(i\int \cos^2(x)\sin(x)\,dx\right)+\int \cos(x)\sin^2(x)\,dx\\
&= i\,\mathrm{Subst}\left(\int x^2\,dx,x,\cos(x)\right)+\mathrm{Subst}\left(\int x^2\,dx,x,\sin(x)\right)\\
&= \frac{1}{3}i\cos^3(x)+\frac{\sin^3(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.01213, size = 33, normalized size = 1.74

$$\frac{\sin(x)}{4}-\frac{1}{12}\sin(3x)+\frac{1}{4}i\cos(x)+\frac{1}{12}i\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(I + Tan[x]),x]
```

```
[Out] (I/4)*Cos[x] + (I/12)*Cos[3*x] + Sin[x]/4 - Sin[3*x]/12
```

Maple [B] time = 0.035, size = 47, normalized size = 2.5

$$\frac{1}{2}\left(\tan\left(\frac{x}{2}\right)-i\right)^{-1}+i\left(\tan\left(\frac{x}{2}\right)+i\right)^{-2}+\frac{2}{3}\left(\tan\left(\frac{x}{2}\right)+i\right)^{-3}-\frac{1}{2}\left(\tan\left(\frac{x}{2}\right)+i\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(I+tan(x)),x)
```

```
[Out] 1/2/(tan(1/2*x)-I)+I/(tan(1/2*x)+I)^2+2/3/(tan(1/2*x)+I)^3-1/2/(tan(1/2*x)+
I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.99361, size = 47, normalized size = 2.47

$$\frac{1}{12}\left(i\,e^{(4i\,x)}+3i\right)e^{(-i\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*e^(4*I*x) + 3*I)*e^(-I*x)
```

Sympy [A] time = 0.17362, size = 17, normalized size = 0.89

$$\frac{ie^{3ix}}{12} + \frac{ie^{-ix}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(I+tan(x)),x)
```

```
[Out] I*exp(3*I*x)/12 + I*exp(-I*x)/4
```

Giac [B] time = 1.3401, size = 45, normalized size = 2.37

$$-\frac{i}{2\left(-i\tan\left(\frac{1}{2}x\right)-1\right)} - \frac{3\tan\left(\frac{1}{2}x\right)^2-1}{6\left(\tan\left(\frac{1}{2}x\right)+i\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/2*I/(-I*tan(1/2*x) - 1) - 1/6*(3*tan(1/2*x)^2 - 1)/(tan(1/2*x) + I)^3
```

3.5 $\int \frac{\csc(x)}{i+\tan(x)}\,dx$

Optimal. Leaf size=16

$$\sin(x) - i\cos(x) + i\tanh^{-1}(\cos(x))$$

```
[Out] I*ArcTanh[Cos[x]] - I*Cos[x] + Sin[x]
```

Rubi [A] time = 0.0864813, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3518, 3108, 3107, 2637, 2592, 321, 206}

$$\sin(x) - i\cos(x) + i\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(I + Tan[x]),x]
```

```
[Out] I*ArcTanh[Cos[x]] - I*Cos[x] + Sin[x]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
```

```
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{i+\tan(x)}\,dx &= \int \frac{\cot(x)}{i\cos(x)+\sin(x)}\,dx\\ &= -\left(i\int \cot(x)(\cos(x)+i\sin(x))\,dx\right)\\ &= -\left(i\int (i\cos(x)+\cos(x)\cot(x))\,dx\right)\\ &= -\left(i\int \cos(x)\cot(x)\,dx\right)+\int \cos(x)\,dx\\ &= \sin(x)+i\,\mathrm{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\cos(x)\right)\\ &= -i\cos(x)+\sin(x)+i\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(x)\right)\\ &= i\tanh^{-1}(\cos(x))-i\cos(x)+\sin(x)\end{aligned}$$

Mathematica [A] time = 0.015746, size = 31, normalized size = 1.94

$$\sin(x)-i\cos(x)-i\log\left(\sin\left(\frac{x}{2}\right)\right)+i\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(I + Tan[x]),x]
```

```
[Out] (-I)*Cos[x] + I*Log[Cos[x/2]] - I*Log[Sin[x/2]] + Sin[x]
```

Maple [A] time = 0.033, size = 21, normalized size = 1.3

$$2\,\left(\tan\left(x/2\right)+i\right)^{-1}-i\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(I+tan(x)),x)
```

```
[Out] 2/(tan(1/2*x)+I)-I*ln(tan(1/2*x))
```

Maxima [B] time = 1.47837, size = 38, normalized size = 2.38

$$\frac{2}{\frac{\sin(x)}{\cos(x)+1} + i} - i \log\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] 2/(sin(x)/(cos(x) + 1) + I) - I*log(sin(x)/(cos(x) + 1))
```

Fricas [B] time = 2.20494, size = 73, normalized size = 4.56

$$-i\,e^{(i\,x)} + i\,\log\left(e^{(i\,x)} + 1\right) - i\,\log\left(e^{(i\,x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] -I*e^(I*x) + I*log(e^(I*x) + 1) - I*log(e^(I*x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(x)}{\tan(x) + i}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(I+tan(x)),x)
```

```
[Out] Integral(csc(x)/(tan(x) + I), x)
```

Giac [A] time = 1.33305, size = 30, normalized size = 1.88

$$-\frac{2i}{-i\,\tan\left(\frac{1}{2}\,x\right) + 1} - i\,\log\left(-i\,\tan\left(\frac{1}{2}\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -2*I/(-I*tan(1/2*x) + 1) - I*log(-I*tan(1/2*x))
```

3.6 $\int \frac{\csc^2(x)}{i+\tan(x)}\,dx$

Optimal. Leaf size=18

$$ix + i\cot(x) + \log(\tan(x)) + \log(\cos(x))$$

```
[Out] I*x + I*Cot[x] + Log[Cos[x]] + Log[Tan[x]]
```

Rubi [A] time = 0.0333998, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3516, 44}

$$ix + i\cot(x) + \log(\tan(x)) + \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(I + Tan[x]),x]
```

```
[Out] I*x + I*Cot[x] + Log[Cos[x]] + Log[Tan[x]]
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\csc^2(x)}{i+\tan(x)}\,dx = \text{Subst}\left(\int \frac{1}{x^2(i+x)}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{-i-x} - \frac{i}{x^2} + \frac{1}{x}\right) dx, x, \tan(x)\right)$$
$$= ix + i\cot(x) + \log(\cos(x)) + \log(\tan(x))$$

Mathematica [A] time = 0.0191565, size = 15, normalized size = 0.83

$$ix + i\cot(x) + \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(I + Tan[x]),x]
```

```
[Out] I*x + I*Cot[x] + Log[Sin[x]]
```

Maple [A] time = 0.033, size = 20, normalized size = 1.1

$$-\ln\left(i+\tan(x)\right) + \ln\left(\tan(x)\right) + \frac{i}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(I+tan(x)),x)
```

```
[Out] -ln(I+tan(x))+ln(tan(x))+I/tan(x)
```

Maxima [A] time = 1.40432, size = 23, normalized size = 1.28

$$\frac{i}{\tan(x)} - \log\left(\tan(x) + i\right) + \log\left(\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] I/tan(x) - log(tan(x) + I) + log(tan(x))
```

Fricas [A] time = 2.02021, size = 78, normalized size = 4.33

$$\frac{\left(e^{(2ix)}-1\right)\log\left(e^{(2ix)}-1\right)-2}{e^{(2ix)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] ((e^(2*I*x) - 1)*log(e^(2*I*x) - 1) - 2)/(e^(2*I*x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc^2(x)}{\tan(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(I+tan(x)),x)
```

```
[Out] Integral(csc(x)**2/(tan(x) + I), x)
```

Giac [A] time = 1.32035, size = 24, normalized size = 1.33

$$\frac{i}{\tan(x)}-\log(\tan(x)+i)+\log(|\tan(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(I+tan(x)),x, algorithm="giac")
```

```
[Out] I/tan(x) - log(tan(x) + I) + log(abs(tan(x)))
```

3.7 $\int \frac{\csc^3(x)}{i+\tan(x)} \, dx$

Optimal. Leaf size=24

$$-\csc(x)-\frac{1}{2}i\tanh^{-1}(\cos(x))+\frac{1}{2}i\cot(x)\csc(x)$$

```
[Out] (-I/2)*ArcTanh[Cos[x]] - Csc[x] + (I/2)*Cot[x]*Csc[x]
```

Rubi [A] time = 0.13226, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3518, 3108, 3107, 2606, 8, 2611, 3770}

$$-\csc(x)-\frac{1}{2}i\tanh^{-1}(\cos(x))+\frac{1}{2}i\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(I + Tan[x]),x]
```

```
[Out] (-I/2)*ArcTanh[Cos[x]] - Csc[x] + (I/2)*Cot[x]*Csc[x]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
```

```
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^3(x)}{i+\tan(x)}\,dx &= \int \frac{\cot(x)\csc^2(x)}{i\cos(x)+\sin(x)}\,dx\\
&= -\left(i\int \cot(x)\csc^2(x)(\cos(x)+i\sin(x))\,dx\right)\\
&= -\left(i\int \left(i\cot(x)\csc(x)+\cot^2(x)\csc(x)\right)\,dx\right)\\
&= -\left(i\int \cot^2(x)\csc(x)\,dx\right)+\int \cot(x)\csc(x)\,dx\\
&= \frac{1}{2}i\cot(x)\csc(x)+\frac{1}{2}i\int \csc(x)\,dx-\mathrm{Subst}(\int 1\,dx,x,\csc(x))\\
&= -\frac{1}{2}i\tanh^{-1}(\cos(x))-\csc(x)+\frac{1}{2}i\cot(x)\csc(x)
\end{aligned}$$

Mathematica [B] time = 0.021679, size = 75, normalized size = 3.12

$$-\frac{1}{2}\tan\left(\frac{x}{2}\right)-\frac{1}{2}\cot\left(\frac{x}{2}\right)+\frac{1}{8}i\csc^2\left(\frac{x}{2}\right)-\frac{1}{8}i\sec^2\left(\frac{x}{2}\right)+\frac{1}{2}i\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{2}i\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(I + Tan[x]),x]
```

```
[Out] -Cot[x/2]/2 + (I/8)*Csc[x/2]^2 - (I/2)*Log[Cos[x/2]] + (I/2)*Log[Sin[x/2]]
- (I/8)*Sec[x/2]^2 - Tan[x/2]/2
```

Maple [B] time = 0.039, size = 42, normalized size = 1.8

$$-\frac{1}{2}\tan\left(\frac{x}{2}\right)-\frac{i}{8}\left(\tan\left(\frac{x}{2}\right)\right)^2+\frac{i}{8}\left(\tan\left(\frac{x}{2}\right)\right)^{-2}-\frac{1}{2}\left(\tan\left(\frac{x}{2}\right)\right)^{-1}+\frac{i}{2}\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(I+tan(x)),x)
```

```
[Out] -1/2*tan(1/2*x)-1/8*I*tan(1/2*x)^2+1/8*I/tan(1/2*x)^2-1/2/tan(1/2*x)+1/2*I*
ln(tan(1/2*x))
```

Maxima [B] time = 1.24577, size = 80, normalized size = 3.33

$$-\frac{\left(\frac{4\sin(x)}{\cos(x)+1}-i\right)(\cos(x)+1)^2}{8\sin(x)^2}-\frac{\sin(x)}{2(\cos(x)+1)}-\frac{i\sin(x)^2}{8(\cos(x)+1)^2}+\frac{1}{2}i\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*sin(x)/(cos(x) + 1) - I)*(cos(x) + 1)^2/sin(x)^2 - 1/2*sin(x)/(cos(
x) + 1) - 1/8*I*sin(x)^2/(cos(x) + 1)^2 + 1/2*I*log(sin(x)/(cos(x) + 1))
```

Fricas [B] time = 2.13258, size = 232, normalized size = 9.67

$$\frac{\left(-i\,e^{(4i\,x)}+2i\,e^{(2i\,x)}-i\right)\log\left(e^{(i\,x)}+1\right)+\left(i\,e^{(4i\,x)}-2i\,e^{(2i\,x)}+i\right)\log\left(e^{(i\,x)}-1\right)-6i\,e^{(3i\,x)}+2i\,e^{(i\,x)}}{2\left(e^{(4i\,x)}-2\,e^{(2i\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] 1/2*((-I*e^(4*I*x) + 2*I*e^(2*I*x) - I)*log(e^(I*x) + 1) + (I*e^(4*I*x) - 2
*I*e^(2*I*x) + I)*log(e^(I*x) - 1) - 6*I*e^(3*I*x) + 2*I*e^(I*x))/(e^(4*I*x
) - 2*e^(2*I*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(x)}{\tan(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(I+tan(x)),x)
```

```
[Out] Integral(csc(x)**3/(tan(x) + I), x)
```

Giac [B] time = 1.3854, size = 63, normalized size = 2.62

$$-\frac{1}{8}i\,\tan\left(\frac{1}{2}\,x\right)^2-\frac{6i\,\tan\left(\frac{1}{2}\,x\right)^2+4\,\tan\left(\frac{1}{2}\,x\right)-i}{8\,\tan\left(\frac{1}{2}\,x\right)^2}+\frac{1}{2}i\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)-\frac{1}{2}\,\tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/8*I*tan(1/2*x)^2 - 1/8*(6*I*tan(1/2*x)^2 + 4*tan(1/2*x) - I)/tan(1/2*x)^
2 + 1/2*I*log(abs(tan(1/2*x))) - 1/2*tan(1/2*x)
```

3.8 $\int \frac{\csc^4(x)}{i+\tan(x)} \, dx$

Optimal. Leaf size=19

$$-\frac{\cot^2(x)}{2}+\frac{1}{3} i \cot^3(x)$$

```
[Out] -Cot[x]^2/2 + (I/3)*Cot[x]^3
```

Rubi [A] time = 0.038832, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3516, 848, 43}

$$-\frac{\cot^2(x)}{2}+\frac{1}{3} i \cot^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(I + Tan[x]),x]
```

```
[Out] -Cot[x]^2/2 + (I/3)*Cot[x]^3
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^4(x)}{i+\tan(x)}\,dx &= \text{Subst}\left(\int \frac{1+x^2}{x^4(i+x)}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \frac{-i+x}{x^4}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{i}{x^4}+\frac{1}{x^3}\right)dx, x, \tan(x)\right)\\
&= -\frac{1}{2}\cot^2(x)+\frac{1}{3}i\cot^3(x)
\end{aligned}
$$

Mathematica [A] time = 0.0175136, size = 29, normalized size = 1.53

$$
-\frac{1}{3}i\cot(x)-\frac{\csc^2(x)}{2}+\frac{1}{3}i\cot(x)\csc^2(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(I + Tan[x]),x]
```

```
[Out] (-I/3)*Cot[x] - Csc[x]^2/2 + (I/3)*Cot[x]*Csc[x]^2
```

Maple [A] time = 0.038, size = 15, normalized size = 0.8

$$
-\frac{1}{2\left(\tan(x)\right)^2}+\frac{\frac{i}{3}}{\left(\tan(x)\right)^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(I+tan(x)),x)
```

```
[Out] -1/2/tan(x)^2+1/3*I/tan(x)^3
```

Maxima [A] time = 1.36622, size = 16, normalized size = 0.84

$$
-\frac{i\left(-3i\tan(x)-2\right)}{6\tan(x)^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] -1/6*I*(-3*I*tan(x) - 2)/tan(x)^3
```

Fricas [B] time = 1.99596, size = 109, normalized size = 5.74

$$\frac{2\left(6e^{(4ix)} - 3e^{(2ix)} + 1\right)}{3\left(e^{(6ix)} - 3e^{(4ix)} + 3e^{(2ix)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] 2/3*(6*e^(4*I*x) - 3*e^(2*I*x) + 1)/(e^(6*I*x) - 3*e^(4*I*x) + 3*e^(2*I*x)
- 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(I+tan(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41731, size = 16, normalized size = 0.84

$$-\frac{3\tan(x) - 2i}{6\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/6*(3*tan(x) - 2*I)/tan(x)^3
```

3.9 $\int \frac{\csc^5(x)}{i+\tan(x)}\,dx$

Optimal. Leaf size=40

$$-\frac{\csc^3(x)}{3}-\frac{1}{8}i\tanh^{-1}(\cos(x))+\frac{1}{4}i\cot(x)\csc^3(x)-\frac{1}{8}i\cot(x)\csc(x)$$

```
[Out] (-I/8)*ArcTanh[Cos[x]] - (I/8)*Cot[x]*Csc[x] - Csc[x]^3/3 + (I/4)*Cot[x]*Cs
c[x]^3
```

Rubi [A] time = 0.150726, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3518, 3108, 3107, 2606, 30, 2611, 3768, 3770}

$$-\frac{\csc^3(x)}{3}-\frac{1}{8}i\tanh^{-1}(\cos(x))+\frac{1}{4}i\cot(x)\csc^3(x)-\frac{1}{8}i\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^5/(I + Tan[x]),x]
```

```
[Out] (-I/8)*ArcTanh[Cos[x]] - (I/8)*Cot[x]*Csc[x] - Csc[x]^3/3 + (I/4)*Cot[x]*Cs
c[x]^3
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^5(x)}{i+\tan(x)}\,dx &= \int \frac{\cot(x)\csc^4(x)}{i\cos(x)+\sin(x)}\,dx\\
&= -\left(i\int \cot(x)\csc^4(x)(\cos(x)+i\sin(x))\,dx\right)\\
&= -\left(i\int \left(i\cot(x)\csc^3(x)+\cot^2(x)\csc^3(x)\right)\,dx\right)\\
&= -\left(i\int \cot^2(x)\csc^3(x)\,dx\right)+\int \cot(x)\csc^3(x)\,dx\\
&= \frac{1}{4}i\cot(x)\csc^3(x)+\frac{1}{4}i\int \csc^3(x)\,dx-\mathrm{Subst}\left(\int x^2\,dx,x,\csc(x)\right)\\
&= -\frac{1}{8}i\cot(x)\csc(x)-\frac{\csc^3(x)}{3}+\frac{1}{4}i\cot(x)\csc^3(x)+\frac{1}{8}i\int \csc(x)\,dx\\
&= -\frac{1}{8}i\tanh^{-1}(\cos(x))-\frac{1}{8}i\cot(x)\csc(x)-\frac{\csc^3(x)}{3}+\frac{1}{4}i\cot(x)\csc^3(x)\end{aligned}$$

Mathematica [B] time = 0.0233137, size = 139, normalized size = 3.48

$$-\frac{1}{12}\tan\left(\frac{x}{2}\right)-\frac{1}{12}\cot\left(\frac{x}{2}\right)+\frac{1}{64}i\csc^4\left(\frac{x}{2}\right)-\frac{1}{32}i\csc^2\left(\frac{x}{2}\right)-\frac{1}{64}i\sec^4\left(\frac{x}{2}\right)+\frac{1}{32}i\sec^2\left(\frac{x}{2}\right)+\frac{1}{8}i\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{8}i\log\left(\mathrm{co}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^5/(I + Tan[x]),x]
```

```
[Out] -Cot[x/2]/12 - (I/32)*Csc[x/2]^2 - (Cot[x/2]*Csc[x/2]^2)/24 + (I/64)*Csc[x/
2]^4 - (I/8)*Log[Cos[x/2]] + (I/8)*Log[Sin[x/2]] + (I/32)*Sec[x/2]^2 - (I/6
4)*Sec[x/2]^4 - Tan[x/2]/12 - (Sec[x/2]^2*Tan[x/2])/24
```

Maple [A] time = 0.043, size = 58, normalized size = 1.5

$$-\frac{1}{8}\tan\left(\frac{x}{2}\right)-\frac{i}{64}\left(\tan\left(\frac{x}{2}\right)\right)^4-\frac{1}{24}\left(\tan\left(\frac{x}{2}\right)\right)^3+\frac{i}{64}\left(\tan\left(\frac{x}{2}\right)\right)^{-4}-\frac{1}{8}\left(\tan\left(\frac{x}{2}\right)\right)^{-1}-\frac{1}{24}\left(\tan\left(\frac{x}{2}\right)\right)^{-3}+\frac{i}{8}\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^5/(I+tan(x)),x)
```

```
[Out] -1/8*tan(1/2*x)-1/64*I*tan(1/2*x)^4-1/24*tan(1/2*x)^3+1/64*I/tan(1/2*x)^4-1
/8/tan(1/2*x)-1/24/tan(1/2*x)^3+1/8*I*ln(tan(1/2*x))
```

Maxima [B] time = 1.30085, size = 112, normalized size = 2.8

$$-\frac{\left(\frac{8\sin(x)}{\cos(x)+1}+\frac{24\sin(x)^3}{(\cos(x)+1)^3}-3i\right)(\cos(x)+1)^4}{192\sin(x)^4}-\frac{\sin(x)}{8(\cos(x)+1)}-\frac{\sin(x)^3}{24(\cos(x)+1)^3}-\frac{i\sin(x)^4}{64(\cos(x)+1)^4}+\frac{1}{8}i\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*sin(x)/(cos(x) + 1) + 24*sin(x)^3/(cos(x) + 1)^3 - 3*I)*(cos(x) +
 1)^4/sin(x)^4 - 1/8*sin(x)/(cos(x) + 1) - 1/24*sin(x)^3/(cos(x) + 1)^3 - 1
/64*I*sin(x)^4/(cos(x) + 1)^4 + 1/8*I*log(sin(x)/(cos(x) + 1))
```

Fricas [B] time = 2.13878, size = 424, normalized size = 10.6

$$\frac{\left(-3ie^{(8ix)}+12ie^{(6ix)}-18ie^{(4ix)}+12ie^{(2ix)}-3i\right)\log\left(e^{(ix)}+1\right)+\left(3ie^{(8ix)}-12ie^{(6ix)}+18ie^{(4ix)}-12ie^{(2ix)}+3i\right)\log\left(e^{(ix)}-1\right)+\ldots}{24\left(e^{(8ix)}-4e^{(6ix)}+6e^{(4ix)}-4e^{(2ix)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] 1/24*((-3*I*e^(8*I*x) + 12*I*e^(6*I*x) - 18*I*e^(4*I*x) + 12*I*e^(2*I*x) -
3*I)*log(e^(I*x) + 1) + (3*I*e^(8*I*x) - 12*I*e^(6*I*x) + 18*I*e^(4*I*x) -
12*I*e^(2*I*x) + 3*I)*log(e^(I*x) - 1) + 6*I*e^(7*I*x) + 106*I*e^(5*I*x) -
22*I*e^(3*I*x) + 6*I*e^(I*x))/(e^(8*I*x) - 4*e^(6*I*x) + 6*e^(4*I*x) - 4*e^
(2*I*x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**5/(I+tan(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.40169, size = 85, normalized size = 2.12

$$-\frac{1}{64}i\tan\left(\frac{1}{2}x\right)^4-\frac{1}{24}\tan\left(\frac{1}{2}x\right)^3-\frac{50i\tan\left(\frac{1}{2}x\right)^4+24\tan\left(\frac{1}{2}x\right)^3+8\tan\left(\frac{1}{2}x\right)-3i}{192\tan\left(\frac{1}{2}x\right)^4}+\frac{1}{8}i\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)-\frac{1}{8}\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/64*I*tan(1/2*x)^4 - 1/24*tan(1/2*x)^3 - 1/192*(50*I*tan(1/2*x)^4 + 24*ta
n(1/2*x)^3 + 8*tan(1/2*x) - 3*I)/tan(1/2*x)^4 + 1/8*I*log(abs(tan(1/2*x)))
- 1/8*tan(1/2*x)
```

3.10 $\int \frac{\csc^6(x)}{i+\tan(x)}\,dx$

Optimal. Leaf size=37

$$\frac{1}{5}i\cot^5(x) - \frac{\cot^4(x)}{4} + \frac{1}{3}i\cot^3(x) - \frac{\cot^2(x)}{2}$$

```
[Out] -Cot[x]^2/2 + (I/3)*Cot[x]^3 - Cot[x]^4/4 + (I/5)*Cot[x]^5
```

Rubi [A] time = 0.0469125, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3516, 848, 75}

$$\frac{1}{5}i\cot^5(x) - \frac{\cot^4(x)}{4} + \frac{1}{3}i\cot^3(x) - \frac{\cot^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^6/(I + Tan[x]),x]
```

```
[Out] -Cot[x]^2/2 + (I/3)*Cot[x]^3 - Cot[x]^4/4 + (I/5)*Cot[x]^5
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^6(x)}{i+\tan(x)}\,dx &= \text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6(i+x)}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \frac{(-i+x)^2(i+x)}{x^6}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{i}{x^6}+\frac{1}{x^5}-\frac{i}{x^4}+\frac{1}{x^3}\right)dx, x, \tan(x)\right)\\ &= -\frac{1}{2}\cot^2(x)+\frac{1}{3}i\cot^3(x)-\frac{\cot^4(x)}{4}+\frac{1}{5}i\cot^5(x)\end{aligned}$$

Mathematica [A] time = 0.018521, size = 41, normalized size = 1.11

$$-\frac{2}{15}i\cot(x)-\frac{\csc^4(x)}{4}+\frac{1}{5}i\cot(x)\csc^4(x)-\frac{1}{15}i\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^6/(I + Tan[x]),x]
```

```
[Out] ((-2*I)/15)*Cot[x] - (I/15)*Cot[x]*Csc[x]^2 - Csc[x]^4/4 + (I/5)*Cot[x]*Csc
[x]^4
```

Maple [A] time = 0.041, size = 28, normalized size = 0.8

$$-\frac{1}{4\,(\tan(x))^4}-\frac{1}{2\,(\tan(x))^2}+\frac{\frac{i}{3}}{(\tan(x))^3}+\frac{\frac{i}{5}}{(\tan(x))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^6/(I+tan(x)),x)
```

```
[Out] -1/4/tan(x)^4-1/2/tan(x)^2+1/3*I/tan(x)^3+1/5*I/tan(x)^5
```

Maxima [A] time = 1.36274, size = 32, normalized size = 0.86

$$\frac{i\left(30i\tan\left(x\right)^{3}+20\tan\left(x\right)^{2}+15i\tan\left(x\right)+12\right)}{60\tan\left(x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^6/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] 1/60*I*(30*I*tan(x)^3 + 20*tan(x)^2 + 15*I*tan(x) + 12)/tan(x)^5
```

Fricas [B] time = 1.98741, size = 176, normalized size = 4.76

$$-\frac{4\left(30e^{(6ix)}-10e^{(4ix)}+5e^{(2ix)}-1\right)}{15\left(e^{(10ix)}-5e^{(8ix)}+10e^{(6ix)}-10e^{(4ix)}+5e^{(2ix)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^6/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] -4/15*(30*e^(6*I*x) - 10*e^(4*I*x) + 5*e^(2*I*x) - 1)/(e^(10*I*x) - 5*e^(8*
I*x) + 10*e^(6*I*x) - 10*e^(4*I*x) + 5*e^(2*I*x) - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**6/(I+tan(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38777, size = 32, normalized size = 0.86

$$-\frac{30\tan\left(x\right)^{3}-20i\tan\left(x\right)^{2}+15\tan\left(x\right)-12i}{60\tan\left(x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^6/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/60*(30*tan(x)^3 - 20*I*tan(x)^2 + 15*tan(x) - 12*I)/tan(x)^5
```

3.11 $\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=101

$$-\frac{a\cos^5(c+dx)}{5d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\sin^5(c+dx)}{5d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d + (2*a*Cos[c + d*x]^3)/(3*
d) - (a*Cos[c + d*x]^5)/(5*d) - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*
d) - (b*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0764194, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3517, 2633, 2592, 302, 206}

$$-\frac{a\cos^5(c+dx)}{5d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\sin^5(c+dx)}{5d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d + (2*a*Cos[c + d*x]^3)/(3*
d) - (a*Cos[c + d*x]^5)/(5*d) - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*
d) - (b*Sin[c + d*x]^5)/(5*d)
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx = \int \left(a\sin^5(c+dx)+b\sin^5(c+dx)\tan(c+dx)\right)\,dx$$
$$= a\int \sin^5(c+dx)\,dx + b\int \sin^5(c+dx)\tan(c+dx)\,dx$$
$$= -\frac{a\,\mathrm{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \cos(c+dx)\right)}{d} + \frac{b\,\mathrm{Subst}\left(\int \frac{x^6}{1-x^2}\,dx, x, \sin(c+d$$
$$= -\frac{a\cos(c+dx)}{d} + \frac{2a\cos^3(c+dx)}{3d} - \frac{a\cos^5(c+dx)}{5d} + \frac{b\,\mathrm{Subst}\left(\int \left(-1-x^2-x^4+\right.\right.}{d}$$
$$= -\frac{a\cos(c+dx)}{d} + \frac{2a\cos^3(c+dx)}{3d} - \frac{a\cos^5(c+dx)}{5d} - \frac{b\sin(c+dx)}{d} - \frac{b\sin^3(c+d}{3d}$$
$$= \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cos(c+dx)}{d} + \frac{2a\cos^3(c+dx)}{3d} - \frac{a\cos^5(c+dx)}{5d} - \frac{b\sin}{}$$

Mathematica [A] time = 0.0317254, size = 103, normalized size = 1.02

$$-\frac{5a\cos(c+dx)}{8d} + \frac{5a\cos(3(c+dx))}{48d} - \frac{a\cos(5(c+dx))}{80d} - \frac{b\sin^5(c+dx)}{5d} - \frac{b\sin^3(c+dx)}{3d} - \frac{b\sin(c+dx)}{d} + \frac{b\tanh^{-1}(\sin}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (5*a*Cos[c + d*x])/(8*d) + (5*a*Cos[3*(c + d*
x)])/(48*d) - (a*Cos[5*(c + d*x)])/(80*d) - (b*Sin[c + d*x])/d - (b*Sin[c +
 d*x]^3)/(3*d) - (b*Sin[c + d*x]^5)/(5*d)
```

Maple [A] time = 0.041, size = 113, normalized size = 1.1

$$-\frac{b(\sin(dx+c))^5}{5d}-\frac{b(\sin(dx+c))^3}{3d}-\frac{b\sin(dx+c)}{d}+\frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{8a\cos(dx+c)}{15d}-\frac{a\cos(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5*(a+b*tan(d*x+c)),x)
```

```
[Out] -1/5*b*sin(d*x+c)^5/d-1/3*b*sin(d*x+c)^3/d-b*sin(d*x+c)/d+1/d*b*ln(sec(d*x+
c)+tan(d*x+c))-8/15*a*cos(d*x+c)/d-1/5/d*a*cos(d*x+c)*sin(d*x+c)^4-4/15/d*a
*cos(d*x+c)*sin(d*x+c)^2
```

Maxima [A] time = 1.38921, size = 123, normalized size = 1.22

$$-\frac{2\left(3\cos(dx+c)^5-10\cos(dx+c)^3+15\cos(dx+c)\right)a+\left(6\sin(dx+c)^5+10\sin(dx+c)^3-15\log(\sin(dx+c)+\right.}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(2*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*a + (6*si
n(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*
x + c) - 1) + 30*sin(d*x + c))*b)/d
```

Fricas [A] time = 2.27656, size = 267, normalized size = 2.64

$$-\frac{6a\cos(dx+c)^5-20a\cos(dx+c)^3+30a\cos(dx+c)-15b\log(\sin(dx+c)+1)+15b\log(-\sin(dx+c)+1)+2}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/30*(6*a*cos(d*x + c)^5 - 20*a*cos(d*x + c)^3 + 30*a*cos(d*x + c) - 15*b*
log(sin(d*x + c) + 1) + 15*b*log(-sin(d*x + c) + 1) + 2*(3*b*cos(d*x + c)^4
 - 11*b*cos(d*x + c)^2 + 23*b)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5*(a+b*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.12 $\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=83

$$-\frac{\sin^3(c+dx)\cos(c+dx)(a+b\tan(c+dx))}{4d}-\frac{\sin(c+dx)\cos(c+dx)(3a+4b\tan(c+dx))}{8d}+\frac{3ax}{8}-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] (3*a*x)/8 - (b*Log[Cos[c + d*x]])/d - (Cos[c + d*x]*Sin[c + d*x]^3*(a + b*T
an[c + d*x]))/(4*d) - (Cos[c + d*x]*Sin[c + d*x]*(3*a + 4*b*Tan[c + d*x]))/
(8*d)
```

Rubi [A] time = 0.168075, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {819, 635, 203, 260}

$$-\frac{\sin^3(c+dx)\cos(c+dx)(a+b\tan(c+dx))}{4d}-\frac{\sin(c+dx)\cos(c+dx)(3a+4b\tan(c+dx))}{8d}+\frac{3ax}{8}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - (b*Log[Cos[c + d*x]])/d - (Cos[c + d*x]*Sin[c + d*x]^3*(a + b*T
an[c + d*x]))/(4*d) - (Cos[c + d*x]*Sin[c + d*x]*(3*a + 4*b*Tan[c + d*x]))/
(8*d)
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+bx)}{\left(1+x^2\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sin^3(c+dx)(a+b\tan(c+dx))}{4d} + \frac{\text{Subst}\left(\int \frac{x^2(3a+4bx)}{\left(1+x^2\right)^2}\,dx, x, \tan(c+\right.}{4d}$$

$$= -\frac{\cos(c+dx)\sin^3(c+dx)(a+b\tan(c+dx))}{4d} - \frac{\cos(c+dx)\sin(c+dx)(3a+4b\,\text{t}}{8d}$$

$$= -\frac{\cos(c+dx)\sin^3(c+dx)(a+b\tan(c+dx))}{4d} - \frac{\cos(c+dx)\sin(c+dx)(3a+4b\,\text{t}}{8d}$$

$$= \frac{3ax}{8} - \frac{b\log(\cos(c+dx))}{d} - \frac{\cos(c+dx)\sin^3(c+dx)(a+b\tan(c+dx))}{4d} - \frac{\cos(c\,-}{}$$

Mathematica [A] time = 0.078587, size = 82, normalized size = 0.99

$$\frac{3a(c+dx)}{8d} - \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d} - \frac{b\left(\frac{1}{4}\cos^4(c+dx) - \cos^2(c+dx) + \log(\cos(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) - (b*(-Cos[c + d*x]^2 + Cos[c + d*x]^4/4 + Log[Cos[c
+ d*x]]))/d - (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.033, size = 92, normalized size = 1.1

$$-\frac{b(\sin(dx+c))^4}{4d}-\frac{b(\sin(dx+c))^2}{2d}-\frac{b\ln(\cos(dx+c))}{d}-\frac{a\cos(dx+c)(\sin(dx+c))^3}{4d}-\frac{3a\sin(dx+c)\cos(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(a+b*tan(d*x+c)),x)
```

```
[Out] -1/4/d*b*sin(d*x+c)^4-1/2/d*b*sin(d*x+c)^2-b*ln(cos(d*x+c))/d-1/4/d*a*cos(d
*x+c)*sin(d*x+c)^3-3/8/d*a*sin(d*x+c)*cos(d*x+c)+3/8*a*x+3/8/d*a*c
```

Maxima [A] time = 2.15915, size = 117, normalized size = 1.41

$$\frac{3(dx+c)a+4b\log\left(\tan(dx+c)^2+1\right)-\frac{5a\tan(dx+c)^3-8b\tan(dx+c)^2+3a\tan(dx+c)-6b}{\tan(dx+c)^4+2\tan(dx+c)^2+1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(3*(d*x + c)*a + 4*b*log(tan(d*x + c)^2 + 1) - (5*a*tan(d*x + c)^3 - 8*
b*tan(d*x + c)^2 + 3*a*tan(d*x + c) - 6*b)/(tan(d*x + c)^4 + 2*tan(d*x + c)
^2 + 1))/d
```

Fricas [A] time = 2.35933, size = 189, normalized size = 2.28

$$-\frac{2b\cos(dx+c)^4-3adx-8b\cos(dx+c)^2+8b\log(-\cos(dx+c))-\left(2a\cos(dx+c)^3-5a\cos(dx+c)\right)\sin(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*cos(d*x + c)^4 - 3*a*d*x - 8*b*cos(d*x + c)^2 + 8*b*log(-cos(d*x
+ c)) - (2*a*cos(d*x + c)^3 - 5*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(a+b\tan(c+dx))\sin^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**4, x)
```

Giac [B] time = 1.69996, size = 1439, normalized size = 17.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(12*a*d*x*tan(d*x)^4*tan(c)^4 - 16*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*
tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d
*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 + 24*a*d*x*tan(d*x)^4*tan(c)^2 + 24*a*
d*x*tan(d*x)^2*tan(c)^4 + 11*b*tan(d*x)^4*tan(c)^4 - 32*b*log(4*(tan(c)^2 +
 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(
d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^2 + 12*a*tan(d*x)^4*tan(
c)^3 - 32*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
 + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*ta
n(c)^4 + 12*a*tan(d*x)^3*tan(c)^4 + 12*a*d*x*tan(d*x)^4 + 48*a*d*x*tan(d*x)
^2*tan(c)^2 + 6*b*tan(d*x)^4*tan(c)^2 - 32*b*tan(d*x)^3*tan(c)^3 + 12*a*d*x
*tan(c)^4 + 6*b*tan(d*x)^2*tan(c)^4 - 16*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4
*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(
d*x)*tan(c) + 1))*tan(d*x)^4 + 20*a*tan(d*x)^4*tan(c) - 64*b*log(4*(tan(c)^
2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + t
an(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 24*a*tan(d*x)^3*t
an(c)^2 + 24*a*tan(d*x)^2*tan(c)^3 - 16*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*
tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d
*x)*tan(c) + 1))*tan(c)^4 + 20*a*tan(d*x)*tan(c)^4 + 24*a*d*x*tan(d*x)^2 -
13*b*tan(d*x)^4 - 64*b*tan(d*x)^3*tan(c) + 24*a*d*x*tan(c)^2 - 36*b*tan(d*x
)^2*tan(c)^2 - 64*b*tan(d*x)*tan(c)^3 - 13*b*tan(c)^4 - 32*b*log(4*(tan(c)^
2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + t
an(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2 - 20*a*tan(d*x)^3 - 24*a*tan
```

```
(d*x)^2*tan(c) - 32*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x
)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan
(c)^2 - 24*a*tan(d*x)*tan(c)^2 - 20*a*tan(c)^3 + 12*a*d*x + 6*b*tan(d*x)^2
- 32*b*tan(d*x)*tan(c) + 6*b*tan(c)^2 - 16*b*log(4*(tan(c)^2 + 1)/(tan(d*x)
^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*ta
n(d*x)*tan(c) + 1)) - 12*a*tan(d*x) - 12*a*tan(c) + 11*b)/(d*tan(d*x)^4*tan
(c)^4 + 2*d*tan(d*x)^4*tan(c)^2 + 2*d*tan(d*x)^2*tan(c)^4 + d*tan(d*x)^4 +
4*d*tan(d*x)^2*tan(c)^2 + d*tan(c)^4 + 2*d*tan(d*x)^2 + 2*d*tan(c)^2 + d)
```

3.13 $\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=69

$$\frac{a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d + (a*Cos[c + d*x]^3)/(3*d)
 - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0679413, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3517, 2633, 2592, 302, 206}

$$\frac{a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d + (a*Cos[c + d*x]^3)/(3*d)
 - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d)
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx &= \int \left(a\sin^3(c+dx)+b\sin^3(c+dx)\tan(c+dx)\right)\,dx \\
&= a\int \sin^3(c+dx)\,dx + b\int \sin^3(c+dx)\tan(c+dx)\,dx \\
&= -\frac{a\,\mathrm{Subst}\left(\int\left(1-x^2\right)\,dx, x, \cos(c+dx)\right)}{d} + \frac{b\,\mathrm{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} \\
&= -\frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} + \frac{b\,\mathrm{Subst}\left(\int\left(-1-x^2+\frac{1}{1-x^2}\right)\,dx, x, \sin(c+d\right.}{d} \\
&= -\frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} - \frac{b\sin(c+dx)}{d} - \frac{b\sin^3(c+dx)}{3d} + \frac{b\,\mathrm{Subst}\left(\int\right.}{} \\
&= \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} - \frac{b\sin(c+dx)}{d} - \frac{b\sin}{}
\end{aligned}$$

Mathematica [A] time = 0.0267371, size = 71, normalized size = 1.03

$$-\frac{3a\cos(c+dx)}{4d}+\frac{a\cos(3(c+dx))}{12d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)
])/(12*d) - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.035, size = 79, normalized size = 1.1

$$-\frac{b\left(\sin\left(dx+c\right)\right)^{3}}{3\,d}-\frac{b\sin\left(dx+c\right)}{d}+\frac{b\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}-\frac{a\cos\left(dx+c\right)\left(\sin\left(dx+c\right)\right)^{2}}{3\,d}-\frac{2\,a\cos\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c)),x)
```

```
[Out] -1/3*b*sin(d*x+c)^3/d-b*sin(d*x+c)/d+1/d*b*ln(sec(d*x+c)+tan(d*x+c))-1/3/d*
a*cos(d*x+c)*sin(d*x+c)^2-2/3*a*cos(d*x+c)/d
```

Maxima [A] time = 1.36148, size = 95, normalized size = 1.38

$$\frac{2\left(\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)a-\left(2\,\sin\left(dx+c\right)^{3}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)+6\,\sin\left(dx+c\right)\right)b}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(cos(d*x + c)^3 - 3*cos(d*x + c))*a - (2*sin(d*x + c)^3 - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*b)/d
```

Fricas [A] time = 2.30315, size = 197, normalized size = 2.86

$$\frac{2\,a\cos\left(dx+c\right)^{3}-6\,a\cos\left(dx+c\right)+3\,b\log\left(\sin\left(dx+c\right)+1\right)-3\,b\log\left(-\sin\left(dx+c\right)+1\right)+2\left(b\cos\left(dx+c\right)^{2}-4\,b\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*cos(d*x + c)^3 - 6*a*cos(d*x + c) + 3*b*log(sin(d*x + c) + 1) - 3*
b*log(-sin(d*x + c) + 1) + 2*(b*cos(d*x + c)^2 - 4*b)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\tan(c + dx))\sin^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**3, x)
```

Giac [B] time = 2.78391, size = 7223, normalized size = 104.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(
1/2*d*x)^6*tan(1/2*c)^6 - 3*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(
1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + ta
n(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2
*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*t
an(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 4*a*tan(1/2*d*x)^6*tan(1/2*c)
^6 + 9*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1
/2*d*x)^6*tan(1/2*c)^4 - 9*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan
(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*
d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*ta
n(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 12*b*tan(1/2*d*x)^6*tan(1/2*c)
^5 + 9*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1
/2*d*x)^4*tan(1/2*c)^6 - 9*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1
```

/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan
(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*
d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*ta
n(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 12*b*tan(1/2*d*x)^5*tan(1/2*c)
^6 + 12*a*tan(1/2*d*x)^6*tan(1/2*c)^4 + 12*a*tan(1/2*d*x)^4*tan(1/2*c)^6 +
9*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*
x)^6*tan(1/2*c)^2 - 9*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)
^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*
d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2
*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 40*b*tan(1/2*d*x)^6*tan(1/2*c)^3 +
27*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)
^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*
d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d
*x)^4*tan(1/2*c)^4 - 27*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/
2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1
/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 60*b*tan(1/2*d*x)^5*tan(1/2*c)^4
- 60*b*tan(1/2*d*x)^4*tan(1/2*c)^5 + 9*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*
d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/
2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3
+ 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2
*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 9*b*log(2*(tan(1/2
*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*t
an(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6
- 40*b*tan(1/2*d*x)^3*tan(1/2*c)^6 - 12*a*tan(1/2*d*x)^6*tan(1/2*c)^2 - 96*
a*tan(1/2*d*x)^5*tan(1/2*c)^3 - 108*a*tan(1/2*d*x)^4*tan(1/2*c)^4 - 96*a*ta
n(1/2*d*x)^3*tan(1/2*c)^5 - 12*a*tan(1/2*d*x)^2*tan(1/2*c)^6 + 3*b*log(2*(t
an(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c)
 + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
+ tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^6 - 3*b*l
og(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan
(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^6
- 12*b*tan(1/2*d*x)^6*tan(1/2*c) + 27*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d
*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2

```
*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 +
 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*
d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 - 27*b*log(2*(tan(1/2
*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*t
an(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2
+ 60*b*tan(1/2*d*x)^5*tan(1/2*c)^2 + 120*b*tan(1/2*d*x)^4*tan(1/2*c)^3 + 27
*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4
*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x
)^2*tan(1/2*c)^4 - 27*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)
^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*
d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2
*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 120*b*tan(1/2*d*x)^3*tan(1/2*c)^4 +
 60*b*tan(1/2*d*x)^2*tan(1/2*c)^5 + 3*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d
*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2
*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 +
 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*
d*x) - 2*tan(1/2*c) + 1))*tan(1/2*c)^6 - 3*b*log(2*(tan(1/2*c)^2 + 1)/(tan(
1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*ta
n(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan
(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*c)^6 - 12*b*tan(1/2*d*x)*tan(1/2*c)^
6 - 4*a*tan(1/2*d*x)^6 + 108*a*tan(1/2*d*x)^4*tan(1/2*c)^2 + 128*a*tan(1/2*
d*x)^3*tan(1/2*c)^3 + 108*a*tan(1/2*d*x)^2*tan(1/2*c)^4 - 4*a*tan(1/2*c)^6
+ 9*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*
d*x)^4 - 9*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*t
an(1/2*d*x)^4 + 12*b*tan(1/2*d*x)^5 - 60*b*tan(1/2*d*x)^4*tan(1/2*c) + 27*b
*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*t
an(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^
2*tan(1/2*c)^2 - 27*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*
x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c
```

```
) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 120*b*tan(1/2*d*x)^3*tan(1/2*c)^2 - 1
20*b*tan(1/2*d*x)^2*tan(1/2*c)^3 + 9*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*
x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*
c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 +
2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 2*tan(1/2*c) + 1))*tan(1/2*c)^4 - 9*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1
/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan
(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)
^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(
1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*c)^4 - 60*b*tan(1/2*d*x)*tan(1/2*c)^4
 + 12*b*tan(1/2*c)^5 - 12*a*tan(1/2*d*x)^4 - 96*a*tan(1/2*d*x)^3*tan(1/2*c)
 - 108*a*tan(1/2*d*x)^2*tan(1/2*c)^2 - 96*a*tan(1/2*d*x)*tan(1/2*c)^3 - 12*
a*tan(1/2*c)^4 + 9*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x
)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c)
 + 1))*tan(1/2*d*x)^2 - 9*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/
2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(
1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d
*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan
(1/2*c) + 1))*tan(1/2*d*x)^2 + 40*b*tan(1/2*d*x)^3 + 60*b*tan(1/2*d*x)^2*ta
n(1/2*c) + 9*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 +
2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))
*tan(1/2*c)^2 - 9*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)
^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c)
+ 1))*tan(1/2*c)^2 + 60*b*tan(1/2*d*x)*tan(1/2*c)^2 + 40*b*tan(1/2*c)^3 + 1
2*a*tan(1/2*d*x)^2 + 12*a*tan(1/2*c)^2 + 3*b*log(2*(tan(1/2*c)^2 + 1)/(tan(
1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*ta
n(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan
(1/2*d*x) - 2*tan(1/2*c) + 1)) - 3*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)
^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)
^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*
tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) + 2*tan(1/2*c) + 1)) + 12*b*tan(1/2*d*x) + 12*b*tan(1/2*c) + 4*a)/(d*tan(
1/2*d*x)^6*tan(1/2*c)^6 + 3*d*tan(1/2*d*x)^6*tan(1/2*c)^4 + 3*d*tan(1/2*d*x
)^4*tan(1/2*c)^6 + 3*d*tan(1/2*d*x)^6*tan(1/2*c)^2 + 9*d*tan(1/2*d*x)^4*tan
(1/2*c)^4 + 3*d*tan(1/2*d*x)^2*tan(1/2*c)^6 + d*tan(1/2*d*x)^6 + 9*d*tan(1/
2*d*x)^4*tan(1/2*c)^2 + 9*d*tan(1/2*d*x)^2*tan(1/2*c)^4 + d*tan(1/2*c)^6 +
3*d*tan(1/2*d*x)^4 + 9*d*tan(1/2*d*x)^2*tan(1/2*c)^2 + 3*d*tan(1/2*c)^4 + 3
*d*tan(1/2*d*x)^2 + 3*d*tan(1/2*c)^2 + d)
```

3.14 $\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=49

$$-\frac{\sin(c+dx)\cos(c+dx)(a+b\tan(c+dx))}{2d}+\frac{ax}{2}-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] (a*x)/2 - (b*Log[Cos[c + d*x]])/d - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c
 + d*x]))/(2*d)
```

Rubi [A] time = 0.0844788, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {819, 635, 203, 260}

$$-\frac{\sin(c+dx)\cos(c+dx)(a+b\tan(c+dx))}{2d}+\frac{ax}{2}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*x)/2 - (b*Log[Cos[c + d*x]])/d - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c
 + d*x]))/(2*d)
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))}{2d} + \frac{\text{Subst}\left(\int \frac{a+2bx}{1+x^2}\,dx, x, \tan(c+dx\right.}{2d}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))}{2d} + \frac{a\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+d\right.}{2d}$$

$$= \frac{ax}{2} - \frac{b\log(\cos(c+dx))}{d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))}{2d}$$

Mathematica [A] time = 0.0480753, size = 56, normalized size = 1.14

$$\frac{a(c+dx)}{2d} - \frac{a\sin(2(c+dx))}{4d} - \frac{b\left(\log(\cos(c+dx)) - \frac{1}{2}\cos^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(2*d) - (b*(-Cos[c + d*x]^2/2 + Log[Cos[c + d*x]]))/d - (a*Si
n[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.035, size = 58, normalized size = 1.2

$$-\frac{b(\sin(dx+c))^2}{2d} - \frac{b\ln(\cos(dx+c))}{d} - \frac{a\sin(dx+c)\cos(dx+c)}{2d} + \frac{ax}{2} + \frac{ac}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c)),x)
```

```
[Out] -1/2/d*b*sin(d*x+c)^2-b*ln(cos(d*x+c))/d-1/2/d*a*sin(d*x+c)*cos(d*x+c)+1/2*
a*x+1/2/d*a*c
```

Maxima [A] time = 2.51148, size = 70, normalized size = 1.43

$$\frac{(dx+c)a+b\log\left(\tan\left(dx+c\right)^2+1\right)-\frac{a\tan(dx+c)-b}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((d*x + c)*a + b*log(tan(d*x + c)^2 + 1) - (a*tan(d*x + c) - b)/(tan(d*
x + c)^2 + 1))/d
```

Fricas [A] time = 2.25526, size = 120, normalized size = 2.45

$$\frac{adx+b\cos\left(dx+c\right)^2-a\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,b\log\left(-\cos\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x + b*cos(d*x + c)^2 - a*cos(d*x + c)*sin(d*x + c) - 2*b*log(-cos(
d*x + c)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan\left(c+dx\right)\right)\sin^2\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**2, x)
```

Giac [B] time = 1.43679, size = 558, normalized size = 11.39

$$\frac{2\,adx\tan\left(dx\right)^{2}\tan\left(c\right)^{2}-2\,b\log\left(\frac{4\left(\tan(c)^{2}+1\right)}{\tan(dx)^{4}\tan(c)^{2}-2\,\tan(dx)^{3}\tan(c)+\tan(dx)^{2}\tan(c)^{2}+\tan(dx)^{2}-2\,\tan(dx)\tan(c)+1}\right)\tan\left(dx\right)^{2}\tan\left(c\right)^{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(2*a*d*x*tan(d*x)^2*tan(c)^2 - 2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan
(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)
*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 2*a*d*x*tan(d*x)^2 + 2*a*d*x*tan(c)^2 +
 b*tan(d*x)^2*tan(c)^2 - 2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*
tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
1))*tan(d*x)^2 + 2*a*tan(d*x)^2*tan(c) - 2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)
^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*ta
n(d*x)*tan(c) + 1))*tan(c)^2 + 2*a*tan(d*x)*tan(c)^2 + 2*a*d*x - b*tan(d*x)
^2 - 4*b*tan(d*x)*tan(c) - b*tan(c)^2 - 2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^
4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan
(d*x)*tan(c) + 1)) - 2*a*tan(d*x) - 2*a*tan(c) + b)/(d*tan(d*x)^2*tan(c)^2
+ d*tan(d*x)^2 + d*tan(c)^2 + d)
```

3.15 $\int \sin(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=37

$$-\frac{a\cos(c+dx)}{d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d - (b*Sin[c + d*x])/d
```

Rubi [A] time = 0.0357625, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3517, 2638, 2592, 321, 206}

$$-\frac{a\cos(c+dx)}{d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d - (b*Sin[c + d*x])/d
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sin(c+dx)(a+b\tan(c+dx))\,dx &= \int (a\sin(c+dx)+b\sin(c+dx)\tan(c+dx))\,dx \\
&= a\int \sin(c+dx)\,dx + b\int \sin(c+dx)\tan(c+dx)\,dx \\
&= -\frac{a\cos(c+dx)}{d} + \frac{b\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} \\
&= -\frac{a\cos(c+dx)}{d} - \frac{b\sin(c+dx)}{d} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} \\
&= \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cos(c+dx)}{d} - \frac{b\sin(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.019747, size = 48, normalized size = 1.3

$$\frac{a\sin(c)\sin(dx)}{d} - \frac{a\cos(c)\cos(dx)}{d} - \frac{b\sin(c+dx)}{d} + \frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c]*Cos[d*x])/d + (a*Sin[c]*Sin[d*x])/d
 - (b*Sin[c + d*x])/d
```

Maple [A] time = 0.027, size = 45, normalized size = 1.2

$$-\frac{a\cos(dx+c)}{d}-\frac{b\sin(dx+c)}{d}+\frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c)),x)
```

```
[Out] -a*cos(d*x+c)/d-b*sin(d*x+c)/d+1/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.52106, size = 62, normalized size = 1.68

$$\frac{b(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\sin(dx+c))-2a\cos(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) - 2
*a*cos(d*x + c))/d
```

Fricas [A] time = 2.29999, size = 134, normalized size = 3.62

$$-\frac{2a\cos(dx+c)-b\log(\sin(dx+c)+1)+b\log(-\sin(dx+c)+1)+2b\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*cos(d*x + c) - b*log(sin(d*x + c) + 1) + b*log(-sin(d*x + c) + 1)
 + 2*b*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(a+b\tan(c+dx))\sin(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x), x)
```

Giac [B] time = 1.65799, size = 1669, normalized size = 45.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/
2*d*x)^2*tan(1/2*c)^2 - b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/
2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1
/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4
*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x
)^2 - b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d
*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/
2*d*x)^2 - 4*b*tan(1/2*d*x)^2*tan(1/2*c) + b*log(2*(tan(1/2*c)^2 + 1)/(tan(
1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*ta
n(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan
(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*c)^2 - b*log(2*(tan(1/2*c)^2 + 1)/(t
an(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3
*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*c)^2 - 4*b*tan(1/2*d*x)*tan(1/2*c
)^2 - 2*a*tan(1/2*d*x)^2 - 8*a*tan(1/2*d*x)*tan(1/2*c) - 2*a*tan(1/2*c)^2 +
 b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4
```

```
*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)) - b*log(2*(
tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c
) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)) + 4*b*tan(1/2*d*x) +
4*b*tan(1/2*c) + 2*a)/(d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(1/2*d*x)^2 + d
*tan(1/2*c)^2 + d)
```

3.16 $\int \csc(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=26

$$\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (b*ArcTanh[Sin[c + d*x]])/d
```

Rubi [A] time = 0.029023, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3517, 3770}

$$\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (b*ArcTanh[Sin[c + d*x]])/d
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)(a+b\tan(c+dx))\,dx &= \int (a\csc(c+dx)+b\sec(c+dx))\,dx\\ &= a\int \csc(c+dx)\,dx+b\int \sec(c+dx)\,dx\\ &= -\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.0237081, size = 52, normalized size = 2.

$$\frac{a\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Log[Cos[c/2 + (d*x)/2]])/d + (a*Log[Sin[c/
2 + (d*x)/2]])/d
```

Maple [A] time = 0.036, size = 42, normalized size = 1.6

$$\frac{a\ln(\csc(dx+c)-\cot(dx+c))}{d}+\frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d*a*ln(csc(d*x+c)-cot(d*x+c))+1/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.53495, size = 62, normalized size = 2.38

$$\frac{b(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))-2\,a\log(\cot(dx+c)+\csc(dx+c))}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 2*a*log(cot(d*x +
c) + csc(d*x + c)))/d
```

Fricas [B] time = 2.35074, size = 170, normalized size = 6.54

$$-\frac{a\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)-a\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)-b\log(\sin(dx+c)+1)+b\log(-\sin(dx+c)+1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*log(1/2*cos(d*x + c) + 1/2) - a*log(-1/2*cos(d*x + c) + 1/2) - b*lo
g(sin(d*x + c) + 1) + b*log(-sin(d*x + c) + 1))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\tan(c + dx))\csc(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x), x)
```

Giac [A] time = 1.41334, size = 66, normalized size = 2.54

$$\frac{b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) + a*log(abs(tan(1/2*d*x + 1/2*c))))/d
```

3.17 $\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=25

$$\frac{b\log(\tan(c+dx))}{d}-\frac{a\cot(c+dx)}{d}$$

```
[Out] -((a*Cot[c + d*x])/d) + (b*Log[Tan[c + d*x]])/d
```

Rubi [A] time = 0.0783231, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{b\log(\tan(c+dx))}{d}-\frac{a\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) + (b*Log[Tan[c + d*x]])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{a+bx}{x^2}\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a}{x^2}+\frac{b}{x}\right)dx,x,\tan(c+dx)\right)}{d}\\ &= -\frac{a\cot(c+dx)}{d}+\frac{b\log(\tan(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.0594411, size = 36, normalized size = 1.44

$$-\frac{a\cot(c+dx)}{d}-\frac{b(\log(\cos(c+dx))-\log(\sin(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (b*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]]))/d
```

Maple [A] time = 0.077, size = 26, normalized size = 1.

$$-\frac{\cot(dx+c)\,a}{d}+\frac{b\ln(\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c)),x)
```

```
[Out] -a*cot(d*x+c)/d+b*ln(tan(d*x+c))/d
```

Maxima [A] time = 1.38836, size = 34, normalized size = 1.36

$$\frac{b\log(\tan(dx+c))-\frac{a}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b*log(tan(d*x + c)) - a/tan(d*x + c))/d
```

Fricas [B] time = 2.49515, size = 171, normalized size = 6.84

$$-\frac{b\log\left(\cos(dx+c)^2\right)\sin(dx+c)-b\log\left(-\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\right)\sin(dx+c)+2\,a\cos(dx+c)}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*log(cos(d*x + c)^2)*sin(d*x + c) - b*log(-1/4*cos(d*x + c)^2 + 1/4)
*sin(d*x + c) + 2*a*cos(d*x + c))/(d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan (c + dx)) \csc^2 (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x)**2, x)
```

Giac [A] time = 1.36389, size = 47, normalized size = 1.88

$$\frac{b \log (|\tan (dx + c)|) - \frac{b \tan(dx+c)+a}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (b*log(abs(tan(d*x + c))) - (b*tan(d*x + c) + a)/tan(d*x + c))/d
```

3.18 $\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=60

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] -(a*ArcTanh[Cos[c + d*x]])/(2*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Csc[c +
 d*x])/d - (a*Cot[c + d*x]*Csc[c + d*x])/(2*d)
```

Rubi [A] time = 0.0679689, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3517, 3768, 3770, 2621, 321, 207}

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```


```
[Out] -(a*ArcTanh[Cos[c + d*x]])/(2*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Csc[c +
 d*x])/d - (a*Cot[c + d*x]*Csc[c + d*x])/(2*d)
```


Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx &= \int \left(a\csc^3(c+dx)+b\csc^2(c+dx)\sec(c+dx)\right)\,dx\\ &= a\int \csc^3(c+dx)\,dx+b\int \csc^2(c+dx)\sec(c+dx)\,dx\\ &= -\frac{a\cot(c+dx)\csc(c+dx)}{2d}+\frac{1}{2}a\int \csc(c+dx)\,dx-\frac{b\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\csc(\right.}{d}\\ &= -\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b\csc(c+dx)}{d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-\frac{b\,\mathrm{Subst}\left(\right.}{}\\ &= -\frac{a\tanh^{-1}(\cos(c+dx))}{2d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{b\csc(c+dx)}{d}-\frac{a\cot(c+dx)}{2}\end{aligned}$$

Mathematica [C] time = 0.0265519, size = 107, normalized size = 1.78

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{b\csc(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] -(a*Csc[(c + d*x)/2]^2)/(8*d) - (b*Csc[c + d*x]*Hypergeometric2F1[-1/2, 1,
1/2, Sin[c + d*x]^2])/d - (a*Log[Cos[(c + d*x)/2]])/(2*d) + (a*Log[Sin[(c +
 d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A] time = 0.082, size = 75, normalized size = 1.3

$$-\frac{b}{d\sin(dx+c)}+\frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{\cot(dx+c)\,a\csc(dx+c)}{2d}+\frac{a\ln(\csc(dx+c)-\cot(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c)),x)
```

```
[Out] -1/d*b/sin(d*x+c)+1/d*b*ln(sec(d*x+c)+tan(d*x+c))-1/2*a*cot(d*x+c)*csc(d*x+
c)/d+1/2/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.14193, size = 112, normalized size = 1.87

$$\frac{a\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-2\,b\left(\frac{2}{\sin(dx+c)}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log(c
os(d*x + c) - 1)) - 2*b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d
*x + c) - 1)))/d
```

Fricas [B] time = 2.89013, size = 367, normalized size = 6.12

$$\frac{2\,a\cos(dx+c)-\left(a\cos(dx+c)^2-a\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+\left(a\cos(dx+c)^2-a\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2\left(b\right.}{4\left(d\cos(dx+c)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*cos(d*x + c) - (a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2)
+ (a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2) + 2*(b*cos(d*x + c)^2
 - b)*log(sin(d*x + c) + 1) - 2*(b*cos(d*x + c)^2 - b)*log(-sin(d*x + c) +
1) + 4*b*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\tan(c + dx))\csc^3(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x)**3, x)
```

Giac [B] time = 1.51231, size = 159, normalized size = 2.65

$$\frac{a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 8\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 8\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + 4\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - 4\,b}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 + 8*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*
b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a*log(abs(tan(1/2*d*x + 1/2*c))) -
 4*b*tan(1/2*d*x + 1/2*c) - (6*a*tan(1/2*d*x + 1/2*c)^2 + 4*b*tan(1/2*d*x +
 1/2*c) + a)/tan(1/2*d*x + 1/2*c)^2)/d
```

3.19 $\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=57

$$-\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{2d}+\frac{b\log(\tan(c+dx))}{d}$$

```
[Out] -((a*Cot[c + d*x])/d) - (b*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(3*d)
 + (b*Log[Tan[c + d*x]])/d
```

Rubi [A] time = 0.0879247, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {766}

$$-\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{2d}+\frac{b\log(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (b*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(3*d)
 + (b*Log[Tan[c + d*x]])/d
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)(1+x^2)}{x^4}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a}{x^4}+\frac{b}{x^3}+\frac{a}{x^2}+\frac{b}{x}\right)dx, x, \tan(c+dx)\right)}{d}\\ &= -\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+\frac{b\log(\tan(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.244585, size = 72, normalized size = 1.26

$$-\frac{2a\cot(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}-\frac{b\left(\csc^2(c+dx)-2\log(\sin(c+dx))+2\log(\cos(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] (-2*a*Cot[c + d*x])/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) - (b*(Csc
[c + d*x]^2 + 2*Log[Cos[c + d*x]] - 2*Log[Sin[c + d*x]]))/(2*d)
```

Maple [A] time = 0.081, size = 60, normalized size = 1.1

$$-\frac{b}{2\,d\,(\sin(dx+c))^2}+\frac{b\ln(\tan(dx+c))}{d}-\frac{2\,\cot(dx+c)\,a}{3\,d}-\frac{\cot(dx+c)\,a\,(\csc(dx+c))^2}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c)),x)
```

```
[Out] -1/2/d*b/sin(d*x+c)^2+b*ln(tan(d*x+c))/d-2/3*a*cot(d*x+c)/d-1/3/d*a*cot(d*x
+c)*csc(d*x+c)^2
```

Maxima [A] time = 1.12848, size = 68, normalized size = 1.19

$$\frac{6\,b\log(\tan(dx+c))-\frac{6\,a\tan(dx+c)^2+3\,b\tan(dx+c)+2\,a}{\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*b*log(tan(d*x + c)) - (6*a*tan(d*x + c)^2 + 3*b*tan(d*x + c) + 2*a)/
tan(d*x + c)^3)/d
```

Fricas [B] time = 2.79451, size = 316, normalized size = 5.54

$$-\frac{4\,a\cos\left(dx+c\right)^{3}+3\,\left(b\cos\left(dx+c\right)^{2}-b\right)\log\left(\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)-3\,\left(b\cos\left(dx+c\right)^{2}-b\right)\log\left(-\frac{1}{4}\,\cos\left(dx+c\right)^{2}\right.}{6\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(4*a*cos(d*x + c)^3 + 3*(b*cos(d*x + c)^2 - b)*log(cos(d*x + c)^2)*sin
(d*x + c) - 3*(b*cos(d*x + c)^2 - b)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x
 + c) - 6*a*cos(d*x + c) - 3*b*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*
x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan\left(c+dx\right)\right)\csc^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x)**4, x)
```

Giac [A] time = 1.36171, size = 84, normalized size = 1.47

$$\frac{6\,b\log\left(\left|\tan\left(dx+c\right)\right|\right)-\frac{11\,b\tan\left(dx+c\right)^{3}+6\,a\tan\left(dx+c\right)^{2}+3\,b\tan\left(dx+c\right)+2\,a}{\tan\left(dx+c\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*b*log(abs(tan(d*x + c))) - (11*b*tan(d*x + c)^3 + 6*a*tan(d*x + c)^2
 + 3*b*tan(d*x + c) + 2*a)/tan(d*x + c)^3)/d
```

3.20 $\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=98

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}+\frac{b}{}$$

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(8*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Csc[c
 + d*x])/d - (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (b*Csc[c + d*x]^3)/(3*
d) - (a*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.0922901, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3517, 3768, 3770, 2621, 302, 207}

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}+\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```


```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(8*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Csc[c
 + d*x])/d - (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (b*Csc[c + d*x]^3)/(3*
d) - (a*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```


Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx &= \int \left(a\csc^5(c+dx)+b\csc^4(c+dx)\sec(c+dx)\right)dx \\
&= a\int \csc^5(c+dx)\,dx + b\int \csc^4(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{1}{4}(3a)\int \csc^3(c+dx)\,dx - \frac{b\,\mathrm{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx,\right.}{d} \\
&= -\frac{3a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{1}{8}(3a)\int \csc(c+dx \\
&= -\frac{3a\tanh^{-1}(\cos(c+dx))}{8d} - \frac{b\csc(c+dx)}{d} - \frac{3a\cot(c+dx)\csc(c+dx)}{8d} - \frac{b\csc^3}{} \\
&= -\frac{3a\tanh^{-1}(\cos(c+dx))}{8d} + \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{b\csc(c+dx)}{d} - \frac{3a\cot(c+}{}
\end{aligned}$$

Mathematica [C] time = 0.0286861, size = 151, normalized size = 1.54

$$-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} - \frac{3a\,\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] (-3*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (b*Csc[c
 + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[c + d*x]^2])/(3*d) - (3*a*Lo
g[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) + (3*a*Sec[(
c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

Maple [A] time = 0.084, size = 109, normalized size = 1.1

$$-\frac{b}{3\,d\,(\sin(dx+c))^3}-\frac{b}{d\sin(dx+c)}+\frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{\cot(dx+c)\,a\,(\csc(dx+c))^3}{4\,d}-\frac{3\,\cot(dx+c)\,a}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+b*tan(d*x+c)),x)
```

```
[Out] -1/3/d*b/sin(d*x+c)^3-1/d*b/sin(d*x+c)+1/d*b*ln(sec(d*x+c)+tan(d*x+c))-1/4*
a*cot(d*x+c)*csc(d*x+c)^3/d-3/8*a*cot(d*x+c)*csc(d*x+c)/d+3/8/d*a*ln(csc(d*
x+c)-cot(d*x+c))
```

Maxima [A] time = 1.26038, size = 166, normalized size = 1.69

$$\frac{3\,a\left(\frac{2\left(3\,\cos(dx+c)^3-5\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-3\,\log(\cos(dx+c)+1)+3\,\log(\cos(dx+c)-1)\right)-8\,b\left(\frac{2\left(3\,\sin(dx+c)^2+1\right)}{\sin(dx+c)^3}-3\,\log(\sin(dx+\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(3*a*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*
x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 8*b*(2
*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(si
n(d*x + c) - 1)))/d
```

Fricas [B] time = 2.72285, size = 585, normalized size = 5.97

$$18\,a\cos\left(dx+c\right)^3-30\,a\cos\left(dx+c\right)-9\,\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+9\,\left(a\cos\left(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(18*a*cos(d*x + c)^3 - 30*a*cos(d*x + c) - 9*(a*cos(d*x + c)^4 - 2*a*c
os(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2) + 9*(a*cos(d*x + c)^4 - 2*a*
cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2) + 24*(b*cos(d*x + c)^4 - 2
*b*cos(d*x + c)^2 + b)*log(sin(d*x + c) + 1) - 24*(b*cos(d*x + c)^4 - 2*b*c
os(d*x + c)^2 + b)*log(-sin(d*x + c) + 1) + 16*(3*b*cos(d*x + c)^2 - 4*b)*s
in(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+b*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42746, size = 239, normalized size = 2.44

$$3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-8\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+192\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-192\,b\log\left(\left|\mathrm{t}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 - 8*b*tan(1/2*d*x + 1/2*c)^3 + 24*a*tan(1
/2*d*x + 1/2*c)^2 + 192*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 192*b*log(ab
```

```
s(tan(1/2*d*x + 1/2*c) - 1)) + 72*a*log(abs(tan(1/2*d*x + 1/2*c))) - 120*b*
tan(1/2*d*x + 1/2*c) - (150*a*tan(1/2*d*x + 1/2*c)^4 + 120*b*tan(1/2*d*x +
1/2*c)^3 + 24*a*tan(1/2*d*x + 1/2*c)^2 + 8*b*tan(1/2*d*x + 1/2*c) + 3*a)/ta
n(1/2*d*x + 1/2*c)^4)/d
```

3.21 $\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=87

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\cot^4(c+dx)}{4d}-\frac{b\cot^2(c+dx)}{d}+\frac{b\log(\tan(c+dx))}{d}$$

```
[Out] -((a*Cot[c + d*x])/d) - (b*Cot[c + d*x]^2)/d - (2*a*Cot[c + d*x]^3)/(3*d) -
 (b*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5*d) + (b*Log[Tan[c + d*x]]
)/d
```

Rubi [A] time = 0.107578, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {766}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\cot^4(c+dx)}{4d}-\frac{b\cot^2(c+dx)}{d}+\frac{b\log(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (b*Cot[c + d*x]^2)/d - (2*a*Cot[c + d*x]^3)/(3*d) -
 (b*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5*d) + (b*Log[Tan[c + d*x]]
)/d
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)\left(1+x^2\right)^2}{x^6}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a}{x^6}+\frac{b}{x^5}+\frac{2a}{x^4}+\frac{2b}{x^3}+\frac{a}{x^2}+\frac{b}{x}\right)dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{b\cot^4(c+dx)}{4d}-\frac{a\cot^5(c+}{5d}$$

Mathematica [A] time = 0.545438, size = 104, normalized size = 1.2

$$-\frac{8a\cot(c+dx)}{15d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}-\frac{4a\cot(c+dx)\csc^2(c+dx)}{15d}-\frac{b\left(\csc^4(c+dx)+2\csc^2(c+dx)-4\log(\sin\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

```
[Out] (-8*a*Cot[c + d*x])/(15*d) - (4*a*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) - (a*
Cot[c + d*x]*Csc[c + d*x]^4)/(5*d) - (b*(2*Csc[c + d*x]^2 + Csc[c + d*x]^4
+ 4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]]))/(4*d)
```

Maple [A] time = 0.085, size = 94, normalized size = 1.1

$$-\frac{b}{4d(\sin(dx+c))^4}-\frac{b}{2d(\sin(dx+c))^2}+\frac{b\ln(\tan(dx+c))}{d}-\frac{8\cot(dx+c)a}{15d}-\frac{\cot(dx+c)a(\csc(dx+c))^4}{5d}-\frac{4\cot(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*tan(d*x+c)),x)
```

```
[Out] -1/4/d*b/sin(d*x+c)^4-1/2/d*b/sin(d*x+c)^2+b*ln(tan(d*x+c))/d-8/15*a*cot(d*
x+c)/d-1/5/d*a*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a*cot(d*x+c)*csc(d*x+c)^2
```

Maxima [A] time = 1.13162, size = 97, normalized size = 1.11

$$\frac{60b\log(\tan(dx+c))-\frac{60a\tan(dx+c)^4+60b\tan(dx+c)^3+40a\tan(dx+c)^2+15b\tan(dx+c)+12a}{\tan(dx+c)^5}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*b*log(tan(d*x + c)) - (60*a*tan(d*x + c)^4 + 60*b*tan(d*x + c)^3 +
 40*a*tan(d*x + c)^2 + 15*b*tan(d*x + c) + 12*a)/tan(d*x + c)^5)/d
```

Fricas [B] time = 2.10032, size = 473, normalized size = 5.44

$$-\frac{32\,a\cos\left(dx+c\right)^{5}-80\,a\cos\left(dx+c\right)^{3}+30\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+b\right)\log\left(\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)-30}{60\left(d\cos\left(dx+c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(32*a*cos(d*x + c)^5 - 80*a*cos(d*x + c)^3 + 30*(b*cos(d*x + c)^4 - 2
*b*cos(d*x + c)^2 + b)*log(cos(d*x + c)^2)*sin(d*x + c) - 30*(b*cos(d*x + c
)^4 - 2*b*cos(d*x + c)^2 + b)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) +
 60*a*cos(d*x + c) - 15*(2*b*cos(d*x + c)^2 - 3*b)*sin(d*x + c))/((d*cos(d*
x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38364, size = 113, normalized size = 1.3

$$\frac{60\,b\log\left(\left|\tan\left(dx+c\right)\right|\right)-\frac{137\,b\tan(dx+c)^{5}+60\,a\tan(dx+c)^{4}+60\,b\tan(dx+c)^{3}+40\,a\tan(dx+c)^{2}+15\,b\tan(dx+c)+12\,a}{\tan(dx+c)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(60*b*log(abs(tan(d*x + c))) - (137*b*tan(d*x + c)^5 + 60*a*tan(d*x +
c)^4 + 60*b*tan(d*x + c)^3 + 40*a*tan(d*x + c)^2 + 15*b*tan(d*x + c) + 12*a
)/tan(d*x + c)^5)/d
```

3.22 $\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=113

$$\frac{3}{8}x\left(a^2-5b^2\right)+\frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{\sin(c+dx)\cos^3(c+dx}{4d}$$

```
[Out] (3*(a^2 - 5*b^2)*x)/8 - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Tan[c + d*x])/d
+ (Cos[c + d*x]^2*(7*b - 5*a*Tan[c + d*x])*(a + b*Tan[c + d*x]))/(8*d) + (C
os[c + d*x]^3*Sin[c + d*x]*(a + b*Tan[c + d*x])^2)/(4*d)
```

Rubi [A] time = 0.185813, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1645, 1810, 635, 203, 260}

$$\frac{3}{8}x\left(a^2-5b^2\right)+\frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{\sin(c+dx)\cos^3(c+dx}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (3*(a^2 - 5*b^2)*x)/8 - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Tan[c + d*x])/d
+ (Cos[c + d*x]^2*(7*b - 5*a*Tan[c + d*x])*(a + b*Tan[c + d*x]))/(8*d) + (C
os[c + d*x]^3*Sin[c + d*x]*(a + b*Tan[c + d*x])^2)/(4*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
```

```
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^4(a+x)^2}{\left(b^2+x^2\right)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{4d} - \frac{\text{Subst}\left(\int \frac{(a+x)\left(ab^4+3b^4x-4ab^2x^2-4\right.}{\left(b^2+x^2\right)^2}\right.}{4bd}$$

$$= \frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d} + \frac{\cos^3(c+dx)\sin(c+d}{4}$$

$$= \frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d} + \frac{\cos^3(c+dx)\sin(c+d}{4}$$

$$= \frac{b^2\tan(c+dx)}{d} + \frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d} + \cos^3$$

$$= \frac{b^2\tan(c+dx)}{d} + \frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d} + \cos^3$$

$$= \frac{3}{8}\left(a^2-5b^2\right)x - \frac{2ab\log(\cos(c+dx))}{d} + \frac{b^2\tan(c+dx)}{d} + \frac{\cos^2(c+dx)(7b-5a}{}$$

Mathematica [B] time = 3.19675, size = 240, normalized size = 2.12

$$b\left(\frac{2\left(3b^2-2a^2\right)\sin(2(c+dx))}{b} + \frac{4\left(3b^2-2a^2\right)\tan^{-1}(\tan(c+dx))}{b} + 4\left(\frac{a^2-3b^2}{\sqrt{-b^2}}+2a\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right) + 4\left(\frac{3b^2-a^2}{\sqrt{-b^2}}+2a\right)\log\left(\sqrt{-}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b*((4*(-2*a^2 + 3*b^2)*ArcTan[Tan[c + d*x]])/b + 16*a*Cos[c + d*x]^2 - 4*a
*Cos[c + d*x]^4 + 4*(2*a + (a^2 - 3*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan
[c + d*x]] + 4*(2*a + (-a^2 + 3*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c +
 d*x]] + (2*(a^2 - b^2)*Cos[c + d*x]^3*Sin[c + d*x])/b + (2*(-2*a^2 + 3*b^2
)*Sin[2*(c + d*x)])/b + (3*(a^2 - b^2)*(2*ArcTan[Tan[c + d*x]] + Sin[2*(c +
 d*x)]))/(2*b) + 8*b*Tan[c + d*x]))/(8*d)
```

Maple [A] time = 0.04, size = 204, normalized size = 1.8

$$\frac{b^2 (\sin (dx+c))^7}{d\cos (dx+c)}+\frac{b^2\cos (dx+c) (\sin (dx+c))^5}{d}+\frac{5\,b^2\cos (dx+c) (\sin (dx+c))^3}{4\,d}+\frac{15\,b^2\cos (dx+c)\sin (dx+c)}{8\,d}-\frac{15}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*b^2*sin(d*x+c)^7/cos(d*x+c)+1/d*b^2*cos(d*x+c)*sin(d*x+c)^5+5/4/d*b^2*c
os(d*x+c)*sin(d*x+c)^3+15/8/d*b^2*cos(d*x+c)*sin(d*x+c)-15/8*b^2*x-15/8/d*b
^2*c-1/2/d*a*b*sin(d*x+c)^4-1/d*a*b*sin(d*x+c)^2-2*a*b*ln(cos(d*x+c))/d-1/4
/d*a^2*cos(d*x+c)*sin(d*x+c)^3-3/8/d*a^2*cos(d*x+c)*sin(d*x+c)+3/8*a^2*x+3/
8/d*a^2*c
```

Maxima [A] time = 1.47685, size = 173, normalized size = 1.53

$$\frac{8\,ab\log\left(\tan (dx+c)^2+1\right)+8\,b^2\tan (dx+c)+3\left(a^2-5\,b^2\right)(dx+c)+\frac{16\,ab\tan(dx+c)^2-\left(5\,a^2-9\,b^2\right)\tan(dx+c)^3+12\,ab-\left(3\,a^2-7\,b^2\right)\tan}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(8*a*b*log(tan(d*x + c)^2 + 1) + 8*b^2*tan(d*x + c) + 3*(a^2 - 5*b^2)*(
d*x + c) + (16*a*b*tan(d*x + c)^2 - (5*a^2 - 9*b^2)*tan(d*x + c)^3 + 12*a*b
 - (3*a^2 - 7*b^2)*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 2.06341, size = 333, normalized size = 2.95

$$-\frac{8\,ab\cos (dx+c)^5-32\,ab\cos (dx+c)^3+32\,ab\cos (dx+c)\log (-\cos (dx+c))-\left(6\left(a^2-5\,b^2\right)dx-13\,ab\right)\cos (dx+c)-}{16\,d\cos (dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*a*b*cos(d*x + c)^5 - 32*a*b*cos(d*x + c)^3 + 32*a*b*cos(d*x + c)*l
og(-cos(d*x + c)) - (6*(a^2 - 5*b^2)*d*x - 13*a*b)*cos(d*x + c) - 2*(2*(a^2
```

```
 - b^2)*cos(d*x + c)^4 - (5*a^2 - 9*b^2)*cos(d*x + c)^2 + 8*b^2)*sin(d*x +
c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 6.66882, size = 7808, normalized size = 69.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*
tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^5 + 24*a^2*d*x
*tan(d*x)^5*tan(c)^5 - 120*b^2*d*x*tan(d*x)^5*tan(c)^5 + 3*pi*b^2*sgn(-2*ta
n(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*t
an(c)^5 + 6*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c)
+ 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^3 - 3*pi*b
^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan
(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 6*pi*b^2*sgn(2*tan(d*x
)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*
x) - 2*tan(c))*tan(d*x)^3*tan(c)^5 + 6*b^2*arctan((tan(d*x) + tan(c))/(tan(
d*x)*tan(c) - 1))*tan(d*x)^5*tan(c)^5 - 6*b^2*arctan(-(tan(d*x) - tan(c))/(
tan(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c)^5 - 64*a*b*log(4*(tan(c)^2 + 1)/(ta
n(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2
- 2*tan(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c)^5 + 48*a^2*d*x*tan(d*x)^5*tan(c
)^3 - 240*b^2*d*x*tan(d*x)^5*tan(c)^3 + 6*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) +
 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^3 - 24*a^2*
d*x*tan(d*x)^4*tan(c)^4 + 120*b^2*d*x*tan(d*x)^4*tan(c)^4 - 3*pi*b^2*sgn(-2
*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^
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4*tan(c)^4 + 48*a^2*d*x*tan(d*x)^3*tan(c)^5 - 240*b^2*d*x*tan(d*x)^3*tan(c)
^5 + 6*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) -
 2*tan(c))*tan(d*x)^3*tan(c)^5 + 44*a*b*tan(d*x)^5*tan(c)^5 + 3*pi*b^2*sgn(
2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 +
 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c) - 6*pi*b^2*sgn(2*tan(d*x)^2*tan(c
)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*ta
n(c))*tan(d*x)^4*tan(c)^2 + 12*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2
*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^
3*tan(c)^3 + 12*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d
*x)^5*tan(c)^3 - 12*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*
tan(d*x)^5*tan(c)^3 - 128*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
 1))*tan(d*x)^5*tan(c)^3 - 6*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*t
an(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*
tan(c)^4 - 6*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)
^4*tan(c)^4 + 6*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(
d*x)^4*tan(c)^4 + 64*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(
d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*
tan(d*x)^4*tan(c)^4 + 24*a^2*tan(d*x)^5*tan(c)^4 - 120*b^2*tan(d*x)^5*tan(c
)^4 + 3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*
tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c)^5 + 12*b^2*arcta
n((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^3*tan(c)^5 - 12*b^2*a
rctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^5 - 128
*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(
d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^5
+ 24*a^2*tan(d*x)^4*tan(c)^5 - 120*b^2*tan(d*x)^4*tan(c)^5 + 24*a^2*d*x*tan
(d*x)^5*tan(c) - 120*b^2*d*x*tan(d*x)^5*tan(c) + 3*pi*b^2*sgn(-2*tan(d*x)^2
*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c) -
48*a^2*d*x*tan(d*x)^4*tan(c)^2 + 240*b^2*d*x*tan(d*x)^4*tan(c)^2 - 6*pi*b^2
*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*ta
n(d*x)^4*tan(c)^2 + 96*a^2*d*x*tan(d*x)^3*tan(c)^3 - 480*b^2*d*x*tan(d*x)^3
*tan(c)^3 + 12*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*ta
n(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^3 + 24*a*b*tan(d*x)^5*tan(c)^3 - 48*a^
2*d*x*tan(d*x)^2*tan(c)^4 + 240*b^2*d*x*tan(d*x)^2*tan(c)^4 - 6*pi*b^2*sgn(
-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x
)^2*tan(c)^4 - 172*a*b*tan(d*x)^4*tan(c)^4 + 24*a^2*d*x*tan(d*x)*tan(c)^5 -
 120*b^2*d*x*tan(d*x)*tan(c)^5 + 3*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(
d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c)^5 + 24*a*b*tan(d*x)^
3*tan(c)^5 - 3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(
c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 6*pi*b^2*sgn
(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2
+ 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c) + 6*b^2*arctan((tan(d*x) + tan(c
))/(tan(d*x)*tan(c) - 1))*tan(d*x)^5*tan(c) - 6*b^2*arctan(-(tan(d*x) - tan
(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c) - 64*a*b*log(4*(tan(c)^2 + 1)
/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x
```

)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c) - 12*pi*b^2*sgn(2*tan(d*x)^
2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x)
 - 2*tan(c))*tan(d*x)^2*tan(c)^2 - 12*b^2*arctan((tan(d*x) + tan(c))/(tan(d
*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^2 + 12*b^2*arctan(-(tan(d*x) - tan(c))/(
tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^2 + 128*a*b*log(4*(tan(c)^2 + 1)/(t
an(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2
 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^2 + 40*a^2*tan(d*x)^5*tan(c)^2
 - 200*b^2*tan(d*x)^5*tan(c)^2 + 6*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sg
n(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d
*x)*tan(c)^3 + 24*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan
(d*x)^3*tan(c)^3 - 24*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1)
)*tan(d*x)^3*tan(c)^3 - 256*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 -
 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
 + 1))*tan(d*x)^3*tan(c)^3 + 24*a^2*tan(d*x)^4*tan(c)^3 - 120*b^2*tan(d*x)^
4*tan(c)^3 - 3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(
c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 - 12*b^2*arctan(
(tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^4 + 12*b^2*arc
tan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^4 + 128*a
*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*
x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^4 +
24*a^2*tan(d*x)^3*tan(c)^4 - 120*b^2*tan(d*x)^3*tan(c)^4 + 6*b^2*arctan((ta
n(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c)^5 - 6*b^2*arctan(-(
tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)*tan(c)^5 - 64*a*b*log(4*
(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(
c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)*tan(c)^5 + 40*a^2*tan(
d*x)^2*tan(c)^5 - 200*b^2*tan(d*x)^2*tan(c)^5 - 24*a^2*d*x*tan(d*x)^4 + 120
*b^2*d*x*tan(d*x)^4 - 3*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)
^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 48*a^2*d*x*tan(d*x)^3*tan(c) - 240
*b^2*d*x*tan(d*x)^3*tan(c) + 6*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)
*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c) - 52*a*b*tan(d*x)^5*ta
n(c) - 96*a^2*d*x*tan(d*x)^2*tan(c)^2 + 480*b^2*d*x*tan(d*x)^2*tan(c)^2 - 1
2*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*ta
n(c))*tan(d*x)^2*tan(c)^2 - 280*a*b*tan(d*x)^4*tan(c)^2 + 48*a^2*d*x*tan(d*
x)*tan(c)^3 - 240*b^2*d*x*tan(d*x)*tan(c)^3 + 6*pi*b^2*sgn(-2*tan(d*x)^2*ta
n(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c)^3 - 16*
a*b*tan(d*x)^3*tan(c)^3 - 24*a^2*d*x*tan(c)^4 + 120*b^2*d*x*tan(c)^4 - 3*pi
*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)
)*tan(c)^4 - 280*a*b*tan(d*x)^2*tan(c)^4 - 52*a*b*tan(d*x)*tan(c)^5 - 6*pi*
b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*ta
n(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2 - 6*b^2*arctan((tan(d*x) + tan(c
))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4 + 6*b^2*arctan(-(tan(d*x) - tan(c))/(t
an(d*x)*tan(c) + 1))*tan(d*x)^4 + 64*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*t
an(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*
x)*tan(c) + 1))*tan(d*x)^4 - 64*b^2*tan(d*x)^5 + 3*pi*b^2*sgn(2*tan(d*x)^2*
tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) -

```
 2*tan(c))*tan(d*x)*tan(c) + 12*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*ta
n(c) - 1))*tan(d*x)^3*tan(c) - 12*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)
*tan(c) + 1))*tan(d*x)^3*tan(c) - 128*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*
tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d
*x)*tan(c) + 1))*tan(d*x)^3*tan(c) - 80*a^2*tan(d*x)^4*tan(c) + 80*b^2*tan(
d*x)^4*tan(c) - 6*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*t
an(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 - 24*b^2*arct
an((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^2 + 24*b^2*
arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 25
6*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan
(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2
 - 96*a^2*tan(d*x)^3*tan(c)^2 - 160*b^2*tan(d*x)^3*tan(c)^2 + 12*b^2*arctan
((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c)^3 - 12*b^2*arct
an(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)*tan(c)^3 - 128*a*b*
log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^
2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)*tan(c)^3 - 96*a^
2*tan(d*x)^2*tan(c)^3 - 160*b^2*tan(d*x)^2*tan(c)^3 - 6*b^2*arctan((tan(d*x
) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(c)^4 + 6*b^2*arctan(-(tan(d*x) - tan
(c))/(tan(d*x)*tan(c) + 1))*tan(c)^4 + 64*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x
)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*t
an(d*x)*tan(c) + 1))*tan(c)^4 - 80*a^2*tan(d*x)*tan(c)^4 + 80*b^2*tan(d*x)*
tan(c)^4 - 64*b^2*tan(c)^5 - 48*a^2*d*x*tan(d*x)^2 + 240*b^2*d*x*tan(d*x)^2
 - 6*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2
*tan(c))*tan(d*x)^2 + 52*a*b*tan(d*x)^4 + 24*a^2*d*x*tan(d*x)*tan(c) - 120*
b^2*d*x*tan(d*x)*tan(c) + 3*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*ta
n(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c) + 280*a*b*tan(d*x)^3*tan(c)
 - 48*a^2*d*x*tan(c)^2 + 240*b^2*d*x*tan(c)^2 - 6*pi*b^2*sgn(-2*tan(d*x)^2*
tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 + 16*a*b*tan
(d*x)^2*tan(c)^2 + 280*a*b*tan(d*x)*tan(c)^3 + 52*a*b*tan(c)^4 - 3*pi*b^2*s
gn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^
2 + 2*tan(d*x) - 2*tan(c)) - 12*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*ta
n(c) - 1))*tan(d*x)^2 + 12*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c)
 + 1))*tan(d*x)^2 + 128*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*t
an(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1
))*tan(d*x)^2 + 40*a^2*tan(d*x)^3 - 200*b^2*tan(d*x)^3 + 6*b^2*arctan((tan(
d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c) - 6*b^2*arctan(-(tan(
d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)*tan(c) - 64*a*b*log(4*(tan(c
)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 +
 tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)*tan(c) + 24*a^2*tan(d*x)^2*t
an(c) - 120*b^2*tan(d*x)^2*tan(c) - 12*b^2*arctan((tan(d*x) + tan(c))/(tan(
d*x)*tan(c) - 1))*tan(c)^2 + 12*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*t
an(c) + 1))*tan(c)^2 + 128*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 -
2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
+ 1))*tan(c)^2 + 24*a^2*tan(d*x)*tan(c)^2 - 120*b^2*tan(d*x)*tan(c)^2 + 40*
a^2*tan(c)^3 - 200*b^2*tan(c)^3 - 24*a^2*d*x + 120*b^2*d*x - 3*pi*b^2*sgn(-
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2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)) - 24*a*b
*tan(d*x)^2 + 172*a*b*tan(d*x)*tan(c) - 24*a*b*tan(c)^2 - 6*b^2*arctan((tan
(d*x) + tan(c))/(tan(d*x)*tan(c) - 1)) + 6*b^2*arctan(-(tan(d*x) - tan(c))/
(tan(d*x)*tan(c) + 1)) + 64*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 -
 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
 + 1)) + 24*a^2*tan(d*x) - 120*b^2*tan(d*x) + 24*a^2*tan(c) - 120*b^2*tan(c
) - 44*a*b)/(d*tan(d*x)^5*tan(c)^5 + 2*d*tan(d*x)^5*tan(c)^3 - d*tan(d*x)^4
*tan(c)^4 + 2*d*tan(d*x)^3*tan(c)^5 + d*tan(d*x)^5*tan(c) - 2*d*tan(d*x)^4*
tan(c)^2 + 4*d*tan(d*x)^3*tan(c)^3 - 2*d*tan(d*x)^2*tan(c)^4 + d*tan(d*x)*t
an(c)^5 - d*tan(d*x)^4 + 2*d*tan(d*x)^3*tan(c) - 4*d*tan(d*x)^2*tan(c)^2 +
2*d*tan(d*x)*tan(c)^3 - d*tan(c)^4 - 2*d*tan(d*x)^2 + d*tan(d*x)*tan(c) - 2
*d*tan(c)^2 - d)
```

3.23 $\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=122

$$\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos(c+dx)}{d}-\frac{2ab\sin^3(c+dx)}{3d}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2\cos^3(c+dx)}{3d}+\frac{2b}{}$$

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - (a^2*Cos[c + d*x])/d + (2*b^2*Cos[c + d*x
])/d + (a^2*Cos[c + d*x]^3)/(3*d) - (b^2*Cos[c + d*x]^3)/(3*d) + (b^2*Sec[c
 + d*x])/d - (2*a*b*Sin[c + d*x])/d - (2*a*b*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.130693, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3517, 2633, 2592, 302, 206, 2590, 270}

$$\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos(c+dx)}{d}-\frac{2ab\sin^3(c+dx)}{3d}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2\cos^3(c+dx)}{3d}+\frac{2b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - (a^2*Cos[c + d*x])/d + (2*b^2*Cos[c + d*x
])/d + (a^2*Cos[c + d*x]^3)/(3*d) - (b^2*Cos[c + d*x]^3)/(3*d) + (b^2*Sec[c
 + d*x])/d - (2*a*b*Sin[c + d*x])/d - (2*a*b*Sin[c + d*x]^3)/(3*d)
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a^2\sin^3(c+dx)+2ab\sin^3(c+dx)\tan(c+dx)+b^2\sin^3(c+dx)\tan^2(c+dx\right.$$

$$= a^2\int \sin^3(c+dx)\,dx+(2ab)\int \sin^3(c+dx)\tan(c+dx)\,dx+b^2\int \sin^3(c+dx)$$

$$= -\frac{a^2\,\text{Subst}\left(\int \left(1-x^2\right)dx,x,\cos(c+dx)\right)}{d}+\frac{(2ab)\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx,x,\sin(c+d\right.}{d}$$

$$= -\frac{a^2\cos(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{(2ab)\,\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx,x,\sin\right.}{d}$$

$$= -\frac{a^2\cos(c+dx)}{d}+\frac{2b^2\cos(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{b^2\cos^3(c+dx)}{3d}+\frac{b^2\sec}{}$$

$$= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2\cos(c+dx)}{d}+\frac{2b^2\cos(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}$$

Mathematica [A] time = 1.02334, size = 152, normalized size = 1.25

$$\frac{\sec(c+dx)\left(\left(20b^2-8a^2\right)\cos(2(c+dx))+\left(a^2-b^2\right)\cos(4(c+dx))-9a^2-28ab\sin(2(c+dx))+2ab\sin(4(c+dx))-48\right.}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(-9*a^2 + 45*b^2 + (-8*a^2 + 20*b^2)*Cos[2*(c + d*x)] + (a^2
- b^2)*Cos[4*(c + d*x)] - 48*a*b*Cos[c + d*x]*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] + 48*a*b*Cos[c + d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
- 28*a*b*Sin[2*(c + d*x)] + 2*a*b*Sin[4*(c + d*x)]))/(24*d)
```

Maple [A] time = 0.041, size = 167, normalized size = 1.4

$$\frac{b^2(\sin(dx+c))^6}{d\cos(dx+c)}+\frac{8\,b^2\cos(dx+c)}{3\,d}+\frac{b^2\cos(dx+c)(\sin(dx+c))^4}{d}+\frac{4\,b^2\cos(dx+c)(\sin(dx+c))^2}{3\,d}-\frac{2\,ab\,(\sin(dx+}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*b^2*sin(d*x+c)^6/cos(d*x+c)+8/3*b^2*cos(d*x+c)/d+1/d*b^2*cos(d*x+c)*sin
(d*x+c)^4+4/3/d*b^2*cos(d*x+c)*sin(d*x+c)^2-2/3*a*b*sin(d*x+c)^3/d-2*a*b*si
n(d*x+c)/d+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))-1/3/d*cos(d*x+c)*sin(d*x+c)^2*
a^2-2/3*a^2*cos(d*x+c)/d
```

Maxima [A] time = 0.973981, size = 140, normalized size = 1.15

$$\frac{\left(\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)a^{2}-\left(2\,\sin\left(dx+c\right)^{3}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)+6\,\sin\left(dx+c\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((cos(d*x + c)^3 - 3*cos(d*x + c))*a^2 - (2*sin(d*x + c)^3 - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*a*b - (cos(d*x +
c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*b^2)/d
```

Fricas [A] time = 1.99257, size = 321, normalized size = 2.63

$$\frac{\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{4}+3\,ab\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,ab\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)-3\left(a^{2}-2\,b^{2}\right.}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((a^2 - b^2)*cos(d*x + c)^4 + 3*a*b*cos(d*x + c)*log(sin(d*x + c) + 1)
- 3*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - 3*(a^2 - 2*b^2)*cos(d*x + c)^
2 + 3*b^2 + 2*(a*b*cos(d*x + c)^3 - 4*a*b*cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan\left(c+dx\right)\right)^{2}\sin^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.24 $\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=76

$$\frac{1}{2}x\left(a^2-3b^2\right)-\frac{2ab\log(\cos(c+dx))}{d}-\frac{\sin(c+dx)\cos(c+dx)(a+b\tan(c+dx))^2}{2d}+\frac{3b^2\tan(c+dx)}{2d}$$

```
[Out] ((a^2 - 3*b^2)*x)/2 - (2*a*b*Log[Cos[c + d*x]])/d + (3*b^2*Tan[c + d*x])/(2
*d) - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rubi [A] time = 0.113079, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1645, 774, 635, 203, 260}

$$\frac{1}{2}x\left(a^2-3b^2\right)-\frac{2ab\log(\cos(c+dx))}{d}-\frac{\sin(c+dx)\cos(c+dx)(a+b\tan(c+dx))^2}{2d}+\frac{3b^2\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 - 3*b^2)*x)/2 - (2*a*b*Log[Cos[c + d*x]])/d + (3*b^2*Tan[c + d*x])/(2
*d) - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^2(a+x)^2}{\left(b^2+x^2\right)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{2d} - \frac{\text{Subst}\left(\int \frac{(a+x)\left(-ab^2-3b^2x\right)}{b^2+x^2}\,dx, x, b\right.}{2bd}$$

$$= \frac{3b^2\tan(c+dx)}{2d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{2d} - \frac{\text{Subst}\left(\int \frac{-a^2b^2}{\ }\right.}{\ }$$

$$= \frac{3b^2\tan(c+dx)}{2d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{2d} + \frac{(2ab)\,\text{Subst}\left(\int\right.}{\ }$$

$$= \frac{1}{2}\left(a^2-3b^2\right)x - \frac{2ab\log(\cos(c+dx))}{d} + \frac{3b^2\tan(c+dx)}{2d} - \frac{\cos(c+dx)\sin(c+a}{\ }$$

Mathematica [B] time = 2.36223, size = 162, normalized size = 2.13

$$\frac{b\left(\frac{(b^2-a^2)\sin(2(c+dx))}{2b}+\frac{(b^2-a^2)\tan^{-1}(\tan(c+dx))}{b}+\left(\frac{a^2-2b^2}{\sqrt{-b^2}}+2a\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+\left(\frac{2b^2-a^2}{\sqrt{-b^2}}+2a\right)\log\left(\sqrt{-b^2}+b\tan\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b*(((-a^2 + b^2)*ArcTan[Tan[c + d*x]])/b + 2*a*Cos[c + d*x]^2 + (2*a + (a^
2 - 2*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + (2*a + (-a^2 + 2*
b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + ((-a^2 + b^2)*Sin[2*(c
+ d*x)])/(2*b) + 2*b*Tan[c + d*x]))/(2*d)
```

Maple [B] time = 0.039, size = 145, normalized size = 1.9

$$\frac{b^2(\sin(dx+c))^5}{d\cos(dx+c)}+\frac{b^2\cos(dx+c)(\sin(dx+c))^3}{d}+\frac{3b^2\cos(dx+c)\sin(dx+c)}{2d}-\frac{3b^2x}{2}-\frac{3b^2c}{2d}-\frac{ab(\sin(dx+c))^2}{d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*b^2*sin(d*x+c)^5/cos(d*x+c)+1/d*b^2*cos(d*x+c)*sin(d*x+c)^3+3/2/d*b^2*c
os(d*x+c)*sin(d*x+c)-3/2*b^2*x-3/2/d*b^2*c-1/d*a*b*sin(d*x+c)^2-2*a*b*ln(co
s(d*x+c))/d-1/2/d*a^2*cos(d*x+c)*sin(d*x+c)+1/2*a^2*x+1/2/d*a^2*c
```

Maxima [A] time = 1.47601, size = 111, normalized size = 1.46

$$\frac{2ab\log\left(\tan(dx+c)^2+1\right)+2b^2\tan(dx+c)+\left(a^2-3b^2\right)(dx+c)+\frac{2ab-\left(a^2-b^2\right)\tan(dx+c)}{\tan(dx+c)^2+1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*b*log(tan(d*x + c)^2 + 1) + 2*b^2*tan(d*x + c) + (a^2 - 3*b^2)*(d*
x + c) + (2*a*b - (a^2 - b^2)*tan(d*x + c))/(tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 2.01632, size = 240, normalized size = 3.16

$$\frac{2\,ab\cos\left(dx+c\right)^{3}-4\,ab\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)+\left(\left(a^{2}-3\,b^{2}\right)dx-ab\right)\cos\left(dx+c\right)-\left(\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}-2\right.}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*cos(d*x + c)^3 - 4*a*b*cos(d*x + c)*log(-cos(d*x + c)) + ((a^2 -
 3*b^2)*d*x - a*b)*cos(d*x + c) - ((a^2 - b^2)*cos(d*x + c)^2 - 2*b^2)*sin(
d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\tan\left(c+dx\right)\right)^{2}\sin^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x)**2, x)
```

Giac [B] time = 1.87871, size = 1432, normalized size = 18.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*d*x*tan(d*x)^3*tan(c)^3 - 3*b^2*d*x*tan(d*x)^3*tan(c)^3 - 2*a*b*lo
g(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*
tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 + a^2*d
*x*tan(d*x)^3*tan(c) - 3*b^2*d*x*tan(d*x)^3*tan(c) - a^2*d*x*tan(d*x)^2*tan
(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(c)^2 + a^2*d*x*tan(d*x)*tan(c)^3 - 3*b^2*d
*x*tan(d*x)*tan(c)^3 + a*b*tan(d*x)^3*tan(c)^3 - 2*a*b*log(4*(tan(c)^2 + 1)
```

```
/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x
)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c) + 2*a*b*log(4*(tan(c)^2 + 1
)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*
x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + a^2*tan(d*x)^3*tan(c)^
2 - 3*b^2*tan(d*x)^3*tan(c)^2 - 2*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(
c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*
tan(c) + 1))*tan(d*x)*tan(c)^3 + a^2*tan(d*x)^2*tan(c)^3 - 3*b^2*tan(d*x)^2
*tan(c)^3 - a^2*d*x*tan(d*x)^2 + 3*b^2*d*x*tan(d*x)^2 + a^2*d*x*tan(d*x)*ta
n(c) - 3*b^2*d*x*tan(d*x)*tan(c) - a*b*tan(d*x)^3*tan(c) - a^2*d*x*tan(c)^2
 + 3*b^2*d*x*tan(c)^2 - 5*a*b*tan(d*x)^2*tan(c)^2 - a*b*tan(d*x)*tan(c)^3 +
 2*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + ta
n(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2 - 2*b^2
*tan(d*x)^3 - 2*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^
3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d
*x)*tan(c) - 2*a^2*tan(d*x)^2*tan(c) + 2*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)
^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*ta
n(d*x)*tan(c) + 1))*tan(c)^2 - 2*a^2*tan(d*x)*tan(c)^2 - 2*b^2*tan(c)^3 - a
^2*d*x + 3*b^2*d*x + a*b*tan(d*x)^2 + 5*a*b*tan(d*x)*tan(c) + a*b*tan(c)^2
+ 2*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) + a^2*tan(d*x) -
3*b^2*tan(d*x) + a^2*tan(c) - 3*b^2*tan(c) - a*b)/(d*tan(d*x)^3*tan(c)^3 +
d*tan(d*x)^3*tan(c) - d*tan(d*x)^2*tan(c)^2 + d*tan(d*x)*tan(c)^3 - d*tan(d
*x)^2 + d*tan(d*x)*tan(c) - d*tan(c)^2 - d)
```

3.25 $\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=68

$$-\frac{a^2\cos(c+dx)}{d}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\cos(c+dx)}{d}+\frac{b^2\sec(c+dx)}{d}$$

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - (a^2*Cos[c + d*x])/d + (b^2*Cos[c + d*x])
/d + (b^2*Sec[c + d*x])/d - (2*a*b*Sin[c + d*x])/d
```

Rubi [A] time = 0.0736639, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3517, 2638, 2592, 321, 206, 2590, 14}

$$-\frac{a^2\cos(c+dx)}{d}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\cos(c+dx)}{d}+\frac{b^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - (a^2*Cos[c + d*x])/d + (b^2*Cos[c + d*x])
/d + (b^2*Sec[c + d*x])/d - (2*a*b*Sin[c + d*x])/d
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx &= \int \left(a^2\sin(c+dx)+2ab\sin(c+dx)\tan(c+dx)+b^2\sin(c+dx)\tan^2(c+dx)\right)\\
&= a^2\int \sin(c+dx)\,dx+(2ab)\int \sin(c+dx)\tan(c+dx)\,dx+b^2\int \sin(c+dx)\,\mathrm{ta}\\
&= -\frac{a^2\cos(c+dx)}{d}+\frac{(2ab)\,\mathrm{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d}-\frac{b^2\,\mathrm{Subst}\left(\int \frac{1-x^2}{x^2}\,d\right.}{d}\\
&= -\frac{a^2\cos(c+dx)}{d}-\frac{2ab\sin(c+dx)}{d}+\frac{(2ab)\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d}-\\
&= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2\cos(c+dx)}{d}+\frac{b^2\cos(c+dx)}{d}+\frac{b^2\sec(c+dx)}{d}-\end{aligned}$$

Mathematica [A] time = 0.439056, size = 111, normalized size = 1.63

$$-\frac{\sec(c+dx)\left(\left(a^2-b^2\right)\cos(2(c+dx))+a^2+2ab\sin(2(c+dx))+4ab\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(Sec[c + d*x]*(a^2 - 3*b^2 + (a^2 - b^2)*Cos[2*(c + d*x)] + 4*a*b*Cos[c +
d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]]) + 2*a*b*Sin[2*(c + d*x)]))/(2*d)
```

Maple [A] time = 0.036, size = 108, normalized size = 1.6

$$\frac{b^2(\sin(dx+c))^4}{d\cos(dx+c)}+\frac{b^2\cos(dx+c)(\sin(dx+c))^2}{d}+2\frac{b^2\cos(dx+c)}{d}-2\frac{ab\sin(dx+c)}{d}+2\frac{ab\ln(\sec(dx+c)+\tan(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*b^2*sin(d*x+c)^4/cos(d*x+c)+1/d*b^2*cos(d*x+c)*sin(d*x+c)^2+2*b^2*cos(d
*x+c)/d-2*a*b*sin(d*x+c)/d+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))-a^2*cos(d*x+c)
/d
```

Maxima [A] time = 1.11944, size = 90, normalized size = 1.32

$$\frac{b^2\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)+ab(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\sin(dx+c))-a^2\cos(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (b^2*(1/cos(d*x + c) + cos(d*x + c)) + a*b*(log(sin(d*x + c) + 1) - log(sin
(d*x + c) - 1) - 2*sin(d*x + c)) - a^2*cos(d*x + c))/d
```

Fricas [A] time = 2.32235, size = 230, normalized size = 3.38

$$\frac{ab\cos(dx+c)\log(\sin(dx+c)+1) - ab\cos(dx+c)\log(-\sin(dx+c)+1) - 2\,ab\cos(dx+c)\sin(dx+c) - \left(a^2-b^2\right)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a*b*cos(d*x + c)*log(sin(d*x + c) + 1) - a*b*cos(d*x + c)*log(-sin(d*x + c
) + 1) - 2*a*b*cos(d*x + c)*sin(d*x + c) - (a^2 - b^2)*cos(d*x + c)^2 + b^2
)/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\tan(c+dx))^2 \sin(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x), x)
```

Giac [B] time = 4.53757, size = 3830, normalized size = 56.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*
d*x)^4*tan(1/2*c)^4 - a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/
```

2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1
/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + a^2*tan(1/2*d*x)^4*tan(1/2*c)^4 -
2*b^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/
2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(
1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^
3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 4*a*b*log(2*(tan
(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) -
2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^3*tan(1/2*c
)^3 - 4*a*b*tan(1/2*d*x)^4*tan(1/2*c)^3 - 4*a*b*tan(1/2*d*x)^3*tan(1/2*c)^4
- 2*a^2*tan(1/2*d*x)^4*tan(1/2*c)^2 - 8*a^2*tan(1/2*d*x)^3*tan(1/2*c)^3 +
8*b^2*tan(1/2*d*x)^3*tan(1/2*c)^3 - 2*a^2*tan(1/2*d*x)^2*tan(1/2*c)^4 - a*b
log(2(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*t
an(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^
4 + a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d
*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/
2*d*x)^4 - 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4
+ 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1
))*tan(1/2*d*x)^3*tan(1/2*c) + 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)
^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)
^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*
tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^3*tan(1/2*c) + 4*a*b*tan(1/2*d*x)^4*tan
(1/2*c) + 24*a*b*tan(1/2*d*x)^3*tan(1/2*c)^2 - 4*a*b*log(2*(tan(1/2*c)^2 +
1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d
*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2
- 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c)^3 + 4*a*b*lo
g(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(
1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)*tan
(1/2*c)^3 + 24*a*b*tan(1/2*d*x)^2*tan(1/2*c)^3 - a*b*log(2*(tan(1/2*c)^2 +
1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d
*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2

```
 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*c)^4 + a*b*log(2*(tan(1/2*c)
^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(
1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*c)^4 + 4*a*b*tan(1/2*d*
x)*tan(1/2*c)^4 + a^2*tan(1/2*d*x)^4 - 2*b^2*tan(1/2*d*x)^4 + 8*a^2*tan(1/2
*d*x)^3*tan(1/2*c) - 8*b^2*tan(1/2*d*x)^3*tan(1/2*c) + 20*a^2*tan(1/2*d*x)^
2*tan(1/2*c)^2 - 24*b^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a^2*tan(1/2*d*x)*ta
n(1/2*c)^3 - 8*b^2*tan(1/2*d*x)*tan(1/2*c)^3 + a^2*tan(1/2*c)^4 - 2*b^2*tan
(1/2*c)^4 - 4*a*b*tan(1/2*d*x)^3 - 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*
d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/
2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3
+ 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2
*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c) + 4*a*b*log(2*(tan(1/2*c
)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan
(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/
2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c) - 24*a
*b*tan(1/2*d*x)^2*tan(1/2*c) - 24*a*b*tan(1/2*d*x)*tan(1/2*c)^2 - 4*a*b*tan
(1/2*c)^3 - 2*a^2*tan(1/2*d*x)^2 - 8*a^2*tan(1/2*d*x)*tan(1/2*c) + 8*b^2*ta
n(1/2*d*x)*tan(1/2*c) - 2*a^2*tan(1/2*c)^2 + a*b*log(2*(tan(1/2*c)^2 + 1)/(
tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^
3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2
*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2
*tan(1/2*d*x) - 2*tan(1/2*c) + 1)) - a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*
d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/
2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3
- 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) + 2*tan(1/2*c) + 1)) + 4*a*b*tan(1/2*d*x) + 4*a*b*tan(1/2*c) + a^2 -
2*b^2)/(d*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*d*tan(1/2*d*x)^3*tan(1/2*c)^3 - d
*tan(1/2*d*x)^4 - 4*d*tan(1/2*d*x)^3*tan(1/2*c) - 4*d*tan(1/2*d*x)*tan(1/2*
c)^3 - d*tan(1/2*c)^4 - 4*d*tan(1/2*d*x)*tan(1/2*c) + d)
```

3.26 $\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=43

$$-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

```
[Out] -((a^2*ArcTanh[Cos[c + d*x]])/d) + (2*a*b*ArcTanh[Sin[c + d*x]])/d + (b^2*S
ec[c + d*x])/d
```

Rubi [A] time = 0.0563928, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3517, 3770, 2606, 8}

$$-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((a^2*ArcTanh[Cos[c + d*x]])/d) + (2*a*b*ArcTanh[Sin[c + d*x]])/d + (b^2*S
ec[c + d*x])/d
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a^2\csc(c+dx)+2ab\sec(c+dx)+b^2\sec(c+dx)\tan(c+dx)\right)dx$$
$$= a^2\int \csc(c+dx)\,dx+(2ab)\int \sec(c+dx)\,dx+b^2\int \sec(c+dx)\tan(c+dx)\,dx$$
$$= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\,\mathrm{Subst}(\int 1\,dx,x,\sec(}{d}$$
$$= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

Mathematica [B] time = 0.233457, size = 97, normalized size = 2.26

$$\frac{a\left(a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2b\log\left(\sin\left(\frac{1}{2}(c+dx\right.\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a*(-(a*Log[Cos[(c + d*x)/2]]) - 2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] + a*Log[Sin[(c + d*x)/2]] + 2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
) + b^2*Sec[c + d*x])/d
```

Maple [A] time = 0.044, size = 61, normalized size = 1.4

$$\frac{b^2}{d\cos(dx+c)}+2\,\frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*b^2/cos(d*x+c)+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^2*ln(csc(d*x+c)-
cot(d*x+c))
```

Maxima [A] time = 1.06914, size = 81, normalized size = 1.88

$$\frac{ab(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))-a^2\log(\cot(dx+c)+\csc(dx+c))+\frac{b^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - a^2*log(cot(d*x + c)
 + csc(d*x + c)) + b^2/cos(d*x + c))/d
```

Fricas [B] time = 2.4343, size = 288, normalized size = 6.7

$$-\frac{a^2\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-a^2\cos(dx+c)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2\,ab\cos(dx+c)\log(\sin(dx+c)+\cdots}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2) - a^2*cos(d*x + c)*log(-
1/2*cos(d*x + c) + 1/2) - 2*a*b*cos(d*x + c)*log(sin(d*x + c) + 1) + 2*a*b*
cos(d*x + c)*log(-sin(d*x + c) + 1) - 2*b^2)/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\tan(c+dx))^2\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x), x)
```

Giac [A] time = 1.5719, size = 100, normalized size = 2.33

$$\frac{2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\,b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a*b*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*b^2/(tan(1/2*d*x + 1/2*c
)^2 - 1))/d
```

3.27 $\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=42

$$-\frac{a^2\cot(c+dx)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] -((a^2*Cot[c + d*x])/d) + (2*a*b*Log[Tan[c + d*x]])/d + (b^2*Tan[c + d*x])/
d
```

Rubi [A] time = 0.0474087, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 43}

$$-\frac{a^2\cot(c+dx)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((a^2*Cot[c + d*x])/d) + (2*a*b*Log[Tan[c + d*x]])/d + (b^2*Tan[c + d*x])/
d
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{(a+x)^2}{x^2}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{b\,\mathrm{Subst}\left(\int \left(1+\frac{a^2}{x^2}+\frac{2a}{x}\right)dx, x, b\tan(c+dx)\right)}{d}$$
$$= -\frac{a^2\cot(c+dx)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Mathematica [B] time = 0.544128, size = 91, normalized size = 2.17

$$-\frac{\cos(c+dx)(a+b\tan(c+dx))^2\left(a\cos(c+dx)(a\cot(c+dx)+2b(\log(\cos(c+dx))-\log(\sin(c+dx))))-b^2\sin(c+dx\right.}{d(a\cos(c+dx)+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((Cos[c + d*x]*(a*Cos[c + d*x]*(a*Cot[c + d*x] + 2*b*(Log[Cos[c + d*x]] -
Log[Sin[c + d*x]])) - b^2*Sin[c + d*x])*(a + b*Tan[c + d*x])^2)/(d*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^2))
```

Maple [A] time = 0.042, size = 43, normalized size = 1.

$$-\frac{a^2\cot(dx+c)}{d}+2\,\frac{ab\ln(\tan(dx+c))}{d}+\frac{b^2\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x)
```

```
[Out] -a^2*cot(d*x+c)/d+2*a*b*ln(tan(d*x+c))/d+b^2*tan(d*x+c)/d
```

Maxima [A] time = 1.10725, size = 53, normalized size = 1.26

$$\frac{2\,ab\log(\tan(dx+c))+b^2\tan(dx+c)-\frac{a^2}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*a*b*log(tan(d*x + c)) + b^2*tan(d*x + c) - a^2/tan(d*x + c))/d
```

Fricas [B] time = 2.28245, size = 246, normalized size = 5.86

$$-\frac{ab\cos(dx+c)\log\left(\cos(dx+c)^2\right)\sin(dx+c) - ab\cos(dx+c)\log\left(-\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\right)\sin(dx+c) + \left(a^2+b^2\right)\cos(dx+c)^2 - b^2}{d\cos(dx+c)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a*b*cos(d*x + c)*log(cos(d*x + c)^2)*sin(d*x + c) - a*b*cos(d*x + c)*log(
-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) + (a^2 + b^2)*cos(d*x + c)^2 - b^2)
/(d*cos(d*x + c)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\tan(c+dx))^2\csc^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**2, x)
```

Giac [A] time = 1.53881, size = 69, normalized size = 1.64

$$\frac{2\,ab\log\left(|\tan(dx+c)|\right) + b^2\tan(dx+c) - \frac{2\,ab\tan(dx+c)+a^2}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a*b*log(abs(tan(d*x + c))) + b^2*tan(d*x + c) - (2*a*b*tan(d*x + c) + a^
2)/tan(d*x + c))/d
```

3.28 $\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=95

$$-\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}-\frac{2ab\csc(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}-$$

```
[Out] -(a^2*ArcTanh[Cos[c + d*x]])/(2*d) - (b^2*ArcTanh[Cos[c + d*x]])/d + (2*a*b
*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Csc[c + d*x])/d - (a^2*Cot[c + d*x]*Csc[
c + d*x])/(2*d) + (b^2*Sec[c + d*x])/d
```

Rubi [A] time = 0.117606, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3517, 3768, 3770, 2621, 321, 207, 2622}

$$-\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}-\frac{2ab\csc(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(a^2*ArcTanh[Cos[c + d*x]])/(2*d) - (b^2*ArcTanh[Cos[c + d*x]])/d + (2*a*b
*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Csc[c + d*x])/d - (a^2*Cot[c + d*x]*Csc[
c + d*x])/(2*d) + (b^2*Sec[c + d*x])/d
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a^2\csc^3(c+dx)+2ab\csc^2(c+dx)\sec(c+dx)+b^2\csc(c+dx)\sec^2(c+dx)\right)$$

$$= a^2\int \csc^3(c+dx)\,dx+(2ab)\int \csc^2(c+dx)\sec(c+dx)\,dx+b^2\int \csc(c+dx)\,\mathrm{s}$$

$$= -\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}+\frac{1}{2}a^2\int \csc(c+dx)\,dx-\frac{(2ab)\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,\right.}{d}$$

$$= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{2ab\csc(c+dx)}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}+\frac{b^2\,\mathrm{se}}{}$$

$$= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}$$

Mathematica [B] time = 1.95109, size = 250, normalized size = 2.63

$$4\left(a^2+2b^2\right)\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)-4\left(a^2+2b^2\right)\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)-a^2\csc^2\left(\tfrac{1}{2}(c+dx)\right)+a^2\sec^2\left(\tfrac{1}{2}(c+dx)\right)-8ab\,\mathrm{t}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (8*b^2 - 8*a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 - 4*(a^2 + 2*b^2)*
Log[Cos[(c + d*x)/2]] - 16*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4
*(a^2 + 2*b^2)*Log[Sin[(c + d*x)/2]] + 16*a*b*Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]] + a^2*Sec[(c + d*x)/2]^2 + (8*b^2*Sin[(c + d*x)/2])/(Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]) - (8*b^2*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + S
in[(c + d*x)/2]) - 8*a*b*Tan[(c + d*x)/2])/(8*d)
```

Maple [A] time = 0.051, size = 120, normalized size = 1.3

$$\frac{b^2}{d\cos(dx+c)}+\frac{b^2\ln(\csc(dx+c)-\cot(dx+c))}{d}-2\frac{ab}{d\sin(dx+c)}+2\frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{a^2\cot(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*b^2/cos(d*x+c)+1/d*b^2*ln(csc(d*x+c)-cot(d*x+c))-2/d*a*b/sin(d*x+c)+2/d
*a*b*ln(sec(d*x+c)+tan(d*x+c))-1/2*a^2*cot(d*x+c)*csc(d*x+c)/d+1/2/d*a^2*ln
(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.10054, size = 165, normalized size = 1.74

$$\frac{a^2\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1)\right) + 2b^2\left(\frac{2}{\cos(dx+c)} - \log(\cos(dx+c)+1) + \log(\cos(dx+$$
$$4d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log
(cos(d*x + c) - 1)) + 2*b^2*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(c
os(d*x + c) - 1)) - 4*a*b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin
(d*x + c) - 1)))/d
```

Fricas [B] time = 2.77791, size = 581, normalized size = 6.12

$$8ab\cos(dx+c)\sin(dx+c) + 2\left(a^2+2b^2\right)\cos(dx+c)^2 - 4b^2 - \left(\left(a^2+2b^2\right)\cos(dx+c)^3 - \left(a^2+2b^2\right)\cos(dx+c)\right)\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(8*a*b*cos(d*x + c)*sin(d*x + c) + 2*(a^2 + 2*b^2)*cos(d*x + c)^2 - 4*b
^2 - ((a^2 + 2*b^2)*cos(d*x + c)^3 - (a^2 + 2*b^2)*cos(d*x + c))*log(1/2*co
s(d*x + c) + 1/2) + ((a^2 + 2*b^2)*cos(d*x + c)^3 - (a^2 + 2*b^2)*cos(d*x +
 c))*log(-1/2*cos(d*x + c) + 1/2) + 4*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c
))*log(sin(d*x + c) + 1) - 4*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*log(-s
in(d*x + c) + 1))/(d*cos(d*x + c)^3 - d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\tan(c+dx))^2\csc^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**3, x)
```

Giac [A] time = 1.63598, size = 232, normalized size = 2.44

$$\frac{a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 16\, ab \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - 16\, ab \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) - 8\, ab \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 4\, (\cdots}{8\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*tan(1/2*d*x + 1/2*c)^2 + 16*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1))
 - 16*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 8*a*b*tan(1/2*d*x + 1/2*c) +
 4*(a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) - 16*b^2/(tan(1/2*d*x + 1/2
*c)^2 - 1) - (6*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*tan(1/2*d*x + 1/2*c)^2
+ 8*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c)^2)/d
```

3.29 $\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=79

$$-\frac{\left(a^2+b^2\right)\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{ab\cot^2(c+dx)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] -(((a^2 + b^2)*Cot[c + d*x])/d) - (a*b*Cot[c + d*x]^2)/d - (a^2*Cot[c + d*x
]^3)/(3*d) + (2*a*b*Log[Tan[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.0700101, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 894}

$$-\frac{\left(a^2+b^2\right)\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{ab\cot^2(c+dx)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 + b^2)*Cot[c + d*x])/d) - (a*b*Cot[c + d*x]^2)/d - (a^2*Cot[c + d*x
]^3)/(3*d) + (2*a*b*Log[Tan[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^2(b^2+x^2)}{x^4}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \left(1+\frac{a^2b^2}{x^4}+\frac{2ab^2}{x^3}+\frac{a^2+b^2}{x^2}+\frac{2a}{x}\right)dx, x, b\tan(c+dx)\right)}{d}$$
$$= -\frac{(a^2+b^2)\cot(c+dx)}{d} - \frac{ab\cot^2(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} + \frac{2ab\log(\tan(c+}{d}$$

Mathematica [A] time = 1.40886, size = 127, normalized size = 1.61

$$-\frac{(a+b\tan(c+dx))^2\left(\cos^2(c+dx)\left((2a^2+3b^2)\cot(c+dx)+6ab(\log(\cos(c+dx))-\log(\sin(c+dx)))\right)+a^2\cot^3(c+d}{3d(a\cos(c+dx)+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((3*a*b*Cot[c + d*x]^2 + a^2*Cot[c + d*x]^3 + Cos[c + d*x]^2*((2*a^2 + 3*b
^2)*Cot[c + d*x] + 6*a*b*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])) - (3*b^2*
Sin[2*(c + d*x)])/2)*(a + b*Tan[c + d*x])^2)/(3*d*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^2)
```

Maple [A] time = 0.053, size = 104, normalized size = 1.3

$$\frac{b^2}{d\sin(dx+c)\cos(dx+c)} - 2\frac{b^2\cot(dx+c)}{d} - \frac{ab}{d(\sin(dx+c))^2} + 2\frac{ab\ln(\tan(dx+c))}{d} - \frac{2a^2\cot(dx+c)}{3d} - \frac{a^2\cot(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*b^2/sin(d*x+c)/cos(d*x+c)-2/d*b^2*cot(d*x+c)-1/d*a*b/sin(d*x+c)^2+2*a*b
*ln(tan(d*x+c))/d-2/3*a^2*cot(d*x+c)/d-1/3/d*a^2*cot(d*x+c)*csc(d*x+c)^2
```

Maxima [A] time = 1.08606, size = 93, normalized size = 1.18

$$\frac{6\,ab\log\left(\tan\left(dx+c\right)\right)+3\,b^2\tan\left(dx+c\right)-\frac{3\,ab\tan(dx+c)+3\left(a^2+b^2\right)\tan(dx+c)^2+a^2}{\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(6*a*b*log(tan(d*x + c)) + 3*b^2*tan(d*x + c) - (3*a*b*tan(d*x + c) + 3
*(a^2 + b^2)*tan(d*x + c)^2 + a^2)/tan(d*x + c)^3)/d
```

Fricas [B] time = 2.39439, size = 446, normalized size = 5.65

$$-\frac{2\left(a^2+3\,b^2\right)\cos\left(dx+c\right)^4-3\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)-3\left(a^2+3\,b^2\right)\cos\left(dx+c\right)^2+3\left(ab\cos\left(dx+c\right)^3-ab\cos\left(d\right.\right.}{3\left(d\cos\left(dx+c\right)^3-c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^2 + 3*b^2)*cos(d*x + c)^4 - 3*a*b*cos(d*x + c)*sin(d*x + c) - 3*
(a^2 + 3*b^2)*cos(d*x + c)^2 + 3*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*lo
g(cos(d*x + c)^2)*sin(d*x + c) - 3*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*
log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) + 3*b^2)/((d*cos(d*x + c)^3 - d
*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.61879, size = 123, normalized size = 1.56

$$\frac{6\,ab\log\left(|\tan\left(dx+c\right)|\right)+3\,b^2\tan\left(dx+c\right)-\frac{11\,ab\tan(dx+c)^3+3\,a^2\tan(dx+c)^2+3\,b^2\tan(dx+c)^2+3\,ab\tan(dx+c)+a^2}{\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*a*b*log(abs(tan(d*x + c))) + 3*b^2*tan(d*x + c) - (11*a*b*tan(d*x +
c)^3 + 3*a^2*tan(d*x + c)^2 + 3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a
^2)/tan(d*x + c)^3)/d
```

3.30 $\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=165

$$-\frac{3a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^2\cot(c+dx)\csc(c+dx)}{8d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c+dx)}{d}$$

```
[Out] (-3*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (3*b^2*ArcTanh[Cos[c + d*x]])/(2*d)
+ (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Csc[c + d*x])/d - (3*a^2*Cot[c +
 d*x]*Csc[c + d*x])/(8*d) - (2*a*b*Csc[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x
]*Csc[c + d*x]^3)/(4*d) + (3*b^2*Sec[c + d*x])/(2*d) - (b^2*Csc[c + d*x]^2*
Sec[c + d*x])/(2*d)
```

Rubi [A] time = 0.160082, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3517, 3768, 3770, 2621, 302, 207, 2622, 288, 321}

$$-\frac{3a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^2\cot(c+dx)\csc(c+dx)}{8d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```


```
[Out] (-3*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (3*b^2*ArcTanh[Cos[c + d*x]])/(2*d)
+ (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Csc[c + d*x])/d - (3*a^2*Cot[c +
 d*x]*Csc[c + d*x])/(8*d) - (2*a*b*Csc[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x
]*Csc[c + d*x]^3)/(4*d) + (3*b^2*Sec[c + d*x])/(2*d) - (b^2*Csc[c + d*x]^2*
Sec[c + d*x])/(2*d)
```


Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a^2\csc^5(c+dx)+2ab\csc^4(c+dx)\sec(c+dx)+b^2\csc^3(c+dx)\sec^2(c+d\right.$$
$$= a^2\int \csc^5(c+dx)\,dx + (2ab)\int \csc^4(c+dx)\sec(c+dx)\,dx + b^2\int \csc^3(c+d$$
$$= -\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{1}{4}\left(3a^2\right)\int \csc^3(c+dx)\,dx - \frac{(2ab)\,\mathrm{Subst}\left(\int \right.}{}$$
$$= -\frac{3a^2\cot(c+dx)\csc(c+dx)}{8d} - \frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{b^2\csc^2(c+dx)\mathrm{s}}{2d}$$
$$= -\frac{3a^2\tanh^{-1}(\cos(c+dx))}{8d} - \frac{2ab\csc(c+dx)}{d} - \frac{3a^2\cot(c+dx)\csc(c+dx)}{8d} -$$
$$= -\frac{3a^2\tanh^{-1}(\cos(c+dx))}{8d} - \frac{3b^2\tanh^{-1}(\cos(c+dx))}{2d} + \frac{2ab\tanh^{-1}(\sin(c+d}{d}$$

Mathematica [B] time = 6.19963, size = 994, normalized size = 6.02

$$-\frac{a^2\cos^2(c+dx)(a+b\tan(c+dx))^2\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{\left(-3a^2-4b^2\right)\cos^2(c+dx)(a+b\tan(c+dx))^2\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d(a\cos(c+dx)+b\sin(c+dx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b^2*Cos[c + d*x]^2*(a + b*Tan[c + d*x])^2)/(d*(a*Cos[c + d*x] + b*Sin[c +
d*x])^2) - (7*a*b*Cos[c + d*x]^2*Cot[(c + d*x)/2]*(a + b*Tan[c + d*x])^2)/(
6*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + ((-3*a^2 - 4*b^2)*Cos[c + d*x]^2
*Csc[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^2)/(32*d*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^2) - (a*b*Cos[c + d*x]^2*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(a +
b*Tan[c + d*x])^2)/(12*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (a^2*Cos[c
+ d*x]^2*Csc[(c + d*x)/2]^4*(a + b*Tan[c + d*x])^2)/(64*d*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^2) - (3*(a^2 + 4*b^2)*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2]]
*(a + b*Tan[c + d*x])^2)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (2*a*b
*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x
```

```
])^2)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (3*(a^2 + 4*b^2)*Cos[c + d*
x]^2*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^2)/(8*d*(a*Cos[c + d*x] + b
*Sin[c + d*x])^2) + (2*a*b*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2] + Sin[(c + d
*x)/2]]*(a + b*Tan[c + d*x])^2)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (
(3*a^2 + 4*b^2)*Cos[c + d*x]^2*Sec[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^2)/(
32*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (a^2*Cos[c + d*x]^2*Sec[(c + d*
x)/2]^4*(a + b*Tan[c + d*x])^2)/(64*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
+ (b^2*Cos[c + d*x]^2*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^2)/(d*(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (b^2*Co
s[c + d*x]^2*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^2)/(d*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (7*a*b*Cos[c + d
*x]^2*Tan[(c + d*x)/2]*(a + b*Tan[c + d*x])^2)/(6*d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2) - (a*b*Cos[c + d*x]^2*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]*(a
+ b*Tan[c + d*x])^2)/(12*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
```

Maple [A] time = 0.054, size = 183, normalized size = 1.1

$$-\frac{b^2}{2\,d\,(\sin(dx+c))^2\cos(dx+c)}+\frac{3\,b^2}{2\,d\cos(dx+c)}+\frac{3\,b^2\ln(\csc(dx+c)-\cot(dx+c))}{2\,d}-\frac{2\,ab}{3\,d\,(\sin(dx+c))^3}-2\,\frac{ab}{d\sin(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x)
```

```
[Out] -1/2/d*b^2/sin(d*x+c)^2/cos(d*x+c)+3/2/d*b^2/cos(d*x+c)+3/2/d*b^2*ln(csc(d*
x+c)-cot(d*x+c))-2/3/d*a*b/sin(d*x+c)^3-2/d*a*b/sin(d*x+c)+2/d*a*b*ln(sec(d
*x+c)+tan(d*x+c))-1/4*a^2*cot(d*x+c)*csc(d*x+c)^3/d-3/8*a^2*cot(d*x+c)*csc(
d*x+c)/d+3/8/d*a^2*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.07451, size = 252, normalized size = 1.53

$$\frac{3\,a^2\left(\frac{2\left(3\,\cos(dx+c)^3-5\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-3\,\log(\cos(dx+c)+1)+3\,\log(\cos(dx+c)-1)\right)+12\,b^2\left(\frac{2\left(3\,\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\,\log(\cos}{48\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*a^2*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(
d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 12*b
^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(cos(d*
x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 16*a*b*(2*(3*sin(d*x + c)^2 + 1)/s
in(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Fricas [B] time = 2.97415, size = 856, normalized size = 5.19

$$\frac{18\left(a^2+4b^2\right)\cos\left(dx+c\right)^4-30\left(a^2+4b^2\right)\cos\left(dx+c\right)^2+48b^2-9\left(\left(a^2+4b^2\right)\cos\left(dx+c\right)^5-2\left(a^2+4b^2\right)\cos\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(18*(a^2 + 4*b^2)*cos(d*x + c)^4 - 30*(a^2 + 4*b^2)*cos(d*x + c)^2 + 4
8*b^2 - 9*((a^2 + 4*b^2)*cos(d*x + c)^5 - 2*(a^2 + 4*b^2)*cos(d*x + c)^3 +
(a^2 + 4*b^2)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 9*((a^2 + 4*b^2)*
cos(d*x + c)^5 - 2*(a^2 + 4*b^2)*cos(d*x + c)^3 + (a^2 + 4*b^2)*cos(d*x + c
))*log(-1/2*cos(d*x + c) + 1/2) + 48*(a*b*cos(d*x + c)^5 - 2*a*b*cos(d*x +
c)^3 + a*b*cos(d*x + c))*log(sin(d*x + c) + 1) - 48*(a*b*cos(d*x + c)^5 - 2
*a*b*cos(d*x + c)^3 + a*b*cos(d*x + c))*log(-sin(d*x + c) + 1) + 32*(3*a*b*
cos(d*x + c)^3 - 4*a*b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5 - 2*d*
cos(d*x + c)^3 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.6524, size = 363, normalized size = 2.2

$$3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-16\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+384\,ab\log\left(\left|\tan\left(\frac{1}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 - 16*a*b*tan(1/2*d*x + 1/2*c)^3 + 24*a^
2*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 384*a*b*log(abs(
tan(1/2*d*x + 1/2*c) + 1)) - 384*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2
40*a*b*tan(1/2*d*x + 1/2*c) + 72*(a^2 + 4*b^2)*log(abs(tan(1/2*d*x + 1/2*c)
)) - 384*b^2/(tan(1/2*d*x + 1/2*c)^2 - 1) - (150*a^2*tan(1/2*d*x + 1/2*c)^4
 + 600*b^2*tan(1/2*d*x + 1/2*c)^4 + 240*a*b*tan(1/2*d*x + 1/2*c)^3 + 24*a^2
*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 16*a*b*tan(1/2*d*
x + 1/2*c) + 3*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

3.31 $\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=122

$$-\frac{(2a^2+b^2)\cot^3(c+dx)}{3d}-\frac{(a^2+2b^2)\cot(c+dx)}{d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{ab\cot^4(c+dx)}{2d}-\frac{2ab\cot^2(c+dx)}{d}+\frac{2ab\log(t$$

```
[Out] -(((a^2 + 2*b^2)*Cot[c + d*x])/d) - (2*a*b*Cot[c + d*x]^2)/d - ((2*a^2 + b^
2)*Cot[c + d*x]^3)/(3*d) - (a*b*Cot[c + d*x]^4)/(2*d) - (a^2*Cot[c + d*x]^5
)/(5*d) + (2*a*b*Log[Tan[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.101281, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 948}

$$-\frac{(2a^2+b^2)\cot^3(c+dx)}{3d}-\frac{(a^2+2b^2)\cot(c+dx)}{d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{ab\cot^4(c+dx)}{2d}-\frac{2ab\cot^2(c+dx)}{d}+\frac{2ab\log(t$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 + 2*b^2)*Cot[c + d*x])/d) - (2*a*b*Cot[c + d*x]^2)/d - ((2*a^2 + b^
2)*Cot[c + d*x]^3)/(3*d) - (a*b*Cot[c + d*x]^4)/(2*d) - (a^2*Cot[c + d*x]^5
)/(5*d) + (2*a*b*Log[Tan[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^2\left(b^2+x^2\right)^2}{x^6}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \left(1+\frac{a^2b^4}{x^6}+\frac{2ab^4}{x^5}+\frac{2a^2b^2+b^4}{x^4}+\frac{4ab^2}{x^3}+\frac{a^2+2b^2}{x^2}+\frac{2a}{x}\right)dx, x, b\tan(c+dx)\right)}{d}$$
$$= -\frac{\left(a^2+2b^2\right)\cot(c+dx)}{d} - \frac{2ab\cot^2(c+dx)}{d} - \frac{\left(2a^2+b^2\right)\cot^3(c+dx)}{3d} - \frac{ab\cot^{\cdots}}{\cdots}$$

Mathematica [A] time = 1.5103, size = 114, normalized size = 0.93

$$-\frac{2\cot(c+dx)\left(\left(4a^2+5b^2\right)\csc^2(c+dx)+3a^2\csc^4(c+dx)+8a^2+25b^2\right)+15b\left(a\csc^4(c+dx)+2a\csc^2(c+dx)-4a\,\text{lo}\cdots\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(2*Cot[c + d*x]*(8*a^2 + 25*b^2 + (4*a^2 + 5*b^2)*Csc[c + d*x]^2 + 3*a^2*C
sc[c + d*x]^4) + 15*b*(2*a*Csc[c + d*x]^2 + a*Csc[c + d*x]^4 + 4*a*Log[Cos[
c + d*x]] - 4*a*Log[Sin[c + d*x]] - 2*b*Tan[c + d*x]))/(30*d)
```

Maple [A] time = 0.055, size = 166, normalized size = 1.4

$$-\frac{b^2}{3d\left(\sin(dx+c)\right)^3\cos(dx+c)}+\frac{4b^2}{3d\sin(dx+c)\cos(dx+c)}-\frac{8b^2\cot(dx+c)}{3d}-\frac{ab}{2d\left(\sin(dx+c)\right)^4}-\frac{ab}{d\left(\sin(dx+c)\right)^2}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x)
```

```
[Out] -1/3/d*b^2/sin(d*x+c)^3/cos(d*x+c)+4/3/d*b^2/sin(d*x+c)/cos(d*x+c)-8/3/d*b^
2*cot(d*x+c)-1/2/d*a*b/sin(d*x+c)^4-1/d*a*b/sin(d*x+c)^2+2*a*b*ln(tan(d*x+c
))/d-8/15*a^2*cot(d*x+c)/d-1/5/d*a^2*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a^2*cot
(d*x+c)*csc(d*x+c)^2
```

Maxima [A] time = 1.03974, size = 140, normalized size = 1.15

$$\frac{60\,ab\log\left(\tan\left(dx+c\right)\right)+30\,b^{2}\tan\left(dx+c\right)-\frac{60\,ab\tan(dx+c)^{3}+30\left(a^{2}+2\,b^{2}\right)\tan(dx+c)^{4}+15\,ab\tan(dx+c)+10\left(2\,a^{2}+b^{2}\right)\tan(dx+c)^{2}+6\,a^{2}}{\tan(dx+c)^{5}}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(60*a*b*log(tan(d*x + c)) + 30*b^2*tan(d*x + c) - (60*a*b*tan(d*x + c)
^3 + 30*(a^2 + 2*b^2)*tan(d*x + c)^4 + 15*a*b*tan(d*x + c) + 10*(2*a^2 + b^
2)*tan(d*x + c)^2 + 6*a^2)/tan(d*x + c)^5)/d
```

Fricas [B] time = 2.42819, size = 628, normalized size = 5.15

$$-\frac{16\left(a^{2}+5\,b^{2}\right)\cos\left(dx+c\right)^{6}-40\left(a^{2}+5\,b^{2}\right)\cos\left(dx+c\right)^{4}+30\left(a^{2}+5\,b^{2}\right)\cos\left(dx+c\right)^{2}+30\left(ab\cos\left(dx+c\right)^{5}-2\,ab\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/30*(16*(a^2 + 5*b^2)*cos(d*x + c)^6 - 40*(a^2 + 5*b^2)*cos(d*x + c)^4 +
30*(a^2 + 5*b^2)*cos(d*x + c)^2 + 30*(a*b*cos(d*x + c)^5 - 2*a*b*cos(d*x +
c)^3 + a*b*cos(d*x + c))*log(cos(d*x + c)^2)*sin(d*x + c) - 30*(a*b*cos(d*x
 + c)^5 - 2*a*b*cos(d*x + c)^3 + a*b*cos(d*x + c))*log(-1/4*cos(d*x + c)^2
+ 1/4)*sin(d*x + c) - 30*b^2 - 15*(2*a*b*cos(d*x + c)^3 - 3*a*b*cos(d*x + c
))*sin(d*x + c))/((d*cos(d*x + c)^5 - 2*d*cos(d*x + c)^3 + d*cos(d*x + c))*
sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40844, size = 177, normalized size = 1.45

$$\frac{60\,ab\log\left(|\tan\left(dx+c\right)|\right)+30\,b^2\tan\left(dx+c\right)-\frac{137\,ab\tan(dx+c)^5+30\,a^2\tan(dx+c)^4+60\,b^2\tan(dx+c)^4+60\,ab\tan(dx+c)^3+20\,a^2\tan(dx+c)^2+10}{\tan(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(60*a*b*log(abs(tan(d*x + c))) + 30*b^2*tan(d*x + c) - (137*a*b*tan(d*
x + c)^5 + 30*a^2*tan(d*x + c)^4 + 60*b^2*tan(d*x + c)^4 + 60*a*b*tan(d*x +
 c)^3 + 20*a^2*tan(d*x + c)^2 + 10*b^2*tan(d*x + c)^2 + 15*a*b*tan(d*x + c)
 + 6*a^2)/tan(d*x + c)^5)/d
```

3.32 $\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=205

$$-\frac{a^2b\sin^3(c+dx)}{d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{a^3\cos(c+dx)}{d}-\frac{ab^2\cos^3(c+dx)}{d}$$

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (5*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cos[c + d*x])/d + (6*a*b^2*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d)
 - (a*b^2*Cos[c + d*x]^3)/d + (3*a*b^2*Sec[c + d*x])/d - (3*a^2*b*Sin[c + d
*x])/d + (5*b^3*Sin[c + d*x])/(2*d) - (a^2*b*Sin[c + d*x]^3)/d + (5*b^3*Sin
[c + d*x]^3)/(6*d) + (b^3*Sin[c + d*x]^3*Tan[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.188873, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3517, 2633, 2592, 302, 206, 2590, 270, 288}

$$-\frac{a^2b\sin^3(c+dx)}{d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{a^3\cos(c+dx)}{d}-\frac{ab^2\cos^3(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (5*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cos[c + d*x])/d + (6*a*b^2*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d)
 - (a*b^2*Cos[c + d*x]^3)/d + (3*a*b^2*Sec[c + d*x])/d - (3*a^2*b*Sin[c + d
*x])/d + (5*b^3*Sin[c + d*x])/(2*d) - (a^2*b*Sin[c + d*x]^3)/d + (5*b^3*Sin
[c + d*x]^3)/(6*d) + (b^3*Sin[c + d*x]^3*Tan[c + d*x]^2)/(2*d)
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a^3\sin^3(c+dx)+3a^2b\sin^3(c+dx)\tan(c+dx)+3ab^2\sin^3(c+dx)\tan^2(c$$

$$= a^3\int \sin^3(c+dx)\,dx + \left(3a^2b\right)\int \sin^3(c+dx)\tan(c+dx)\,dx + \left(3ab^2\right)\int \sin^3($$

$$= -\frac{a^3\,\mathrm{Subst}\left(\int\left(1-x^2\right)\,dx, x, \cos(c+dx)\right)}{d} + \frac{\left(3a^2b\right)\mathrm{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \sin(c$$

$$= -\frac{a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{3d} + \frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d} + \frac{\left(3a^2b\right)\mathrm{Su}$$

$$= -\frac{a^3\cos(c+dx)}{d} + \frac{6ab^2\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{3d} - \frac{ab^2\cos^3(c+dx)}{d} + \frac{3a$$

$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a^3\cos(c+dx)}{d} + \frac{6ab^2\cos(c+dx)}{d} + \frac{a^3\cos^3(c+a}{3d}$$

$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d} - \frac{5b^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{a^3\cos(c+dx)}{d} + \frac{6ab^2}{}$$

Mathematica [B] time = 6.26557, size = 771, normalized size = 3.76

$$-\frac{3a\left(a^2-7b^2\right)\cos^4(c+dx)(a+b\tan(c+dx))^3}{4d(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{b\left(3a^2-b^2\right)\sin(3(c+dx))\cos^3(c+dx)(a+b\tan(c+dx))^3}{12d(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{a\left(a^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3) - (3*a*(a^2 - 7*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3)/(4
*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (a*(a^2 - 3*b^2)*Cos[c + d*x]^3*C
os[3*(c + d*x)]*(a + b*Tan[c + d*x])^3)/(12*d*(a*Cos[c + d*x] + b*Sin[c + d
*x])^3) + ((-6*a^2*b + 5*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3
) + ((6*a^2*b - 5*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (b^
3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c + d*x)/2] - Sin[(c +
d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (3*a*b^2*Cos[c + d*x]^3*S
in[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (b^3*Cos[c + d*x]^3*(a + b*Tan
[c + d*x])^3)/(4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x]
```

```
+ b*Sin[c + d*x])^3) - (3*a*b^2*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[
c + d*x])^3)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*S
in[c + d*x])^3) - (3*b*(5*a^2 - 3*b^2)*Cos[c + d*x]^3*Sin[c + d*x]*(a + b*T
an[c + d*x])^3)/(4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (b*(3*a^2 - b^2
)*Cos[c + d*x]^3*Sin[3*(c + d*x)]*(a + b*Tan[c + d*x])^3)/(12*d*(a*Cos[c +
d*x] + b*Sin[c + d*x])^3)
```

Maple [A] time = 0.053, size = 271, normalized size = 1.3

$$\frac{b^3(\sin(dx+c))^7}{2d(\cos(dx+c))^2}+\frac{b^3(\sin(dx+c))^5}{2d}+\frac{5b^3(\sin(dx+c))^3}{6d}+\frac{5b^3\sin(dx+c)}{2d}-\frac{5b^3\ln(\sec(dx+c)+\tan(dx+c))}{2d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/2/d*b^3*sin(d*x+c)^7/cos(d*x+c)^2+1/2/d*b^3*sin(d*x+c)^5+5/6*b^3*sin(d*x+
c)^3/d+5/2*b^3*sin(d*x+c)/d-5/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))+3/d*a*b^2*s
in(d*x+c)^6/cos(d*x+c)+8*a*b^2*cos(d*x+c)/d+3/d*a*b^2*cos(d*x+c)*sin(d*x+c)
^4+4/d*cos(d*x+c)*sin(d*x+c)^2*a*b^2-a^2*b*sin(d*x+c)^3/d-3*a^2*b*sin(d*x+c
)/d+3/d*b*a^2*ln(sec(d*x+c)+tan(d*x+c))-1/3/d*cos(d*x+c)*sin(d*x+c)^2*a^3-2
/3*a^3*cos(d*x+c)/d
```

Maxima [A] time = 1.03738, size = 234, normalized size = 1.14

$$\frac{4\left(\cos(dx+c)^3-3\cos(dx+c)\right)a^3-6\left(2\sin(dx+c)^3-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)+6\sin(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^3 - 6*(2*sin(d*x + c)^3 - 3*log
(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*a^2*b - 12*(
cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a*b^2 + (4*sin(d*x + c)^3
 - 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(
sin(d*x + c) - 1) + 24*sin(d*x + c))*b^3)/d
```

Fricas [A] time = 2.39564, size = 455, normalized size = 2.22

$$\frac{4\left(a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{5}+36\,ab^{2}\cos\left(dx+c\right)-12\left(a^{3}-6\,ab^{2}\right)\cos\left(dx+c\right)^{3}+3\left(6\,a^{2}b-5\,b^{3}\right)\cos\left(dx+c\right)^{2}\log\left(\sin\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*(a^3 - 3*a*b^2)*cos(d*x + c)^5 + 36*a*b^2*cos(d*x + c) - 12*(a^3 -
6*a*b^2)*cos(d*x + c)^3 + 3*(6*a^2*b - 5*b^3)*cos(d*x + c)^2*log(sin(d*x +
c) + 1) - 3*(6*a^2*b - 5*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*
(3*a^2*b - b^3)*cos(d*x + c)^4 + 3*b^3 - 2*(12*a^2*b - 7*b^3)*cos(d*x + c)^
2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.33 $\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=103

$$-\frac{b\left(3a^2-2b^2\right)\log(\cos(c+dx))}{d}+\frac{1}{2}ax\left(a^2-9b^2\right)+\frac{9ab^2\tan(c+dx)}{2d}-\frac{\sin(c+dx)\cos(c+dx)(a+b\tan(c+dx))^3}{2d}+\frac{b^3}{}$$

```
[Out] (a*(a^2 - 9*b^2)*x)/2 - (b*(3*a^2 - 2*b^2)*Log[Cos[c + d*x]])/d + (9*a*b^2*
Tan[c + d*x])/(2*d) + (b^3*Tan[c + d*x]^2)/d - (Cos[c + d*x]*Sin[c + d*x]*(
a + b*Tan[c + d*x])^3)/(2*d)
```

Rubi [A] time = 0.144593, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1645, 801, 635, 203, 260}

$$-\frac{b\left(3a^2-2b^2\right)\log(\cos(c+dx))}{d}+\frac{1}{2}ax\left(a^2-9b^2\right)+\frac{9ab^2\tan(c+dx)}{2d}-\frac{\sin(c+dx)\cos(c+dx)(a+b\tan(c+dx))^3}{2d}+\frac{b^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(a^2 - 9*b^2)*x)/2 - (b*(3*a^2 - 2*b^2)*Log[Cos[c + d*x]])/d + (9*a*b^2*
Tan[c + d*x])/(2*d) + (b^3*Tan[c + d*x]^2)/d - (Cos[c + d*x]*Sin[c + d*x]*(
a + b*Tan[c + d*x])^3)/(2*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
```

```
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^2(a+x)^3}{\left(b^2+x^2\right)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{2d} - \frac{\text{Subst}\left(\int \frac{(a+x)^2\left(-ab^2-4b^2x\right)}{b^2+x^2}\,dx, x, b\right.}{2bd}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{2d} - \frac{\text{Subst}\left(\int \left(-9ab^2-4b^2x-\frac{ab^2(a^2}{}\right.\right.}{}$$

$$= \frac{9ab^2\tan(c+dx)}{2d} + \frac{b^3\tan^2(c+dx)}{d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx)}{2d}$$

$$= \frac{9ab^2\tan(c+dx)}{2d} + \frac{b^3\tan^2(c+dx)}{d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx)}{2d}$$

$$= \frac{1}{2}a\left(a^2-9b^2\right)x - \frac{b\left(3a^2-2b^2\right)\log(\cos(c+dx))}{d} + \frac{9ab^2\tan(c+dx)}{2d} + \frac{b^3\tan^2(}{d}$$

Mathematica [A] time = 4.22673, size = 203, normalized size = 1.97

$$\frac{b\left(-\frac{a\left(a^2-3b^2\right)\sin(2(c+dx))}{2b} + \left(3a^2-b^2\right)\cos^2(c+dx) - \frac{a\left(a^2-3b^2\right)\tan^{-1}(\tan(c+dx))}{b} + \left(\frac{a^3-6ab^2}{\sqrt{-b^2}} + 3a^2-2b^2\right)\log\left(\sqrt{-b^2}-b\tan(c+\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b*(-((a*(a^2 - 3*b^2)*ArcTan[Tan[c + d*x]])/b) + (3*a^2 - b^2)*Cos[c + d*x
]^2 + (3*a^2 - 2*b^2 + (a^3 - 6*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c
 + d*x]] + (3*a^2 - 2*b^2 + (-a^3 + 6*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b
*Tan[c + d*x]] - (a*(a^2 - 3*b^2)*Sin[2*(c + d*x)])/(2*b) + 6*a*b*Tan[c + d
*x] + b^2*Tan[c + d*x]^2))/(2*d)
```

Maple [B] time = 0.052, size = 226, normalized size = 2.2

$$\frac{b^3(\sin(dx+c))^6}{2d(\cos(dx+c))^2} + \frac{b^3(\sin(dx+c))^4}{2d} + \frac{(\sin(dx+c))^2b^3}{d} + 2\frac{b^3\ln(\cos(dx+c))}{d} + 3\frac{ab^2(\sin(dx+c))^5}{d\cos(dx+c)} + 3\frac{ab^2\cos(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/2/d*b^3*sin(d*x+c)^6/cos(d*x+c)^2+1/2/d*b^3*sin(d*x+c)^4+1/d*sin(d*x+c)^2
*b^3+2*b^3*ln(cos(d*x+c))/d+3/d*a*b^2*sin(d*x+c)^5/cos(d*x+c)+3/d*a*b^2*cos
(d*x+c)*sin(d*x+c)^3+9/2/d*sin(d*x+c)*cos(d*x+c)*a*b^2-9/2*a*b^2*x-9/2/d*a*
b^2*c-3/2/d*b*a^2*sin(d*x+c)^2-3/d*b*a^2*ln(cos(d*x+c))-1/2/d*a^3*sin(d*x+c
)*cos(d*x+c)+1/2*a^3*x+1/2/d*a^3*c
```

Maxima [A] time = 1.51541, size = 153, normalized size = 1.49

$$\frac{b^3\tan(dx+c)^2 + 6\,ab^2\tan(dx+c) + \left(a^3-9\,ab^2\right)(dx+c) + \left(3\,a^2b-2\,b^3\right)\log\left(\tan(dx+c)^2+1\right) + \frac{3\,a^2b-b^3-\left(a^3-3\,ab^2\right)\tan}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*tan(d*x + c)^2 + 6*a*b^2*tan(d*x + c) + (a^3 - 9*a*b^2)*(d*x + c)
+ (3*a^2*b - 2*b^3)*log(tan(d*x + c)^2 + 1) + (3*a^2*b - b^3 - (a^3 - 3*a*b
^2)*tan(d*x + c))/(tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 2.16828, size = 342, normalized size = 3.32

$$\frac{2\left(3\,a^2b-b^3\right)\cos(dx+c)^4 - 4\left(3\,a^2b-2\,b^3\right)\cos(dx+c)^2\log(-\cos(dx+c)) + 2\,b^3 - \left(3\,a^2b-b^3-2\left(a^3-9\,ab^2\right)dx\right)\mathrm{co}}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(3*a^2*b - b^3)*cos(d*x + c)^4 - 4*(3*a^2*b - 2*b^3)*cos(d*x + c)^2*
log(-cos(d*x + c)) + 2*b^3 - (3*a^2*b - b^3 - 2*(a^3 - 9*a*b^2)*d*x)*cos(d*
x + c)^2 + 2*(6*a*b^2*cos(d*x + c) - (a^3 - 3*a*b^2)*cos(d*x + c)^3)*sin(d*
x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\tan(c + dx))^3 \sin^2(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sin(c + d*x)**2, x)
```

Giac [B] time = 3.22714, size = 3497, normalized size = 33.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*a^3*d*x*tan(d*x)^4*tan(c)^4 - 18*a*b^2*d*x*tan(d*x)^4*tan(c)^4 - 6*a
^2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(
d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4
+ 4*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)
^4 + 2*a^3*d*x*tan(d*x)^4*tan(c)^2 - 18*a*b^2*d*x*tan(d*x)^4*tan(c)^2 - 4*a
^3*d*x*tan(d*x)^3*tan(c)^3 + 36*a*b^2*d*x*tan(d*x)^3*tan(c)^3 + 2*a^3*d*x*t
an(d*x)^2*tan(c)^4 - 18*a*b^2*d*x*tan(d*x)^2*tan(c)^4 + 3*a^2*b*tan(d*x)^4*
tan(c)^4 + b^3*tan(d*x)^4*tan(c)^4 - 6*a^2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)
^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*ta
n(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^2 + 4*b^3*log(4*(tan(c)^2 + 1)/(tan(d
*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2
*tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^2 + 12*a^2*b*log(4*(tan(c)^2 + 1)/
(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)
^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 - 8*b^3*log(4*(tan(c)^2 +
1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d
*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 + 2*a^3*tan(d*x)^4*tan(
c)^3 - 18*a*b^2*tan(d*x)^4*tan(c)^3 - 6*a^2*b*log(4*(tan(c)^2 + 1)/(tan(d*x
)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*t
an(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^4 + 4*b^3*log(4*(tan(c)^2 + 1)/(tan(
d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 -
2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^4 + 2*a^3*tan(d*x)^3*tan(c)^4 - 1
8*a*b^2*tan(d*x)^3*tan(c)^4 - 4*a^3*d*x*tan(d*x)^3*tan(c) + 36*a*b^2*d*x*ta
```

n(d*x)^3*tan(c) + 4*a^3*d*x*tan(d*x)^2*tan(c)^2 - 36*a*b^2*d*x*tan(d*x)^2*t
an(c)^2 - 3*a^2*b*tan(d*x)^4*tan(c)^2 + 5*b^3*tan(d*x)^4*tan(c)^2 - 4*a^3*d
*x*tan(d*x)*tan(c)^3 + 36*a*b^2*d*x*tan(d*x)*tan(c)^3 - 18*a^2*b*tan(d*x)^3
*tan(c)^3 + 6*b^3*tan(d*x)^3*tan(c)^3 - 3*a^2*b*tan(d*x)^2*tan(c)^4 + 5*b^3
*tan(d*x)^2*tan(c)^4 + 12*a^2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 -
 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
 + 1))*tan(d*x)^3*tan(c) - 8*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2
- 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c
) + 1))*tan(d*x)^3*tan(c) - 12*a*b^2*tan(d*x)^4*tan(c) - 12*a^2*b*log(4*(ta
n(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^
2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 8*b^3*log(4*
(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(
c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 - 6*a^3*tan
(d*x)^3*tan(c)^2 + 18*a*b^2*tan(d*x)^3*tan(c)^2 + 12*a^2*b*log(4*(tan(c)^2
+ 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan
(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)*tan(c)^3 - 8*b^3*log(4*(tan(c)^2
 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + ta
n(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)*tan(c)^3 - 6*a^3*tan(d*x)^2*tan
(c)^3 + 18*a*b^2*tan(d*x)^2*tan(c)^3 - 12*a*b^2*tan(d*x)*tan(c)^4 + 2*a^3*d
*x*tan(d*x)^2 - 18*a*b^2*d*x*tan(d*x)^2 + 2*b^3*tan(d*x)^4 - 4*a^3*d*x*tan(
d*x)*tan(c) + 36*a*b^2*d*x*tan(d*x)*tan(c) + 6*a^2*b*tan(d*x)^3*tan(c) - 2*
b^3*tan(d*x)^3*tan(c) + 2*a^3*d*x*tan(c)^2 - 18*a*b^2*d*x*tan(c)^2 + 30*a^2
*b*tan(d*x)^2*tan(c)^2 - 2*b^3*tan(d*x)^2*tan(c)^2 + 6*a^2*b*tan(d*x)*tan(c
)^3 - 2*b^3*tan(d*x)*tan(c)^3 + 2*b^3*tan(c)^4 - 6*a^2*b*log(4*(tan(c)^2 +
1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d
*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2 + 4*b^3*log(4*(tan(c)^2 + 1)/(ta
n(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2
- 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2 + 12*a*b^2*tan(d*x)^3 + 12*a^2*b*log(4
*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan
(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)*tan(c) - 8*b^3*log(4*
(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(
c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)*tan(c) + 6*a^3*tan(d*x
)^2*tan(c) - 18*a*b^2*tan(d*x)^2*tan(c) - 6*a^2*b*log(4*(tan(c)^2 + 1)/(tan
(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 -
 2*tan(d*x)*tan(c) + 1))*tan(c)^2 + 4*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*
tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d
*x)*tan(c) + 1))*tan(c)^2 + 6*a^3*tan(d*x)*tan(c)^2 - 18*a*b^2*tan(d*x)*tan
(c)^2 + 12*a*b^2*tan(c)^3 + 2*a^3*d*x - 18*a*b^2*d*x - 3*a^2*b*tan(d*x)^2 +
 5*b^3*tan(d*x)^2 - 18*a^2*b*tan(d*x)*tan(c) + 6*b^3*tan(d*x)*tan(c) - 3*a^
2*b*tan(c)^2 + 5*b^3*tan(c)^2 - 6*a^2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*ta
n(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x
)*tan(c) + 1)) + 4*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*
x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) -
2*a^3*tan(d*x) + 18*a*b^2*tan(d*x) - 2*a^3*tan(c) + 18*a*b^2*tan(c) + 3*a^2
*b + b^3)/(d*tan(d*x)^4*tan(c)^4 + d*tan(d*x)^4*tan(c)^2 - 2*d*tan(d*x)^3*t

```
an(c)^3 + d*tan(d*x)^2*tan(c)^4 - 2*d*tan(d*x)^3*tan(c) + 2*d*tan(d*x)^2*ta
n(c)^2 - 2*d*tan(d*x)*tan(c)^3 + d*tan(d*x)^2 - 2*d*tan(d*x)*tan(c) + d*tan
(c)^2 + d)
```

3.34 $\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=133

$$-\frac{3a^2b\sin(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cos(c+dx)}{d}+\frac{3ab^2\cos(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{3b^3\sin(c+d}{2d}$$

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (3*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cos[c + d*x])/d + (3*a*b^2*Cos[c + d*x])/d + (3*a*b^2*Sec[c + d*x])/d -
 (3*a^2*b*Sin[c + d*x])/d + (3*b^3*Sin[c + d*x])/(2*d) + (b^3*Sin[c + d*x]*
Tan[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.116482, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3517, 2638, 2592, 321, 206, 2590, 14, 288}

$$-\frac{3a^2b\sin(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cos(c+dx)}{d}+\frac{3ab^2\cos(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{3b^3\sin(c+d}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (3*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cos[c + d*x])/d + (3*a*b^2*Cos[c + d*x])/d + (3*a*b^2*Sec[c + d*x])/d -
 (3*a^2*b*Sin[c + d*x])/d + (3*b^3*Sin[c + d*x])/(2*d) + (b^3*Sin[c + d*x]*
Tan[c + d*x]^2)/(2*d)
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a^3\sin(c+dx)+3a^2b\sin(c+dx)\tan(c+dx)+3ab^2\sin(c+dx)\tan^2(c+dx\right.$$

$$= a^3\int \sin(c+dx)\,dx + \left(3a^2b\right)\int \sin(c+dx)\tan(c+dx)\,dx + \left(3ab^2\right)\int \sin(c+$$

$$= -\frac{a^3\cos(c+dx)}{d} + \frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} - \frac{\left(3ab^2\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{a^3\cos(c+dx)}{d} - \frac{3a^2b\sin(c+dx)}{d} + \frac{b^3\sin(c+dx)\tan^2(c+dx)}{2d} + \frac{\left(3a^2b\right)\text{Sub}}{}$$

$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a^3\cos(c+dx)}{d} + \frac{3ab^2\cos(c+dx)}{d} + \frac{3ab^2\sec(c+}{d}$$

$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d} - \frac{3b^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{a^3\cos(c+dx)}{d} + \frac{3ab^2\,\text{c}}{}$$

Mathematica [B] time = 6.1559, size = 637, normalized size = 4.79

$$-\frac{a\left(a^2-3b^2\right)\cos^4(c+dx)(a+b\tan(c+dx))^3}{d(a\cos(c+dx)+b\sin(c+dx))^3} - \frac{b\left(3a^2-b^2\right)\sin(c+dx)\cos^3(c+dx)(a+b\tan(c+dx))^3}{d(a\cos(c+dx)+b\sin(c+dx))^3} - \frac{3\left(2a^2b-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3) - (a*(a^2 - 3*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3)/(d*(
a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*(2*a^2*b - b^3)*Cos[c + d*x]^3*Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^3) + (3*(2*a^2*b - b^3)*Cos[c + d*x]^3*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x]
 + b*Sin[c + d*x])^3) + (b^3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
+ (3*a*b^2*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[
(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (b^
3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*a*b^2*Cos[c + d*x]^3*S
in[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (b*(3*a^2 - b^2)*Cos[c + d*x]^
3*Sin[c + d*x]*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])
```

```
^3)
```

Maple [A] time = 0.043, size = 193, normalized size = 1.5

$$\frac{b^3 (\sin(dx+c))^5}{2d(\cos(dx+c))^2} + \frac{b^3(\sin(dx+c))^3}{2d} + \frac{3b^3\sin(dx+c)}{2d} - \frac{3b^3\ln(\sec(dx+c)+\tan(dx+c))}{2d} + 3\frac{ab^2(\sin(dx+c))^4}{d\cos(dx+c)} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/2/d*b^3*sin(d*x+c)^5/cos(d*x+c)^2+1/2*b^3*sin(d*x+c)^3/d+3/2*b^3*sin(d*x+
c)/d-3/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))+3/d*a*b^2*sin(d*x+c)^4/cos(d*x+c)+
3/d*cos(d*x+c)*sin(d*x+c)^2*a*b^2+6*a*b^2*cos(d*x+c)/d-3*a^2*b*sin(d*x+c)/d
+3/d*b*a^2*ln(sec(d*x+c)+tan(d*x+c))-a^3*cos(d*x+c)/d
```

Maxima [A] time = 1.11825, size = 173, normalized size = 1.3

$$-\frac{b^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} + 3\log(\sin(dx+c)+1) - 3\log(\sin(dx+c)-1) - 4\sin(dx+c)\right) - 12ab^2\left(\frac{1}{\cos(dx+c)} + \cos(dx+c)\right) - }{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) -
3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 12*a*b^2*(1/cos(d*x + c) + cos(
d*x + c)) - 6*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(
d*x + c)) + 4*a^3*cos(d*x + c))/d
```

Fricas [A] time = 2.12391, size = 346, normalized size = 2.6

$$\frac{12ab^2\cos(dx+c) - 4\left(a^3 - 3ab^2\right)\cos(dx+c)^3 + 3\left(2a^2b - b^3\right)\cos(dx+c)^2\log(\sin(dx+c)+1) - 3\left(2a^2b - b^3\right)\cos(}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(12*a*b^2*cos(d*x + c) - 4*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 3*(2*a^2*b
- b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*(2*a^2*b - b^3)*cos(d*x + c
)^2*log(-sin(d*x + c) + 1) + 2*(b^3 - 2*(3*a^2*b - b^3)*cos(d*x + c)^2)*sin
(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\tan(c+dx))^3 \sin(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sin(c + d*x), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.35 $\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=86

$$\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}-\frac{b^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^3\tan(c+dx)\sec(c}{2d}$$

```
[Out] -((a^3*ArcTanh[Cos[c + d*x]])/d) + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (b^3
*ArcTanh[Sin[c + d*x]])/(2*d) + (3*a*b^2*Sec[c + d*x])/d + (b^3*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0856801, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3517, 3770, 2606, 8, 2611}

$$\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}-\frac{b^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^3\tan(c+dx)\sec(c}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a^3*ArcTanh[Cos[c + d*x]])/d) + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (b^3
*ArcTanh[Sin[c + d*x]])/(2*d) + (3*a*b^2*Sec[c + d*x])/d + (b^3*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a^3\csc(c+dx)+3a^2b\sec(c+dx)+3ab^2\sec(c+dx)\tan(c+dx)+b^3\sec(c$$

$$= a^3\int\csc(c+dx)\,dx+\left(3a^2b\right)\int\sec(c+dx)\,dx+\left(3ab^2\right)\int\sec(c+dx)\tan(c+$$

$$= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^3\sec(c+dx)\tan(c+}{2d}$$

$$= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^3\tanh^{-1}(\sin(c+dx))}{2d}$$

Mathematica [B] time = 2.25956, size = 241, normalized size = 2.8

$$-12a^2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a^2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)+4a^3\log\left(\sin\left(\frac{1}{2}(c+dx\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (12*a*b^2 - 4*a^3*Log[Cos[(c + d*x)/2]] - 12*a^2*b*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 2*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*a^3*L
og[Sin[(c + d*x)/2]] + 12*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] -
2*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^3/(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])^2 + 24*a*b^2*Sec[c + d*x]*Sin[(c + d*x)/2]^2 - b^3/(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^2)/(4*d)
```

Maple [A] time = 0.061, size = 125, normalized size = 1.5

$$\frac{b^3 (\sin(dx+c))^3}{2\,d\,(\cos(dx+c))^2} + \frac{b^3 \sin(dx+c)}{2\,d} - \frac{b^3 \ln(\sec(dx+c) + \tan(dx+c))}{2\,d} + 3\,\frac{ab^2}{d\cos(dx+c)} + 3\,\frac{ba^2 \ln(\sec(dx+c) + \tan}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/2/d*b^3*sin(d*x+c)^3/cos(d*x+c)^2+1/2*b^3*sin(d*x+c)/d-1/2/d*b^3*ln(sec(d
*x+c)+tan(d*x+c))+3/d*a*b^2/cos(d*x+c)+3/d*b*a^2*ln(sec(d*x+c)+tan(d*x+c))+
1/d*a^3*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.08484, size = 150, normalized size = 1.74

$$-\frac{b^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} + \log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)\right) - 6\,a^2b(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 4}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) - 6*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) -
1)) + 4*a^3*log(cot(d*x + c) + csc(d*x + c)) - 12*a*b^2/cos(d*x + c))/d
```

Fricas [A] time = 2.49657, size = 383, normalized size = 4.45

$$-\frac{2\,a^3\cos(dx+c)^2\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right) - 2\,a^3\cos(dx+c)^2\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right) - 12\,ab^2\cos(dx+c) - \left(6\,a^2b - \right.}{4\,d\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^3*cos(d*x + c)^2*log(1/2*cos(d*x + c) + 1/2) - 2*a^3*cos(d*x + c)
^2*log(-1/2*cos(d*x + c) + 1/2) - 12*a*b^2*cos(d*x + c) - (6*a^2*b - b^3)*c
```

```
os(d*x + c)^2*log(sin(d*x + c) + 1) + (6*a^2*b - b^3)*cos(d*x + c)^2*log(-s
in(d*x + c) + 1) - 2*b^3*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\tan(c + dx))^3 \csc(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x), x)
```

Giac [A] time = 2.06041, size = 194, normalized size = 2.26

$$\frac{2\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) + \left(6\,a^2b - b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - \left(6\,a^2b - b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(b^3\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + (6*a^2*b - b^3)*log(abs(tan(1/2
*d*x + 1/2*c) + 1)) - (6*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) +
2*(b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^2 + b^3*tan(1/
2*d*x + 1/2*c) + 6*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.36 $\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=64

$$\frac{3a^2b\log(\tan(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\tan^2(c+dx)}{2d}$$

```
[Out] -((a^3*Cot[c + d*x])/d) + (3*a^2*b*Log[Tan[c + d*x]])/d + (3*a*b^2*Tan[c +
d*x])/d + (b^3*Tan[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0525311, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 43}

$$\frac{3a^2b\log(\tan(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\tan^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a^3*Cot[c + d*x])/d) + (3*a^2*b*Log[Tan[c + d*x]])/d + (3*a*b^2*Tan[c +
d*x])/d + (b^3*Tan[c + d*x]^2)/(2*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^3}{x^2}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \left(3a+\frac{a^3}{x^2}+\frac{3a^2}{x}+x\right)dx, x, b\tan(c+dx)\right)}{d}$$
$$= -\frac{a^3\cot(c+dx)}{d} + \frac{3a^2b\log(\tan(c+dx))}{d} + \frac{3ab^2\tan(c+dx)}{d} + \frac{b^3\tan^2(c+dx)}{2d}$$

Mathematica [A] time = 0.975801, size = 126, normalized size = 1.97

$$-\frac{\csc(c+dx)\sec^2(c+dx)\left(3a\left(a^2-b^2\right)\cos(c+dx)+\left(a^3+3ab^2\right)\cos(3(c+dx))-2b\sin(c+dx)\left(3a^2\log(\sin(c+dx))\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(Csc[c + d*x]*Sec[c + d*x]^2*(3*a*(a^2 - b^2)*Cos[c + d*x] + (a^3 + 3*a*b^
2)*Cos[3*(c + d*x)] - 2*b*(b^2 - 3*a^2*Log[Cos[c + d*x]] - 3*a^2*Cos[2*(c +
 d*x)]*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]]) + 3*a^2*Log[Sin[c + d*x]])*S
in[c + d*x]))/(4*d)
```

Maple [A] time = 0.056, size = 63, normalized size = 1.

$$\frac{b^3}{2\,d\,(\cos(dx+c))^2}+3\,\frac{ab^2\tan(dx+c)}{d}+3\,\frac{ba^2\ln(\tan(dx+c))}{d}-\frac{a^3\cot(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/2/d*b^3/cos(d*x+c)^2+3*a*b^2*tan(d*x+c)/d+3*a^2*b*ln(tan(d*x+c))/d-a^3*co
t(d*x+c)/d
```

Maxima [A] time = 1.10142, size = 76, normalized size = 1.19

$$\frac{b^3\tan(dx+c)^2+6\,a^2b\log(\tan(dx+c))+6\,ab^2\tan(dx+c)-\frac{2\,a^3}{\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*tan(d*x + c)^2 + 6*a^2*b*log(tan(d*x + c)) + 6*a*b^2*tan(d*x + c)
- 2*a^3/tan(d*x + c))/d
```

Fricas [B] time = 2.04725, size = 327, normalized size = 5.11

$$-\frac{3\,a^2b\cos\left(dx+c\right)^2\log\left(\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)-3\,a^2b\cos\left(dx+c\right)^2\log\left(-\frac{1}{4}\,\cos\left(dx+c\right)^2+\frac{1}{4}\right)\sin\left(dx+c\right)-6\,ab^2\,\text{c}}{2\,d\cos\left(dx+c\right)^2\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^2*b*cos(d*x + c)^2*log(cos(d*x + c)^2)*sin(d*x + c) - 3*a^2*b*cos
(d*x + c)^2*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 6*a*b^2*cos(d*x +
 c) + 2*(a^3 + 3*a*b^2)*cos(d*x + c)^3 - b^3*sin(d*x + c))/(d*cos(d*x + c)^
2*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan\left(c+dx\right)\right)^3\csc^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**2, x)
```

Giac [A] time = 2.06722, size = 95, normalized size = 1.48

$$\frac{b^3\tan\left(dx+c\right)^2+6\,a^2b\log\left(\left|\tan\left(dx+c\right)\right|\right)+6\,ab^2\tan\left(dx+c\right)-\frac{2\left(3\,a^2b\tan(dx+c)+a^3\right)}{\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*tan(d*x + c)^2 + 6*a^2*b*log(abs(tan(d*x + c))) + 6*a*b^2*tan(d*x
+ c) - 2*(3*a^2*b*tan(d*x + c) + a^3)/tan(d*x + c))/d
```

3.37 $\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=141

$$-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}+\frac{3ab^2\sec(c+dx)}{d}$$

```
[Out] -(a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*b^2*ArcTanh[Cos[c + d*x]])/d + (3
*a^2*b*ArcTanh[Sin[c + d*x]])/d + (b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^
2*b*Csc[c + d*x])/d - (a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (3*a*b^2*Sec[
c + d*x])/d + (b^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.134904, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3517, 3768, 3770, 2621, 321, 207, 2622}

$$-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}+\frac{3ab^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```


```
[Out] -(a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*b^2*ArcTanh[Cos[c + d*x]])/d + (3
*a^2*b*ArcTanh[Sin[c + d*x]])/d + (b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^
2*b*Csc[c + d*x])/d - (a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (3*a*b^2*Sec[
c + d*x])/d + (b^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a^3\csc^3(c+dx) + 3a^2b\csc^2(c+dx)\sec(c+dx) + 3ab^2\csc(c+dx)\sec^2(c+
$$
$$= a^3\int \csc^3(c+dx)\,dx + \left(3a^2b\right)\int \csc^2(c+dx)\sec(c+dx)\,dx + \left(3ab^2\right)\int \csc(c+$$
$$= -\frac{a^3\cot(c+dx)\csc(c+dx)}{2d} + \frac{b^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a^3\int \csc(c+dx)\,$$
$$= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d} + \frac{b^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{3a^2b\csc(c+dx)}{d} - \frac{a^3\cot}{}$$
$$= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d} - \frac{3ab^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^2b\tanh^{-1}(\sin(c+dx}{d}$$

Mathematica [B] time = 6.18345, size = 897, normalized size = 6.36

$$-\frac{3a^2b\cos^3(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{3ab^2\cos^3(c+dx)(a+b\tan(c+dx))^3}{d(a\cos(c+dx)+b\sin(c+dx))^3} - \frac{a^3\cos^3(c+dx)\csc^2\left(\right.}{8d(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3) - (3*a^2*b*Cos[c + d*x]^3*Cot[(c + d*x)/2]*(a + b*Tan[c + d*x]
)^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a^3*Cos[c + d*x]^3*Csc[(c
 + d*x)/2]^2*(a + b*Tan[c + d*x])^3)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])
^3) + ((-a^3 - 6*a*b^2)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2]]*(a + b*Tan[c +
 d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + ((-6*a^2*b - b^3)*Cos
[c + d*x]^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3
)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + ((a^3 + 6*a*b^2)*Cos[c + d*x]
^3*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*S
in[c + d*x])^3) + ((6*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*
x])^3) + (a^3*Cos[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^3)/(8*
d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (b^3*Cos[c + d*x]^3*(a + b*Tan[c +
 d*x])^3)/(4*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*
Sin[c + d*x])^3) + (3*a*b^2*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c +
d*x])^3)/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^3) - (b^3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*a*b^2
*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x)/
```

```
2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*a^2*b*Cos[
c + d*x]^3*Tan[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] +
b*Sin[c + d*x])^3)
```

Maple [A] time = 0.065, size = 170, normalized size = 1.2

$$\frac{b^3\sec(dx+c)\tan(dx+c)}{2d}+\frac{b^3\ln(\sec(dx+c)+\tan(dx+c))}{2d}+3\frac{ab^2}{d\cos(dx+c)}+3\frac{ab^2\ln(\csc(dx+c)-\cot(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/2*b^3*sec(d*x+c)*tan(d*x+c)/d+1/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))+3/d*a*b
^2/cos(d*x+c)+3/d*a*b^2*ln(csc(d*x+c)-cot(d*x+c))-3/d*b*a^2/sin(d*x+c)+3/d*
b*a^2*ln(sec(d*x+c)+tan(d*x+c))-1/2*a^3*cot(d*x+c)*csc(d*x+c)/d+1/2/d*a^3*l
n(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.15234, size = 231, normalized size = 1.64

$$\frac{a^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-b^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log
(cos(d*x + c) - 1)) - b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a*b^2*(2/cos(d*x + c) - log(cos(d*
x + c) + 1) + log(cos(d*x + c) - 1)) - 6*a^2*b*(2/sin(d*x + c) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [B] time = 2.73967, size = 720, normalized size = 5.11

$$-\frac{12\,ab^2\cos(dx+c)-2\left(a^3+6\,ab^2\right)\cos(dx+c)^3+\left(\left(a^3+6\,ab^2\right)\cos(dx+c)^4-\left(a^3+6\,ab^2\right)\cos(dx+c)^2\right)\log\left(\frac{1}{2}\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(12*a*b^2*cos(d*x + c) - 2*(a^3 + 6*a*b^2)*cos(d*x + c)^3 + ((a^3 + 6*
a*b^2)*cos(d*x + c)^4 - (a^3 + 6*a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x + c
) + 1/2) - ((a^3 + 6*a*b^2)*cos(d*x + c)^4 - (a^3 + 6*a*b^2)*cos(d*x + c)^2
)*log(-1/2*cos(d*x + c) + 1/2) - ((6*a^2*b + b^3)*cos(d*x + c)^4 - (6*a^2*b
 + b^3)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((6*a^2*b + b^3)*cos(d*x +
c)^4 - (6*a^2*b + b^3)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(b^3 - (6
*a^2*b + b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4 - d*cos(d*x +
 c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.98513, size = 410, normalized size = 2.91

$$a^3 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 12\,a^2 b \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 4\left(6\,a^2 b + b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 4\left(6\,a^2 b + b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,d\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x + 1/2*c) + 4*(6*a^2*
b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 4*(6*a^2*b + b^3)*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 4*(a^3 + 6*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))
- (2*a^3*tan(1/2*d*x + 1/2*c)^6 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 12*a^2*
b*tan(1/2*d*x + 1/2*c)^5 - 8*b^3*tan(1/2*d*x + 1/2*c)^5 - 3*a^3*tan(1/2*d*x
```

```
 + 1/2*c)^4 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 24*a^2*b*tan(1/2*d*x + 1/2*
c)^3 - 8*b^3*tan(1/2*d*x + 1/2*c)^3 - 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 12*
a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1
/2*c))^2)/d
```

3.38 $\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=113

$$-\frac{a\left(a^2+3b^2\right)\cot(c+dx)}{d}+\frac{b\left(3a^2+b^2\right)\log(\tan(c+dx))}{d}-\frac{3a^2b\cot^2(c+dx)}{2d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{3ab^2\tan(c+dx)}{d}+$$

```
[Out] -((a*(a^2 + 3*b^2)*Cot[c + d*x])/d) - (3*a^2*b*Cot[c + d*x]^2)/(2*d) - (a^3
*Cot[c + d*x]^3)/(3*d) + (b*(3*a^2 + b^2)*Log[Tan[c + d*x]])/d + (3*a*b^2*T
an[c + d*x])/d + (b^3*Tan[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0860254, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 894}

$$-\frac{a\left(a^2+3b^2\right)\cot(c+dx)}{d}+\frac{b\left(3a^2+b^2\right)\log(\tan(c+dx))}{d}-\frac{3a^2b\cot^2(c+dx)}{2d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{3ab^2\tan(c+dx)}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a*(a^2 + 3*b^2)*Cot[c + d*x])/d) - (3*a^2*b*Cot[c + d*x]^2)/(2*d) - (a^3
*Cot[c + d*x]^3)/(3*d) + (b*(3*a^2 + b^2)*Log[Tan[c + d*x]])/d + (3*a*b^2*T
an[c + d*x])/d + (b^3*Tan[c + d*x]^2)/(2*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^3(b^2+x^2)}{x^4}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \left(3a+\frac{a^3b^2}{x^4}+\frac{3a^2b^2}{x^3}+\frac{a^3+3ab^2}{x^2}+\frac{3a^2+b^2}{x}+x\right)dx, x, b\tan(c+dx)\right)}{d}$$
$$= -\frac{a\left(a^2+3b^2\right)\cot(c+dx)}{d}-\frac{3a^2b\cot^2(c+dx)}{2d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{b\left(3a^2+b^2\right.}{}$$

Mathematica [A] time = 2.20723, size = 212, normalized size = 1.88

$$\frac{\sec^2(c+dx)(a\cot(c+dx)+b)^3\left(-2\sin(c+dx)\left(6\left(3a^2b+b^3\right)\cos(2(c+dx))-3b\left(3a^2+b^2\right)\cos(4(c+dx))(\log(\cos(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((b + a*Cot[c + d*x])^3*Sec[c + d*x]^2*(-16*a^3*Cos[c + d*x] - 2*Sin[c + d*
x]*(18*a^2*b - 6*b^3 + 6*(3*a^2*b + b^3)*Cos[2*(c + d*x)] + 9*a^2*b*Log[Cos
[c + d*x]] + 3*b^3*Log[Cos[c + d*x]] - 3*b*(3*a^2 + b^2)*Cos[4*(c + d*x)]*(
Log[Cos[c + d*x]] - Log[Sin[c + d*x]]) - 9*a^2*b*Log[Sin[c + d*x]] - 3*b^3*
Log[Sin[c + d*x]] + 2*a^3*Sin[4*(c + d*x)] + 18*a*b^2*Sin[4*(c + d*x)])))/(
48*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
```

Maple [A] time = 0.068, size = 141, normalized size = 1.3

$$\frac{b^3}{2\,d\left(\cos(dx+c)\right)^2}+\frac{b^3\ln(\tan(dx+c))}{d}+3\,\frac{ab^2}{d\sin(dx+c)\cos(dx+c)}-6\,\frac{ab^2\cot(dx+c)}{d}-\frac{3\,ba^2}{2\,d\left(\sin(dx+c)\right)^2}+3\,\frac{ba}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/2/d*b^3/cos(d*x+c)^2+1/d*b^3*ln(tan(d*x+c))+3/d*a*b^2/sin(d*x+c)/cos(d*x+
c)-6/d*a*b^2*cot(d*x+c)-3/2/d*b*a^2/sin(d*x+c)^2+3*a^2*b*ln(tan(d*x+c))/d-2
/3*a^3*cot(d*x+c)/d-1/3/d*a^3*cot(d*x+c)*csc(d*x+c)^2
```

Maxima [A] time = 1.0644, size = 132, normalized size = 1.17

$$\frac{3\,b^3\tan\left(dx+c\right)^2+18\,ab^2\tan\left(dx+c\right)+6\left(3\,a^2b+b^3\right)\log\left(\tan\left(dx+c\right)\right)-\frac{9\,a^2b\tan(dx+c)+2\,a^3+6\left(a^3+3\,ab^2\right)\tan(dx+c)^2}{\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*b^3*tan(d*x + c)^2 + 18*a*b^2*tan(d*x + c) + 6*(3*a^2*b + b^3)*log(t
an(d*x + c)) - (9*a^2*b*tan(d*x + c) + 2*a^3 + 6*(a^3 + 3*a*b^2)*tan(d*x +
c)^2)/tan(d*x + c)^3)/d
```

Fricas [B] time = 2.01836, size = 575, normalized size = 5.09

$$-\frac{4\left(a^3+9\,ab^2\right)\cos\left(dx+c\right)^5+18\,ab^2\cos\left(dx+c\right)-6\left(a^3+9\,ab^2\right)\cos\left(dx+c\right)^3+3\left(\left(3\,a^2b+b^3\right)\cos\left(dx+c\right)^4-\left(3\,a^2b+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(4*(a^3 + 9*a*b^2)*cos(d*x + c)^5 + 18*a*b^2*cos(d*x + c) - 6*(a^3 + 9
*a*b^2)*cos(d*x + c)^3 + 3*((3*a^2*b + b^3)*cos(d*x + c)^4 - (3*a^2*b + b^3
)*cos(d*x + c)^2)*log(cos(d*x + c)^2)*sin(d*x + c) - 3*((3*a^2*b + b^3)*cos
(d*x + c)^4 - (3*a^2*b + b^3)*cos(d*x + c)^2)*log(-1/4*cos(d*x + c)^2 + 1/4
)*sin(d*x + c) + 3*(b^3 - (3*a^2*b + b^3)*cos(d*x + c)^2)*sin(d*x + c))/((d
*cos(d*x + c)^4 - d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**3,x)
```

[Out] Timed out

Giac [A] time = 2.04811, size = 180, normalized size = 1.59

$$\frac{3\,b^{3}\tan\left(dx+c\right)^{2}+18\,ab^{2}\tan\left(dx+c\right)+6\left(3\,a^{2}b+b^{3}\right)\log\left(\left|\tan\left(dx+c\right)\right|\right)-\frac{33\,a^{2}b\tan(dx+c)^{3}+11\,b^{3}\tan(dx+c)^{3}+6\,a^{3}\tan(dx+c)^{2}+}{\tan(dx+c)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*b^3*tan(d*x + c)^2 + 18*a*b^2*tan(d*x + c) + 6*(3*a^2*b + b^3)*log(a
bs(tan(d*x + c))) - (33*a^2*b*tan(d*x + c)^3 + 11*b^3*tan(d*x + c)^3 + 6*a^
3*tan(d*x + c)^2 + 18*a*b^2*tan(d*x + c)^2 + 9*a^2*b*tan(d*x + c) + 2*a^3)/
tan(d*x + c)^3)/d
```

3.39 $\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=229

$$-\frac{a^2b\csc^3(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3\cot(c+dx)\csc^3(c+a}{4d}$$

```
[Out] (-3*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (9*a*b^2*ArcTanh[Cos[c + d*x]])/(2*d
) + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (3*b^3*ArcTanh[Sin[c + d*x]])/(2*d)
 - (3*a^2*b*Csc[c + d*x])/d - (3*b^3*Csc[c + d*x])/(2*d) - (3*a^3*Cot[c + d
*x]*Csc[c + d*x])/(8*d) - (a^2*b*Csc[c + d*x]^3)/d - (a^3*Cot[c + d*x]*Csc[
c + d*x]^3)/(4*d) + (9*a*b^2*Sec[c + d*x])/(2*d) - (3*a*b^2*Csc[c + d*x]^2*
Sec[c + d*x])/(2*d) + (b^3*Csc[c + d*x]*Sec[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.208038, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3517, 3768, 3770, 2621, 302, 207, 2622, 288, 321}

$$-\frac{a^2b\csc^3(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3\cot(c+dx)\csc^3(c+a}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```


```
[Out] (-3*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (9*a*b^2*ArcTanh[Cos[c + d*x]])/(2*d
) + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (3*b^3*ArcTanh[Sin[c + d*x]])/(2*d)
 - (3*a^2*b*Csc[c + d*x])/d - (3*b^3*Csc[c + d*x])/(2*d) - (3*a^3*Cot[c + d
*x]*Csc[c + d*x])/(8*d) - (a^2*b*Csc[c + d*x]^3)/d - (a^3*Cot[c + d*x]*Csc[
c + d*x]^3)/(4*d) + (9*a*b^2*Sec[c + d*x])/(2*d) - (3*a*b^2*Csc[c + d*x]^2*
Sec[c + d*x])/(2*d) + (b^3*Csc[c + d*x]*Sec[c + d*x]^2)/(2*d)
```


Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a^3\csc^5(c+dx)+3a^2b\csc^4(c+dx)\sec(c+dx)+3ab^2\csc^3(c+dx)\sec^2(c+\right.$$

$$= a^3\int \csc^5(c+dx)\,dx+\left(3a^2b\right)\int \csc^4(c+dx)\sec(c+dx)\,dx+\left(3ab^2\right)\int \csc^3(c$$

$$= -\frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{1}{4}\left(3a^3\right)\int \csc^3(c+dx)\,dx-\frac{\left(3a^2b\right)\mathrm{Subst}\left(\int \frac{}{-1}\right.}{}$$

$$= -\frac{3a^3\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3ab^2\csc^2(c+dx)}{2d}$$

$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{3a^2b\csc(c+dx)}{d}-\frac{3b^3\csc(c+dx)}{2d}-\frac{3a^3\cot(c+}{8}$$

$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{9ab^2\tanh^{-1}(\cos(c+dx))}{2d}+\frac{3a^2b\tanh^{-1}(\sin(c+}{d}$$

Mathematica [B] time = 6.20045, size = 1229, normalized size = 5.37

$$-\frac{a^3\cos^3(c+dx)(a+b\tan(c+dx))^3\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{3\left(a^3+4b^2a\right)\cos^3(c+dx)(a+b\tan(c+dx))^3\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d(a\cos(c+dx)+b\sin(c+dx))^3}-$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3) + ((-7*a^2*b*Cos[(c + d*x)/2] - 2*b^3*Cos[(c + d*x)/2])*Cos[c
+ d*x]^3*Csc[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(4*d*(a*Cos[c + d*x] + b*
Sin[c + d*x])^3) - (3*(a^3 + 4*a*b^2)*Cos[c + d*x]^3*Csc[(c + d*x)/2]^2*(a
+ b*Tan[c + d*x])^3)/(32*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a^2*b*Co
s[c + d*x]^3*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^3)/(8
*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a^3*Cos[c + d*x]^3*Csc[(c + d*x)
```

```
/2]^4*(a + b*Tan[c + d*x])^3)/(64*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) -
(3*(a^3 + 12*a*b^2)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2]]*(a + b*Tan[c + d*x
])^3)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*(2*a^2*b + b^3)*Cos[c
+ d*x]^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(
2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (3*(a^3 + 12*a*b^2)*Cos[c + d*x]
^3*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(8*d*(a*Cos[c + d*x] + b*S
in[c + d*x])^3) + (3*(2*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c +
d*x])^3) + (3*(a^3 + 4*a*b^2)*Cos[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + b*Tan[
c + d*x])^3)/(32*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (a^3*Cos[c + d*x]
^3*Sec[(c + d*x)/2]^4*(a + b*Tan[c + d*x])^3)/(64*d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^3) + (b^3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c +
d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (3*a*b
^2*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (b^3*Cos[c
+ d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*a*b^2*Cos[c + d*x]^3*Sin[(c +
d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a
*Cos[c + d*x] + b*Sin[c + d*x])^3) + (Cos[c + d*x]^3*Sec[(c + d*x)/2]*(-7*a
^2*b*Sin[(c + d*x)/2] - 2*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^3)/(4*
d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a^2*b*Cos[c + d*x]^3*Sec[(c + d*x
)/2]^2*Tan[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(8*d*(a*Cos[c + d*x] + b*Si
n[c + d*x])^3)
```

Maple [A] time = 0.068, size = 254, normalized size = 1.1

$$\frac{b^3}{2\,d\sin(dx+c)(\cos(dx+c))^2}-\frac{3\,b^3}{2\,d\sin(dx+c)}+\frac{3\,b^3\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}-\frac{3\,ab^2}{2\,d\,(\sin(dx+c))^2\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/2/d*b^3/sin(d*x+c)/cos(d*x+c)^2-3/2/d*b^3/sin(d*x+c)+3/2/d*b^3*ln(sec(d*x
+c)+tan(d*x+c))-3/2/d*a*b^2/sin(d*x+c)^2/cos(d*x+c)+9/2/d*a*b^2/cos(d*x+c)+
9/2/d*a*b^2*ln(csc(d*x+c)-cot(d*x+c))-1/d*b*a^2/sin(d*x+c)^3-3/d*b*a^2/sin(
d*x+c)+3/d*b*a^2*ln(sec(d*x+c)+tan(d*x+c))-1/4*a^3*cot(d*x+c)*csc(d*x+c)^3/
d-3/8*a^3*cot(d*x+c)*csc(d*x+c)/d+3/8/d*a^3*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.1056, size = 338, normalized size = 1.48

$$a^3\left(\frac{2\left(3\cos(dx+c)^3-5\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)+12\,ab^2\left(\frac{2\left(3\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\log(\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(a^3*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*
x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 12*a*b
^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(cos(d*
x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 4*b^3*(2*(3*sin(d*x + c)^2 - 2)/(s
in(d*x + c)^3 - sin(d*x + c)) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 8*a^2*b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Fricas [B] time = 2.95534, size = 1060, normalized size = 4.63

$$6\left(a^3+12\,ab^2\right)\cos(dx+c)^5+48\,ab^2\cos(dx+c)-10\left(a^3+12\,ab^2\right)\cos(dx+c)^3-3\left(\left(a^3+12\,ab^2\right)\cos(dx+c)^6-2\left(a^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(a^3 + 12*a*b^2)*cos(d*x + c)^5 + 48*a*b^2*cos(d*x + c) - 10*(a^3 +
 12*a*b^2)*cos(d*x + c)^3 - 3*((a^3 + 12*a*b^2)*cos(d*x + c)^6 - 2*(a^3 + 1
2*a*b^2)*cos(d*x + c)^4 + (a^3 + 12*a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x
+ c) + 1/2) + 3*((a^3 + 12*a*b^2)*cos(d*x + c)^6 - 2*(a^3 + 12*a*b^2)*cos(d
*x + c)^4 + (a^3 + 12*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) +
 12*((2*a^2*b + b^3)*cos(d*x + c)^6 - 2*(2*a^2*b + b^3)*cos(d*x + c)^4 + (2
*a^2*b + b^3)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 12*((2*a^2*b + b^3)*c
os(d*x + c)^6 - 2*(2*a^2*b + b^3)*cos(d*x + c)^4 + (2*a^2*b + b^3)*cos(d*x
+ c)^2)*log(-sin(d*x + c) + 1) + 8*(3*(2*a^2*b + b^3)*cos(d*x + c)^4 + b^3
- 4*(2*a^2*b + b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^6 - 2*d*c
os(d*x + c)^4 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.14968, size = 504, normalized size = 2.2

$$a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 - 8a^2b\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 8a^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 24ab^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 120a^2b\tan\left(\frac{1}{2}dx + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(a^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 8*a^3*t
an(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 120*a^2*b*tan(1/2
*d*x + 1/2*c) - 32*b^3*tan(1/2*d*x + 1/2*c) + 96*(2*a^2*b + b^3)*log(abs(ta
n(1/2*d*x + 1/2*c) + 1)) - 96*(2*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + 24*(a^3 + 12*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + 64*(b^3*tan(1/
2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^2 + b^3*tan(1/2*d*x + 1/2*c
) + 6*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - (50*a^3*tan(1/2*d*x + 1/2*c)^
4 + 600*a*b^2*tan(1/2*d*x + 1/2*c)^4 + 120*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 3
2*b^3*tan(1/2*d*x + 1/2*c)^3 + 8*a^3*tan(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(
1/2*d*x + 1/2*c)^2 + 8*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*
c)^4)/d
```

3.40 $\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=167

$$-\frac{a\left(2a^2+3b^2\right)\cot^3(c+dx)}{3d}-\frac{b\left(6a^2+b^2\right)\cot^2(c+dx)}{2d}-\frac{a\left(a^2+6b^2\right)\cot(c+dx)}{d}+\frac{b\left(3a^2+2b^2\right)\log(\tan(c+dx))}{d}-3$$

```
[Out] -((a*(a^2 + 6*b^2)*Cot[c + d*x])/d) - (b*(6*a^2 + b^2)*Cot[c + d*x]^2)/(2*d
) - (a*(2*a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*d) - (3*a^2*b*Cot[c + d*x]^4)/(4*
d) - (a^3*Cot[c + d*x]^5)/(5*d) + (b*(3*a^2 + 2*b^2)*Log[Tan[c + d*x]])/d +
 (3*a*b^2*Tan[c + d*x])/d + (b^3*Tan[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.134132, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 948}

$$-\frac{a\left(2a^2+3b^2\right)\cot^3(c+dx)}{3d}-\frac{b\left(6a^2+b^2\right)\cot^2(c+dx)}{2d}-\frac{a\left(a^2+6b^2\right)\cot(c+dx)}{d}+\frac{b\left(3a^2+2b^2\right)\log(\tan(c+dx))}{d}-3$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a*(a^2 + 6*b^2)*Cot[c + d*x])/d) - (b*(6*a^2 + b^2)*Cot[c + d*x]^2)/(2*d
) - (a*(2*a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*d) - (3*a^2*b*Cot[c + d*x]^4)/(4*
d) - (a^3*Cot[c + d*x]^5)/(5*d) + (b*(3*a^2 + 2*b^2)*Log[Tan[c + d*x]])/d +
 (3*a*b^2*Tan[c + d*x])/d + (b^3*Tan[c + d*x]^2)/(2*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^3\left(b^2+x^2\right)^2}{x^6}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \left(3a+\frac{a^3b^4}{x^6}+\frac{3a^2b^4}{x^5}+\frac{2a^3b^2+3ab^4}{x^4}+\frac{6a^2b^2+b^4}{x^3}+\frac{a^3+6ab^2}{x^2}+\frac{3a^2+2b^2}{x}+x\right)\right.}{d}$$
$$= -\frac{a\left(a^2+6b^2\right)\cot(c+dx)}{d}-\frac{b\left(6a^2+b^2\right)\cot^2(c+dx)}{2d}-\frac{a\left(2a^2+3b^2\right)\cot^3(c+}{3d}$$

Mathematica [B] time = 1.75822, size = 515, normalized size = 3.08

$$-\frac{\csc^5(c+dx)\sec^2(c+dx)\left(40a\left(5a^2+3b^2\right)\cos(c+dx)+8\left(a^3+15ab^2\right)\cos(3(c+dx))+360a^2b\sin(c+dx)+270a^2b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(Csc[c + d*x]^5*Sec[c + d*x]^2*(40*a*(5*a^2 + 3*b^2)*Cos[c + d*x] + 8*(a^3
 + 15*a*b^2)*Cos[3*(c + d*x)] - 24*a^3*Cos[5*(c + d*x)] - 360*a*b^2*Cos[5*(
c + d*x)] + 8*a^3*Cos[7*(c + d*x)] + 120*a*b^2*Cos[7*(c + d*x)] + 360*a^2*b
*Sin[c + d*x] - 240*b^3*Sin[c + d*x] + 225*a^2*b*Log[Cos[c + d*x]]*Sin[c +
d*x] + 150*b^3*Log[Cos[c + d*x]]*Sin[c + d*x] - 225*a^2*b*Log[Sin[c + d*x]]
*Sin[c + d*x] - 150*b^3*Log[Sin[c + d*x]]*Sin[c + d*x] + 270*a^2*b*Sin[3*(c
 + d*x)] + 180*b^3*Sin[3*(c + d*x)] + 45*a^2*b*Log[Cos[c + d*x]]*Sin[3*(c +
 d*x)] + 30*b^3*Log[Cos[c + d*x]]*Sin[3*(c + d*x)] - 45*a^2*b*Log[Sin[c + d
*x]]*Sin[3*(c + d*x)] - 30*b^3*Log[Sin[c + d*x]]*Sin[3*(c + d*x)] - 90*a^2*
b*Sin[5*(c + d*x)] - 60*b^3*Sin[5*(c + d*x)] - 135*a^2*b*Log[Cos[c + d*x]]*
Sin[5*(c + d*x)] - 90*b^3*Log[Cos[c + d*x]]*Sin[5*(c + d*x)] + 135*a^2*b*Lo
g[Sin[c + d*x]]*Sin[5*(c + d*x)] + 90*b^3*Log[Sin[c + d*x]]*Sin[5*(c + d*x)
] + 45*a^2*b*Log[Cos[c + d*x]]*Sin[7*(c + d*x)] + 30*b^3*Log[Cos[c + d*x]]*
Sin[7*(c + d*x)] - 45*a^2*b*Log[Sin[c + d*x]]*Sin[7*(c + d*x)] - 30*b^3*Log
[Sin[c + d*x]]*Sin[7*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.069, size = 230, normalized size = 1.4

$$\frac{b^3}{2d(\sin(dx+c))^2(\cos(dx+c))^2}-\frac{b^3}{d(\sin(dx+c))^2}+2\frac{b^3\ln(\tan(dx+c))}{d}-\frac{ab^2}{d(\sin(dx+c))^3\cos(dx+c)}+4\frac{}{d\sin(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/2/d*b^3/sin(d*x+c)^2/cos(d*x+c)^2-1/d*b^3/sin(d*x+c)^2+2/d*b^3*ln(tan(d*x
+c))-1/d*a*b^2/sin(d*x+c)^3/cos(d*x+c)+4/d*a*b^2/sin(d*x+c)/cos(d*x+c)-8/d*
a*b^2*cot(d*x+c)-3/4/d*b*a^2/sin(d*x+c)^4-3/2/d*b*a^2/sin(d*x+c)^2+3*a^2*b*
ln(tan(d*x+c))/d-8/15*a^3*cot(d*x+c)/d-1/5/d*a^3*cot(d*x+c)*csc(d*x+c)^4-4/
15/d*a^3*cot(d*x+c)*csc(d*x+c)^2
```

Maxima [A] time = 1.11848, size = 192, normalized size = 1.15

$$\frac{30\,b^3\tan\left(dx+c\right)^2+180\,ab^2\tan\left(dx+c\right)+60\left(3\,a^2b+2\,b^3\right)\log\left(\tan\left(dx+c\right)\right)-\frac{60\left(a^3+6\,ab^2\right)\tan(dx+c)^4+45\,a^2b\tan(dx+c)+30\left(6\,a\right.}{\tan(}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(30*b^3*tan(d*x + c)^2 + 180*a*b^2*tan(d*x + c) + 60*(3*a^2*b + 2*b^3)
*log(tan(d*x + c)) - (60*(a^3 + 6*a*b^2)*tan(d*x + c)^4 + 45*a^2*b*tan(d*x
+ c) + 30*(6*a^2*b + b^3)*tan(d*x + c)^3 + 12*a^3 + 20*(2*a^3 + 3*a*b^2)*ta
n(d*x + c)^2)/tan(d*x + c)^5)/d
```

Fricas [B] time = 2.28379, size = 834, normalized size = 4.99

$$-\frac{32\left(a^3+15\,ab^2\right)\cos\left(dx+c\right)^7-80\left(a^3+15\,ab^2\right)\cos\left(dx+c\right)^5-180\,ab^2\cos\left(dx+c\right)+60\left(a^3+15\,ab^2\right)\cos\left(dx+c\right)^3+3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(32*(a^3 + 15*a*b^2)*cos(d*x + c)^7 - 80*(a^3 + 15*a*b^2)*cos(d*x + c
)^5 - 180*a*b^2*cos(d*x + c) + 60*(a^3 + 15*a*b^2)*cos(d*x + c)^3 + 30*((3*
a^2*b + 2*b^3)*cos(d*x + c)^6 - 2*(3*a^2*b + 2*b^3)*cos(d*x + c)^4 + (3*a^2
*b + 2*b^3)*cos(d*x + c)^2)*log(cos(d*x + c)^2)*sin(d*x + c) - 30*((3*a^2*b
 + 2*b^3)*cos(d*x + c)^6 - 2*(3*a^2*b + 2*b^3)*cos(d*x + c)^4 + (3*a^2*b +
```

```
2*b^3)*cos(d*x + c)^2)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 15*(2*
(3*a^2*b + 2*b^3)*cos(d*x + c)^4 + 2*b^3 - 3*(3*a^2*b + 2*b^3)*cos(d*x + c)
^2)*sin(d*x + c))/((d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^4 + d*cos(d*x + c)^
2)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.03189, size = 255, normalized size = 1.53

$$\frac{30\,b^3\tan\left(dx+c\right)^2+180\,ab^2\tan\left(dx+c\right)+60\left(3\,a^2b+2\,b^3\right)\log\left(\left|\tan\left(dx+c\right)\right|\right)-\frac{411\,a^2b\tan(dx+c)^5+274\,b^3\tan(dx+c)^5+60\,a^3\,\mathrm{ta}}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(30*b^3*tan(d*x + c)^2 + 180*a*b^2*tan(d*x + c) + 60*(3*a^2*b + 2*b^3)
*log(abs(tan(d*x + c))) - (411*a^2*b*tan(d*x + c)^5 + 274*b^3*tan(d*x + c)^
5 + 60*a^3*tan(d*x + c)^4 + 360*a*b^2*tan(d*x + c)^4 + 180*a^2*b*tan(d*x +
c)^3 + 30*b^3*tan(d*x + c)^3 + 40*a^3*tan(d*x + c)^2 + 60*a*b^2*tan(d*x + c
)^2 + 45*a^2*b*tan(d*x + c) + 12*a^3)/tan(d*x + c)^5)/d
```

3.41 $\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=275

$$-\frac{2a^2b^2\cos^3(c+dx)}{d}+\frac{12a^2b^2\cos(c+dx)}{d}+\frac{6a^2b^2\sec(c+dx)}{d}-\frac{4a^3b\sin^3(c+dx)}{3d}-\frac{4a^3b\sin(c+dx)}{d}+\frac{4a^3b\tanh^{-1}(\mathrm{si}}{d}$$

```
[Out] (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (10*a*b^3*ArcTanh[Sin[c + d*x]])/d - (a
^4*Cos[c + d*x])/d + (12*a^2*b^2*Cos[c + d*x])/d - (3*b^4*Cos[c + d*x])/d +
 (a^4*Cos[c + d*x]^3)/(3*d) - (2*a^2*b^2*Cos[c + d*x]^3)/d + (b^4*Cos[c + d
*x]^3)/(3*d) + (6*a^2*b^2*Sec[c + d*x])/d - (3*b^4*Sec[c + d*x])/d + (b^4*S
ec[c + d*x]^3)/(3*d) - (4*a^3*b*Sin[c + d*x])/d + (10*a*b^3*Sin[c + d*x])/d
 - (4*a^3*b*Sin[c + d*x]^3)/(3*d) + (10*a*b^3*Sin[c + d*x]^3)/(3*d) + (2*a*
b^3*Sin[c + d*x]^3*Tan[c + d*x]^2)/d
```

Rubi [A] time = 0.247616, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3517, 2633, 2592, 302, 206, 2590, 270, 288}

$$-\frac{2a^2b^2\cos^3(c+dx)}{d}+\frac{12a^2b^2\cos(c+dx)}{d}+\frac{6a^2b^2\sec(c+dx)}{d}-\frac{4a^3b\sin^3(c+dx)}{3d}-\frac{4a^3b\sin(c+dx)}{d}+\frac{4a^3b\tanh^{-1}(\mathrm{si}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```



```
[Out] (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (10*a*b^3*ArcTanh[Sin[c + d*x]])/d - (a
^4*Cos[c + d*x])/d + (12*a^2*b^2*Cos[c + d*x])/d - (3*b^4*Cos[c + d*x])/d +
 (a^4*Cos[c + d*x]^3)/(3*d) - (2*a^2*b^2*Cos[c + d*x]^3)/d + (b^4*Cos[c + d
*x]^3)/(3*d) + (6*a^2*b^2*Sec[c + d*x])/d - (3*b^4*Sec[c + d*x])/d + (b^4*S
ec[c + d*x]^3)/(3*d) - (4*a^3*b*Sin[c + d*x])/d + (10*a*b^3*Sin[c + d*x])/d
 - (4*a^3*b*Sin[c + d*x]^3)/(3*d) + (10*a*b^3*Sin[c + d*x]^3)/(3*d) + (2*a*
b^3*Sin[c + d*x]^3*Tan[c + d*x]^2)/d
```


Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a^4\sin^3(c+dx)+4a^3b\sin^3(c+dx)\tan(c+dx)+6a^2b^2\sin^3(c+dx)\tan^2(c\right.$$
$$= a^4\int \sin^3(c+dx)\,dx+\left(4a^3b\right)\int \sin^3(c+dx)\tan(c+dx)\,dx+\left(6a^2b^2\right)\int \sin^3(c$$
$$= -\frac{a^4\,\mathrm{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos(c+dx)\right)}{d}+\frac{\left(4a^3b\right)\mathrm{Subst}\left(\int \frac{x^4}{1-x^2}\,dx,x,\sin(c+\right.}{d}$$
$$= -\frac{a^4\cos(c+dx)}{d}+\frac{a^4\cos^3(c+dx)}{3d}+\frac{2ab^3\sin^3(c+dx)\tan^2(c+dx)}{d}+\frac{\left(4a^3b\right)\mathrm{S}}{}$$
$$= -\frac{a^4\cos(c+dx)}{d}+\frac{12a^2b^2\cos(c+dx)}{d}-\frac{3b^4\cos(c+dx)}{d}+\frac{a^4\cos^3(c+dx)}{3d}-\frac{2a}{}$$
$$= \frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\cos(c+dx)}{d}+\frac{12a^2b^2\cos(c+dx)}{d}-\frac{3b^4\cos(c+}{d}$$
$$= \frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{10ab^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\cos(c+dx)}{d}+\frac{12a^2}{}$$

<u>**Mathematica**</u> **[B]** time = 6.27546, size = 1017, normalized size = 3.7

$$-\frac{\left(3a^4-42b^2a^2+11b^4\right)(a+b\tan(c+dx))^4\cos^5(c+dx)}{4d(a\cos(c+dx)+b\sin(c+dx))^4}+\frac{ab\left(a^2-b^2\right)\sin(3(c+dx))(a+b\tan(c+dx))^4\cos^4(c+dx)}{3d(a\cos(c+dx)+b\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(b^2*(-36*a^2 + 17*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(6*d*(a*Cos
[c + d*x] + b*Sin[c + d*x])^4) - ((3*a^4 - 42*a^2*b^2 + 11*b^4)*Cos[c + d*x
]^5*(a + b*Tan[c + d*x])^4)/(4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((a
^4 - 6*a^2*b^2 + b^4)*Cos[c + d*x]^4*Cos[3*(c + d*x)]*(a + b*Tan[c + d*x])^
4)/(12*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (2*(2*a^3*b - 5*a*b^3)*Cos[
c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)
/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*(2*a^3*b - 5*a*b^3)*Cos[c + d
*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(d*(
a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((12*a*b^3 + b^4)*Cos[c + d*x]^4*(a +
 b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*T
an[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(a*Cos[c + d*x
] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*Tan[c
```

```
+ d*x])^4)/(6*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b
*Sin[c + d*x])^4) + ((-12*a*b^3 + b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^
4)/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^4) + (Cos[c + d*x]^4*(36*a^2*b^2*Sin[(c + d*x)/2] - 17*b^4*Sin[(c +
 d*x)/2])*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
)*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(-36*a^2*b^2*Sin[(
c + d*x)/2] + 17*b^4*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (a*b*
(5*a^2 - 9*b^2)*Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Tan[c + d*x])^4)/(d*(a*C
os[c + d*x] + b*Sin[c + d*x])^4) + (a*b*(a^2 - b^2)*Cos[c + d*x]^4*Sin[3*(c
 + d*x)]*(a + b*Tan[c + d*x])^4)/(3*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

Maple [A] time = 0.059, size = 412, normalized size = 1.5

$$\frac{b^4 (\sin(dx+c))^8}{3d(\cos(dx+c))^3} - \frac{5b^4 (\sin(dx+c))^8}{3d\cos(dx+c)} - \frac{16b^4\cos(dx+c)}{3d} - \frac{5b^4\cos(dx+c)(\sin(dx+c))^6}{3d} - 2\frac{b^4\cos(dx+c)(\sin(\ldots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x)
```

```
[Out] 1/3/d*b^4*sin(d*x+c)^8/cos(d*x+c)^3-5/3/d*b^4*sin(d*x+c)^8/cos(d*x+c)-16/3*
b^4*cos(d*x+c)/d-5/3/d*b^4*cos(d*x+c)*sin(d*x+c)^6-2/d*b^4*cos(d*x+c)*sin(d
*x+c)^4-8/3/d*b^4*cos(d*x+c)*sin(d*x+c)^2+2/d*b^3*a*sin(d*x+c)^7/cos(d*x+c)
^2+2/d*b^3*a*sin(d*x+c)^5+10/3*a*b^3*sin(d*x+c)^3/d+10*a*b^3*sin(d*x+c)/d-1
0/d*b^3*a*ln(sec(d*x+c)+tan(d*x+c))+6/d*a^2*b^2*sin(d*x+c)^6/cos(d*x+c)+16*
a^2*b^2*cos(d*x+c)/d+6/d*a^2*b^2*cos(d*x+c)*sin(d*x+c)^4+8/d*a^2*b^2*cos(d*
x+c)*sin(d*x+c)^2-4/3*a^3*b*sin(d*x+c)^3/d-4*a^3*b*sin(d*x+c)/d+4/d*b*a^3*l
n(sec(d*x+c)+tan(d*x+c))-1/3/d*cos(d*x+c)*sin(d*x+c)^2*a^4-2/3*a^4*cos(d*x+
c)/d
```

Maxima [A] time = 1.14764, size = 294, normalized size = 1.07

$$\frac{\left(\cos(dx+c)^3 - 3\cos(dx+c)\right)a^4 - 2\left(2\sin(dx+c)^3 - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1) + 6\sin(dx+\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*((cos(d*x + c)^3 - 3*cos(d*x + c))*a^4 - 2*(2*sin(d*x + c)^3 - 3*log(si
n(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*a^3*b - 6*(cos(
d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^2*b^2 + (4*sin(d*x + c)^3 -
 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(si
n(d*x + c) - 1) + 24*sin(d*x + c))*a*b^3 + (cos(d*x + c)^3 - (9*cos(d*x + c
)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x + c))*b^4)/d
```

Fricas [A] time = 2.13296, size = 528, normalized size = 1.92

$$\frac{\left(a^4 - 6\,a^2b^2 + b^4\right)\cos\left(dx+c\right)^6 - 3\left(a^4 - 12\,a^2b^2 + 3\,b^4\right)\cos\left(dx+c\right)^4 + 3\left(2\,a^3b - 5\,ab^3\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right) + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*((a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^6 - 3*(a^4 - 12*a^2*b^2 + 3*b^4)*
cos(d*x + c)^4 + 3*(2*a^3*b - 5*a*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1)
 - 3*(2*a^3*b - 5*a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + b^4 + 9*(2
*a^2*b^2 - b^4)*cos(d*x + c)^2 + 2*(2*(a^3*b - a*b^3)*cos(d*x + c)^5 + 3*a*
b^3*cos(d*x + c) - 2*(4*a^3*b - 7*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*c
os(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.42 $\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=139

$$\frac{b^2\left(18a^2-5b^2\right)\tan(c+dx)}{2d}-\frac{4ab\left(a^2-2b^2\right)\log(\cos(c+dx))}{d}+\frac{1}{2}x\left(-18a^2b^2+a^4+5b^4\right)+\frac{4ab^3\tan^2(c+dx)}{d}-\frac{\sin(c+$$

```
[Out] ((a^4 - 18*a^2*b^2 + 5*b^4)*x)/2 - (4*a*b*(a^2 - 2*b^2)*Log[Cos[c + d*x]])/
d + (b^2*(18*a^2 - 5*b^2)*Tan[c + d*x])/(2*d) + (4*a*b^3*Tan[c + d*x]^2)/d
+ (5*b^4*Tan[c + d*x]^3)/(6*d) - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c +
d*x])^4)/(2*d)
```

Rubi [A] time = 0.174975, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1645, 801, 635, 203, 260}

$$\frac{b^2\left(18a^2-5b^2\right)\tan(c+dx)}{2d}-\frac{4ab\left(a^2-2b^2\right)\log(\cos(c+dx))}{d}+\frac{1}{2}x\left(-18a^2b^2+a^4+5b^4\right)+\frac{4ab^3\tan^2(c+dx)}{d}-\frac{\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```


```
[Out] ((a^4 - 18*a^2*b^2 + 5*b^4)*x)/2 - (4*a*b*(a^2 - 2*b^2)*Log[Cos[c + d*x]])/
d + (b^2*(18*a^2 - 5*b^2)*Tan[c + d*x])/(2*d) + (4*a*b^3*Tan[c + d*x]^2)/d
+ (5*b^4*Tan[c + d*x]^3)/(6*d) - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c +
d*x])^4)/(2*d)
```


Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
```

```
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{x^2(a+x)^4}{(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^4}{2d} - \frac{\mathrm{Subst}\left(\int \frac{(a+x)^3(-ab^2-5b^2x)}{b^2+x^2}\,dx, x, b\right.}{2bd}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^4}{2d} - \frac{\mathrm{Subst}\left(\int\left(-18a^2b^2+5b^4-16a\right.\right.}{}$$

$$= \frac{b^2(18a^2-5b^2)\tan(c+dx)}{2d} + \frac{4ab^3\tan^2(c+dx)}{d} + \frac{5b^4\tan^3(c+dx)}{6d} - \frac{\cos(c+}{}$$

$$= \frac{b^2(18a^2-5b^2)\tan(c+dx)}{2d} + \frac{4ab^3\tan^2(c+dx)}{d} + \frac{5b^4\tan^3(c+dx)}{6d} - \frac{\cos(c+}{}$$

$$= \frac{1}{2}(a^4-18a^2b^2+5b^4)x - \frac{4ab(a^2-2b^2)\log(\cos(c+dx))}{d} + \frac{b^2(18a^2-5b^2)\tan}{2d}$$

Mathematica [A] time = 6.26561, size = 263, normalized size = 1.89

$$\frac{b\left(-\frac{(-6a^2b^2+a^4+b^4)\tan^{-1}(\tan(c+dx))}{2b} + 2b(3a^2-b^2)\tan(c+dx) + \frac{1}{2}\left(\frac{-12a^2b^2+a^4+3b^4}{\sqrt{-b^2}} + 4a^3 - 8ab^2\right)\log\left(\sqrt{-b^2} - b\tan(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (b*(-((a^4 - 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]])/(2*b) + 2*a*(a - b)*(a
+ b)*Cos[c + d*x]^2 + ((4*a^3 - 8*a*b^2 + (a^4 - 12*a^2*b^2 + 3*b^4)/Sqrt[-
b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((4*a^3 - 8*a*b^2 - (a^4 - 12*a
^2*b^2 + 3*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/2 - ((a^4 - 6
*a^2*b^2 + b^4)*Cos[c + d*x]*Sin[c + d*x])/(2*b) + 2*b*(3*a^2 - b^2)*Tan[c
+ d*x] + 2*a*b^2*Tan[c + d*x]^2 + (b^3*Tan[c + d*x]^3)/3))/d
```

Maple [B] time = 0.057, size = 368, normalized size = 2.7

$$\frac{b^4(\sin(dx+c))^7}{3d(\cos(dx+c))^3} - \frac{4b^4(\sin(dx+c))^7}{3d\cos(dx+c)} - \frac{4b^4\cos(dx+c)(\sin(dx+c))^5}{3d} - \frac{5b^4\cos(dx+c)(\sin(dx+c))^3}{3d} - \frac{5b^4\sin(d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x)
```

```
[Out] 1/3/d*b^4*sin(d*x+c)^7/cos(d*x+c)^3-4/3/d*b^4*sin(d*x+c)^7/cos(d*x+c)-4/3/d
*b^4*cos(d*x+c)*sin(d*x+c)^5-5/3/d*b^4*cos(d*x+c)*sin(d*x+c)^3-5/2/d*b^4*si
n(d*x+c)*cos(d*x+c)+5/2*b^4*x+5/2/d*b^4*c+2/d*b^3*a*sin(d*x+c)^6/cos(d*x+c)
^2+2/d*b^3*a*sin(d*x+c)^4+4/d*b^3*a*sin(d*x+c)^2+8/d*b^3*a*ln(cos(d*x+c))+6
/d*a^2*b^2*sin(d*x+c)^5/cos(d*x+c)+6/d*a^2*b^2*cos(d*x+c)*sin(d*x+c)^3+9/d*
a^2*b^2*sin(d*x+c)*cos(d*x+c)-9*a^2*b^2*x-9/d*a^2*b^2*c-2/d*b*a^3*sin(d*x+c
)^2-4/d*b*a^3*ln(cos(d*x+c))-1/2/d*a^4*sin(d*x+c)*cos(d*x+c)+1/2*a^4*x+1/2/
d*a^4*c
```

Maxima [A] time = 1.70871, size = 208, normalized size = 1.5

$$\frac{2\,b^4\tan(dx+c)^3+12\,ab^3\tan(dx+c)^2+3\left(a^4-18\,a^2b^2+5\,b^4\right)(dx+c)+12\left(a^3b-2\,ab^3\right)\log\left(\tan(dx+c)^2+1\right)+12}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^4*tan(d*x + c)^3 + 12*a*b^3*tan(d*x + c)^2 + 3*(a^4 - 18*a^2*b^2 +
 5*b^4)*(d*x + c) + 12*(a^3*b - 2*a*b^3)*log(tan(d*x + c)^2 + 1) + 12*(3*a^
2*b^2 - b^4)*tan(d*x + c) + 3*(4*a^3*b - 4*a*b^3 - (a^4 - 6*a^2*b^2 + b^4)*
tan(d*x + c))/(tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 2.24796, size = 431, normalized size = 3.1

$$\frac{12\left(a^3b-ab^3\right)\cos(dx+c)^5+12\,ab^3\cos(dx+c)-24\left(a^3b-2\,ab^3\right)\cos(dx+c)^3\log(-\cos(dx+c))-3\left(2\,a^3b-2\,ab^3\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(12*(a^3*b - a*b^3)*cos(d*x + c)^5 + 12*a*b^3*cos(d*x + c) - 24*(a^3*b
- 2*a*b^3)*cos(d*x + c)^3*log(-cos(d*x + c)) - 3*(2*a^3*b - 2*a*b^3 - (a^4
```

```
- 18*a^2*b^2 + 5*b^4)*d*x)*cos(d*x + c)^3 - (3*(a^4 - 6*a^2*b^2 + b^4)*cos(
d*x + c)^4 - 2*b^4 - 2*(18*a^2*b^2 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(
d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**4,x)

[Out] Timed out
```

Giac [B] time = 7.19152, size = 5307, normalized size = 38.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="giac")

[Out] 1/6*(3*a^4*d*x*tan(d*x)^5*tan(c)^5 - 54*a^2*b^2*d*x*tan(d*x)^5*tan(c)^5 + 1
5*b^4*d*x*tan(d*x)^5*tan(c)^5 - 12*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*t
an(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*
x)*tan(c) + 1))*tan(d*x)^5*tan(c)^5 + 24*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*
x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*
tan(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c)^5 + 3*a^4*d*x*tan(d*x)^5*tan(c)^3 -
 54*a^2*b^2*d*x*tan(d*x)^5*tan(c)^3 + 15*b^4*d*x*tan(d*x)^5*tan(c)^3 - 9*a^
4*d*x*tan(d*x)^4*tan(c)^4 + 162*a^2*b^2*d*x*tan(d*x)^4*tan(c)^4 - 45*b^4*d*
x*tan(d*x)^4*tan(c)^4 + 3*a^4*d*x*tan(d*x)^3*tan(c)^5 - 54*a^2*b^2*d*x*tan(
d*x)^3*tan(c)^5 + 15*b^4*d*x*tan(d*x)^3*tan(c)^5 + 6*a^3*b*tan(d*x)^5*tan(c
)^5 + 6*a*b^3*tan(d*x)^5*tan(c)^5 - 12*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)
^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*ta
n(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c)^3 + 24*a*b^3*log(4*(tan(c)^2 + 1)/(ta
n(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2
- 2*tan(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c)^3 + 36*a^3*b*log(4*(tan(c)^2 +
1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d
*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 - 72*a*b^3*log(4*(tan(c
```

)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 + 3*a^4*tan(d*x)^5*tan(c)^4 - 54*a^2*b^2*tan(d*x)^5*tan(c)^4 + 15*b^4*tan(d*x)^5*tan(c)^4 - 12*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^5 + 24*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^5 + 3*a^4*tan(d*x)^4*tan(c)^5 - 54*a^2*b^2*tan(d*x)^4*tan(c)^5 + 15*b^4*tan(d*x)^4*tan(c)^5 - 9*a^4*d*x*tan(d*x)^4*tan(c)^2 + 162*a^2*b^2*d*x*tan(d*x)^4*tan(c)^2 - 45*b^4*d*x*tan(d*x)^4*tan(c)^2 + 12*a^4*d*x*tan(d*x)^3*tan(c)^3 - 216*a^2*b^2*d*x*tan(d*x)^3*tan(c)^3 + 60*b^4*d*x*tan(d*x)^3*tan(c)^3 - 6*a^3*b*tan(d*x)^5*tan(c)^3 + 30*a*b^3*tan(d*x)^5*tan(c)^3 - 9*a^4*d*x*tan(d*x)^2*tan(c)^4 + 162*a^2*b^2*d*x*tan(d*x)^2*tan(c)^4 - 45*b^4*d*x*tan(d*x)^2*tan(c)^4 - 42*a^3*b*tan(d*x)^4*tan(c)^4 + 30*a*b^3*tan(d*x)^4*tan(c)^4 - 6*a^3*b*tan(d*x)^3*tan(c)^5 + 30*a*b^3*tan(d*x)^3*tan(c)^5 + 36*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^2 - 72*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^2 - 36*a^2*b^2*tan(d*x)^5*tan(c)^2 + 10*b^4*tan(d*x)^5*tan(c)^2 - 48*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 + 96*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 - 12*a^4*tan(d*x)^4*tan(c)^3 + 108*a^2*b^2*tan(d*x)^4*tan(c)^3 - 30*b^4*tan(d*x)^4*tan(c)^3 + 36*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^4 - 72*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^4 - 12*a^4*tan(d*x)^3*tan(c)^4 + 108*a^2*b^2*tan(d*x)^3*tan(c)^4 - 30*b^4*tan(d*x)^3*tan(c)^4 - 36*a^2*b^2*tan(d*x)^2*tan(c)^5 + 10*b^4*tan(d*x)^2*tan(c)^5 + 9*a^4*d*x*tan(d*x)^3*tan(c) - 162*a^2*b^2*d*x*tan(d*x)^3*tan(c) + 45*b^4*d*x*tan(d*x)^3*tan(c) + 12*a*b^3*tan(d*x)^5*tan(c) - 12*a^4*d*x*tan(d*x)^2*tan(c)^2 + 216*a^2*b^2*d*x*tan(d*x)^2*tan(c)^2 - 60*b^4*d*x*tan(d*x)^2*tan(c)^2 + 18*a^3*b*tan(d*x)^4*tan(c)^2 - 42*a*b^3*tan(d*x)^4*tan(c)^2 + 9*a^4*d*x*tan(d*x)*tan(c)^3 - 162*a^2*b^2*d*x*tan(d*x)*tan(c)^3 + 45*b^4*d*x*tan(d*x)*tan(c)^3 + 96*a^3*b*tan(d*x)^3*tan(c)^3 - 48*a*b^3*tan(d*x)^3*tan(c)^3 + 18*a^3*b*tan(d*x)^2*tan(c)^4 - 42*a*b^3*tan(d*x)^2*tan(c)^4 + 12*a*b^3*tan(d*x)*tan(c)^5 - 2*b^4*tan(d*x)^5 - 36*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c) + 72*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c) + 72*a^2*b^2*tan(d*x)^4*tan(c) - 30*b^4*tan(d*x)^4*tan(c) + 48*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)

```
^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 - 96
*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + ta
n(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^
2 + 18*a^4*tan(d*x)^3*tan(c)^2 - 108*a^2*b^2*tan(d*x)^3*tan(c)^2 + 10*b^4*t
an(d*x)^3*tan(c)^2 - 36*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
 1))*tan(d*x)*tan(c)^3 + 72*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2
 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(
c) + 1))*tan(d*x)*tan(c)^3 + 18*a^4*tan(d*x)^2*tan(c)^3 - 108*a^2*b^2*tan(d
*x)^2*tan(c)^3 + 10*b^4*tan(d*x)^2*tan(c)^3 + 72*a^2*b^2*tan(d*x)*tan(c)^4
- 30*b^4*tan(d*x)*tan(c)^4 - 2*b^4*tan(c)^5 - 3*a^4*d*x*tan(d*x)^2 + 54*a^2
*b^2*d*x*tan(d*x)^2 - 15*b^4*d*x*tan(d*x)^2 - 12*a*b^3*tan(d*x)^4 + 9*a^4*d
*x*tan(d*x)*tan(c) - 162*a^2*b^2*d*x*tan(d*x)*tan(c) + 45*b^4*d*x*tan(d*x)*
tan(c) - 18*a^3*b*tan(d*x)^3*tan(c) + 42*a*b^3*tan(d*x)^3*tan(c) - 3*a^4*d*
x*tan(c)^2 + 54*a^2*b^2*d*x*tan(c)^2 - 15*b^4*d*x*tan(c)^2 - 96*a^3*b*tan(d
*x)^2*tan(c)^2 + 48*a*b^3*tan(d*x)^2*tan(c)^2 - 18*a^3*b*tan(d*x)*tan(c)^3
+ 42*a*b^3*tan(d*x)*tan(c)^3 - 12*a*b^3*tan(c)^4 + 12*a^3*b*log(4*(tan(c)^2
 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + ta
n(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2 - 24*a*b^3*log(4*(tan(c)^2 +
1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d
*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2 - 36*a^2*b^2*tan(d*x)^3 + 10*b^4
*tan(d*x)^3 - 36*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*
x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*ta
n(d*x)*tan(c) + 72*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(
d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*
tan(d*x)*tan(c) - 12*a^4*tan(d*x)^2*tan(c) + 108*a^2*b^2*tan(d*x)^2*tan(c)
- 30*b^4*tan(d*x)^2*tan(c) + 12*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(
c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*
tan(c) + 1))*tan(c)^2 - 24*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2
- 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c
) + 1))*tan(c)^2 - 12*a^4*tan(d*x)*tan(c)^2 + 108*a^2*b^2*tan(d*x)*tan(c)^2
 - 30*b^4*tan(d*x)*tan(c)^2 - 36*a^2*b^2*tan(c)^3 + 10*b^4*tan(c)^3 - 3*a^4
*d*x + 54*a^2*b^2*d*x - 15*b^4*d*x + 6*a^3*b*tan(d*x)^2 - 30*a*b^3*tan(d*x)
^2 + 42*a^3*b*tan(d*x)*tan(c) - 30*a*b^3*tan(d*x)*tan(c) + 6*a^3*b*tan(c)^2
 - 30*a*b^3*tan(c)^2 + 12*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 -
 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
 + 1)) - 24*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*
tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) + 3*a^4
*tan(d*x) - 54*a^2*b^2*tan(d*x) + 15*b^4*tan(d*x) + 3*a^4*tan(c) - 54*a^2*b
^2*tan(c) + 15*b^4*tan(c) - 6*a^3*b - 6*a*b^3)/(d*tan(d*x)^5*tan(c)^5 + d*t
an(d*x)^5*tan(c)^3 - 3*d*tan(d*x)^4*tan(c)^4 + d*tan(d*x)^3*tan(c)^5 - 3*d*
tan(d*x)^4*tan(c)^2 + 4*d*tan(d*x)^3*tan(c)^3 - 3*d*tan(d*x)^2*tan(c)^4 + 3
*d*tan(d*x)^3*tan(c) - 4*d*tan(d*x)^2*tan(c)^2 + 3*d*tan(d*x)*tan(c)^3 - d*
tan(d*x)^2 + 3*d*tan(d*x)*tan(c) - d*tan(c)^2 - d)
```

3.43 $\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=180

$$\frac{6a^2b^2\cos(c+dx)}{d}+\frac{6a^2b^2\sec(c+dx)}{d}-\frac{4a^3b\sin(c+dx)}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\cos(c+dx)}{d}+\frac{6ab^3\sin(c+}{d}$$

```
[Out] (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (6*a*b^3*ArcTanh[Sin[c + d*x]])/d - (a^
4*Cos[c + d*x])/d + (6*a^2*b^2*Cos[c + d*x])/d - (b^4*Cos[c + d*x])/d + (6*
a^2*b^2*Sec[c + d*x])/d - (2*b^4*Sec[c + d*x])/d + (b^4*Sec[c + d*x]^3)/(3*
d) - (4*a^3*b*Sin[c + d*x])/d + (6*a*b^3*Sin[c + d*x])/d + (2*a*b^3*Sin[c +
 d*x]*Tan[c + d*x]^2)/d
```

Rubi [A] time = 0.15661, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {3517, 2638, 2592, 321, 206, 2590, 14, 288, 270}

$$\frac{6a^2b^2\cos(c+dx)}{d}+\frac{6a^2b^2\sec(c+dx)}{d}-\frac{4a^3b\sin(c+dx)}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\cos(c+dx)}{d}+\frac{6ab^3\sin(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (6*a*b^3*ArcTanh[Sin[c + d*x]])/d - (a^
4*Cos[c + d*x])/d + (6*a^2*b^2*Cos[c + d*x])/d - (b^4*Cos[c + d*x])/d + (6*
a^2*b^2*Sec[c + d*x])/d - (2*b^4*Sec[c + d*x])/d + (b^4*Sec[c + d*x]^3)/(3*
d) - (4*a^3*b*Sin[c + d*x])/d + (6*a*b^3*Sin[c + d*x])/d + (2*a*b^3*Sin[c +
 d*x]*Tan[c + d*x]^2)/d
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a^4\sin(c+dx)+4a^3b\sin(c+dx)\tan(c+dx)+6a^2b^2\sin(c+dx)\tan^2(c+a\right.$$
$$= a^4\int \sin(c+dx)\,dx+\left(4a^3b\right)\int \sin(c+dx)\tan(c+dx)\,dx+\left(6a^2b^2\right)\int \sin(c+$$
$$= -\frac{a^4\cos(c+dx)}{d}+\frac{\left(4a^3b\right)\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d}-\frac{\left(6a^2b^2\right)\text{Subst}\left(\int\right.}{}$$
$$= -\frac{a^4\cos(c+dx)}{d}-\frac{4a^3b\sin(c+dx)}{d}+\frac{2ab^3\sin(c+dx)\tan^2(c+dx)}{d}+\frac{\left(4a^3b\right)\text{S}}{}$$
$$= \frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\cos(c+dx)}{d}+\frac{6a^2b^2\cos(c+dx)}{d}-\frac{b^4\cos(c+dx}{d}$$
$$= \frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{6ab^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\cos(c+dx)}{d}+\frac{6a^2b^2}{}$$

Mathematica [B] time = 5.45856, size = 383, normalized size = 2.13

$$\frac{-48ab\left(a^2-b^2\right)\sin(c+dx)-12\left(-6a^2b^2+a^4+b^4\right)\cos(c+dx)+\frac{2b^2\left(36a^2-11b^2\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{2b^2\left(11b^2-36a^2\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)}-2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (72*a^2*b^2 - 22*b^4 - 12*(a^4 - 6*a^2*b^2 + b^4)*Cos[c + d*x] - 24*a*b*(2*
a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 24*a*b*(2*a^2 - 3*b
^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (b^3*(12*a + b))/(Cos[(c + d
*x)/2] - Sin[(c + d*x)/2])^2 + (2*b^4*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^3 + (2*b^2*(36*a^2 - 11*b^2)*Sin[(c + d*x)/2])/(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]) - (2*b^4*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^3 + (b^3*(-12*a + b))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2 + (2*b^2*(-36*a^2 + 11*b^2)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]) - 48*a*b*(a^2 - b^2)*Sin[c + d*x])/(12*d)
```

Maple [A] time = 0.049, size = 309, normalized size = 1.7

$$\frac{b^4 (\sin(dx+c))^6}{3\,d(\cos(dx+c))^3} - \frac{b^4 (\sin(dx+c))^6}{d\cos(dx+c)} - \frac{8\,b^4\cos(dx+c)}{3\,d} - \frac{b^4\cos(dx+c)(\sin(dx+c))^4}{d} - \frac{4\,b^4\cos(dx+c)(\sin(dx+c}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c))^4,x)
```

```
[Out] 1/3/d*b^4*sin(d*x+c)^6/cos(d*x+c)^3-1/d*b^4*sin(d*x+c)^6/cos(d*x+c)-8/3*b^4
*cos(d*x+c)/d-1/d*b^4*cos(d*x+c)*sin(d*x+c)^4-4/3/d*b^4*cos(d*x+c)*sin(d*x+
c)^2+2/d*b^3*a*sin(d*x+c)^5/cos(d*x+c)^2+2*a*b^3*sin(d*x+c)^3/d+6*a*b^3*sin
(d*x+c)/d-6/d*b^3*a*ln(sec(d*x+c)+tan(d*x+c))+6/d*a^2*b^2*sin(d*x+c)^4/cos(
d*x+c)+6/d*a^2*b^2*cos(d*x+c)*sin(d*x+c)^2+12*a^2*b^2*cos(d*x+c)/d-4*a^3*b*
sin(d*x+c)/d+4/d*b*a^3*ln(sec(d*x+c)+tan(d*x+c))-a^4*cos(d*x+c)/d
```

Maxima [A] time = 1.09334, size = 224, normalized size = 1.24

$$-\frac{3\,ab^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} + 3\,\log(\sin(dx+c)+1) - 3\,\log(\sin(dx+c)-1) - 4\,\sin(dx+c)\right) - 18\,a^2b^2\left(\frac{1}{\cos(dx+c)} + \cos(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1
) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 18*a^2*b^2*(1/cos(d*x + c)
+ cos(d*x + c)) + b^4*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x +
c)) - 6*a^3*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x +
c)) + 3*a^4*cos(d*x + c))/d
```

Fricas [A] time = 2.06641, size = 414, normalized size = 2.3

$$-\frac{3\left(a^4 - 6\,a^2b^2 + b^4\right)\cos(dx+c)^4 - 3\left(2\,a^3b - 3\,ab^3\right)\cos(dx+c)^3\log(\sin(dx+c)+1) + 3\left(2\,a^3b - 3\,ab^3\right)\cos(dx+c)^3}{3\,d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^4 - 3*(2*a^3*b - 3*a*b^3)*cos(
d*x + c)^3*log(sin(d*x + c) + 1) + 3*(2*a^3*b - 3*a*b^3)*cos(d*x + c)^3*log
(-sin(d*x + c) + 1) - b^4 - 6*(3*a^2*b^2 - b^4)*cos(d*x + c)^2 - 6*(a*b^3*c
os(d*x + c) - 2*(a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x +
c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan (c + dx))^4 \sin (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**4*sin(c + d*x), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.44 $\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=118

$$\frac{6a^2b^2\sec(c+dx)}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\tanh^{-1}(\cos(c+dx))}{d}-\frac{2ab^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ab^3\tan(c+dx)}{d}$$

```
[Out] -((a^4*ArcTanh[Cos[c + d*x]])/d) + (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (2*a
*b^3*ArcTanh[Sin[c + d*x]])/d + (6*a^2*b^2*Sec[c + d*x])/d - (b^4*Sec[c + d
*x])/d + (b^4*Sec[c + d*x]^3)/(3*d) + (2*a*b^3*Sec[c + d*x]*Tan[c + d*x])/d
```

Rubi [A] time = 0.114219, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3517, 3770, 2606, 8, 2611}

$$\frac{6a^2b^2\sec(c+dx)}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\tanh^{-1}(\cos(c+dx))}{d}-\frac{2ab^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ab^3\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -((a^4*ArcTanh[Cos[c + d*x]])/d) + (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (2*a
*b^3*ArcTanh[Sin[c + d*x]])/d + (6*a^2*b^2*Sec[c + d*x])/d - (b^4*Sec[c + d
*x])/d + (b^4*Sec[c + d*x]^3)/(3*d) + (2*a*b^3*Sec[c + d*x]*Tan[c + d*x])/d
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a^4\csc(c+dx)+4a^3b\sec(c+dx)+6a^2b^2\sec(c+dx)\tan(c+dx)+4ab^3\text{se}\right.$$

$$= a^4\int \csc(c+dx)\,dx+\left(4a^3b\right)\int \sec(c+dx)\,dx+\left(6a^2b^2\right)\int \sec(c+dx)\tan(c+$$

$$= -\frac{a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ab^3\sec(c+dx)\tan(c}{d}$$

$$= -\frac{a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab^3\tanh^{-1}(\sin(c+d}{d}$$

Mathematica [B] time = 5.1324, size = 352, normalized size = 2.98

$$2b^2\sin^2\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(\left(36a^2-5b^2\right)\cos(2(c+dx))+36a^2+2b^2\cos(c+dx)-b^2\right)+72a^2b^2-48a^3b\log\left(\cos\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (72*a^2*b^2 - 10*b^4 - 12*a^4*Log[Cos[(c + d*x)/2]] - 48*a^3*b*Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]] + 24*a*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + 12*a^4*Log[Sin[(c + d*x)/2]] + 48*a^3*b*Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] - 24*a*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (12*a*b
^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + b^4/(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^2 + 2*b^2*(36*a^2 - b^2 + 2*b^2*Cos[c + d*x] + (36*a^2 - 5*b^2
```

```
)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*Sin[(c + d*x)/2]^2 - (12*a*b^3)/(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^2 + b^4/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^2)/(12*d)
```

Maple [A] time = 0.069, size = 214, normalized size = 1.8

$$\frac{b^4 (\sin (dx+c))^4}{3\,d\,(\cos (dx+c))^3} - \frac{b^4 (\sin (dx+c))^4}{3\,d\cos (dx+c)} - \frac{b^4 \cos (dx+c) (\sin (dx+c))^2}{3\,d} - \frac{2\,b^4 \cos (dx+c)}{3\,d} + 2\,\frac{b^3 a (\sin (dx+c))^3}{d\,(\cos (dx+c))^2} + 2\,\frac{b^3 a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*tan(d*x+c))^4,x)
```

```
[Out] 1/3/d*b^4*sin(d*x+c)^4/cos(d*x+c)^3-1/3/d*b^4*sin(d*x+c)^4/cos(d*x+c)-1/3/d
*b^4*cos(d*x+c)*sin(d*x+c)^2-2/3*b^4*cos(d*x+c)/d+2/d*b^3*a*sin(d*x+c)^3/co
s(d*x+c)^2+2*a*b^3*sin(d*x+c)/d-2/d*b^3*a*ln(sec(d*x+c)+tan(d*x+c))+6/d*a^2
*b^2/cos(d*x+c)+4/d*b*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^4*ln(csc(d*x+c)-c
ot(d*x+c))
```

Maxima [A] time = 1.10845, size = 188, normalized size = 1.59

$$-\frac{3\,ab^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} + \log (\sin (dx+c)+1) - \log (\sin (dx+c)-1)\right) - 6\,a^3 b(\log (\sin (dx+c)+1) - \log (\sin (dx+c)-1))}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1)) - 6*a^3*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 3*a^4*log(cot(d*x + c) + csc(d*x + c)) - 18*a^2*b^2/cos(d*x + c)
+ (3*cos(d*x + c)^2 - 1)*b^4/cos(d*x + c)^3)/d
```

Fricas [A] time = 2.56534, size = 444, normalized size = 3.76

$$-\frac{3\,a^4 \cos (dx+c)^3 \log \left(\frac{1}{2}\,\cos (dx+c) + \frac{1}{2}\right) - 3\,a^4 \cos (dx+c)^3 \log \left(-\frac{1}{2}\,\cos (dx+c) + \frac{1}{2}\right) - 12\,ab^3 \cos (dx+c) \sin (dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*a^4*cos(d*x + c)^3*log(1/2*cos(d*x + c) + 1/2) - 3*a^4*cos(d*x + c)
^3*log(-1/2*cos(d*x + c) + 1/2) - 12*a*b^3*cos(d*x + c)*sin(d*x + c) - 6*(2
*a^3*b - a*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 6*(2*a^3*b - a*b^3)*
cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*b^4 - 6*(6*a^2*b^2 - b^4)*cos(d*x
 + c)^2)/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.70968, size = 261, normalized size = 2.21

$$\frac{3\,a^4 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) + 6\left(2\,a^3 b - ab^3\right) \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 6\left(2\,a^3 b - ab^3\right) \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) +}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 6*(2*a^3*b - a*b^3)*log(abs(tan
(1/2*d*x + 1/2*c) + 1)) - 6*(2*a^3*b - a*b^3)*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + 4*(3*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^2*b^2*tan(1/2*d*x + 1/2*c)^
4 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*b^4*tan(1/2*d*x + 1/2*c)^2 - 3*a*
b^3*tan(1/2*d*x + 1/2*c) - 9*a^2*b^2 + b^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)
/d
```

3.45 $\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=83

$$\frac{6a^2b^2\tan(c+dx)}{d}+\frac{4a^3b\log(\tan(c+dx))}{d}-\frac{a^4\cot(c+dx)}{d}+\frac{2ab^3\tan^2(c+dx)}{d}+\frac{b^4\tan^3(c+dx)}{3d}$$

```
[Out] -((a^4*Cot[c + d*x])/d) + (4*a^3*b*Log[Tan[c + d*x]])/d + (6*a^2*b^2*Tan[c
+ d*x])/d + (2*a*b^3*Tan[c + d*x]^2)/d + (b^4*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0581838, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 43}

$$\frac{6a^2b^2\tan(c+dx)}{d}+\frac{4a^3b\log(\tan(c+dx))}{d}-\frac{a^4\cot(c+dx)}{d}+\frac{2ab^3\tan^2(c+dx)}{d}+\frac{b^4\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -((a^4*Cot[c + d*x])/d) + (4*a^3*b*Log[Tan[c + d*x]])/d + (6*a^2*b^2*Tan[c
+ d*x])/d + (2*a*b^3*Tan[c + d*x]^2)/d + (b^4*Tan[c + d*x]^3)/(3*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^4}{x^2}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \left(6a^2+\frac{a^4}{x^2}+\frac{4a^3}{x}+4ax+x^2\right)dx, x, b\tan(c+dx)\right)}{d}$$
$$= -\frac{a^4\cot(c+dx)}{d}+\frac{4a^3b\log(\tan(c+dx))}{d}+\frac{6a^2b^2\tan(c+dx)}{d}+\frac{2ab^3\tan^2(c}{d}$$

Mathematica [A] time = 1.16045, size = 162, normalized size = 1.95

$$-\frac{\csc(c+dx)\sec^3(c+dx)\left(4\left(3a^4+b^4\right)\cos(2(c+dx))+\left(18a^2b^2+3a^4-b^4\right)\cos(4(c+dx))+3\left(8ab\sin(2(c+dx))\left(-a\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(Csc[c + d*x]*Sec[c + d*x]^3*(4*(3*a^4 + b^4)*Cos[2*(c + d*x)] + (3*a^4 +
18*a^2*b^2 - b^4)*Cos[4*(c + d*x)] + 3*(3*a^4 - 6*a^2*b^2 - b^4 + 8*a*b*(-b
^2 + a^2*Log[Cos[c + d*x]] - a^2*Log[Sin[c + d*x]])*Sin[2*(c + d*x)] + 4*a^
3*b*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])*Sin[4*(c + d*x)])))/(24*d)
```

Maple [A] time = 0.063, size = 90, normalized size = 1.1

$$\frac{b^4(\sin(dx+c))^3}{3d(\cos(dx+c))^3}+2\frac{b^3a}{d(\cos(dx+c))^2}+6\frac{a^2b^2\tan(dx+c)}{d}+4\frac{ba^3\ln(\tan(dx+c))}{d}-\frac{a^4\cot(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x)
```

```
[Out] 1/3/d*b^4*sin(d*x+c)^3/cos(d*x+c)^3+2/d*b^3*a/cos(d*x+c)^2+6*a^2*b^2*tan(d*
x+c)/d+4*a^3*b*ln(tan(d*x+c))/d-a^4*cot(d*x+c)/d
```

Maxima [A] time = 1.12037, size = 97, normalized size = 1.17

$$\frac{b^4\tan(dx+c)^3+6ab^3\tan(dx+c)^2+12a^3b\log(\tan(dx+c))+18a^2b^2\tan(dx+c)-\frac{3a^4}{\tan(dx+c)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 12*a^3*b*log(tan(d*x + c
)) + 18*a^2*b^2*tan(d*x + c) - 3*a^4/tan(d*x + c))/d
```

Fricas [A] time = 2.04927, size = 389, normalized size = 4.69

$$-\frac{6\,a^3b\cos\left(dx+c\right)^3\log\left(\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)-6\,a^3b\cos\left(dx+c\right)^3\log\left(-\frac{1}{4}\,\cos\left(dx+c\right)^2+\frac{1}{4}\right)\sin\left(dx+c\right)-6\,ab^3\cos}{3\,d\cos\left(dx+c\right)^3\sin\left(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a^3*b*cos(d*x + c)^3*log(cos(d*x + c)^2)*sin(d*x + c) - 6*a^3*b*cos
(d*x + c)^3*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 6*a*b^3*cos(d*x +
 c)*sin(d*x + c) + (3*a^4 + 18*a^2*b^2 - b^4)*cos(d*x + c)^4 - b^4 - 2*(9*a
^2*b^2 - b^4)*cos(d*x + c)^2)/(d*cos(d*x + c)^3*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.64793, size = 116, normalized size = 1.4

$$\frac{b^4\tan\left(dx+c\right)^3+6\,ab^3\tan\left(dx+c\right)^2+12\,a^3b\log\left(\left|\tan\left(dx+c\right)\right|\right)+18\,a^2b^2\tan\left(dx+c\right)-\frac{3\left(4\,a^3b\tan\left(dx+c\right)+a^4\right)}{\tan\left(dx+c\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 12*a^3*b*log(abs(tan(d*x
 + c))) + 18*a^2*b^2*tan(d*x + c) - 3*(4*a^3*b*tan(d*x + c) + a^4)/tan(d*x
+ c))/d
```

3.46 $\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=161

$$\frac{6a^2b^2\sec(c+dx)}{d}-\frac{6a^2b^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{4a^3b\csc(c+dx)}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\tanh^{-1}(\cos(c+dx))}{2d}$$

```
[Out] -(a^4*ArcTanh[Cos[c + d*x]])/(2*d) - (6*a^2*b^2*ArcTanh[Cos[c + d*x]])/d +
(4*a^3*b*ArcTanh[Sin[c + d*x]])/d + (2*a*b^3*ArcTanh[Sin[c + d*x]])/d - (4*
a^3*b*Csc[c + d*x])/d - (a^4*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (6*a^2*b^2*
Sec[c + d*x])/d + (b^4*Sec[c + d*x]^3)/(3*d) + (2*a*b^3*Sec[c + d*x]*Tan[c
+ d*x])/d
```

Rubi [A] time = 0.157433, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3517, 3768, 3770, 2621, 321, 207, 2622, 2606, 30}

$$\frac{6a^2b^2\sec(c+dx)}{d}-\frac{6a^2b^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{4a^3b\csc(c+dx)}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\tanh^{-1}(\cos(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(a^4*ArcTanh[Cos[c + d*x]])/(2*d) - (6*a^2*b^2*ArcTanh[Cos[c + d*x]])/d +
(4*a^3*b*ArcTanh[Sin[c + d*x]])/d + (2*a*b^3*ArcTanh[Sin[c + d*x]])/d - (4*
a^3*b*Csc[c + d*x])/d - (a^4*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (6*a^2*b^2*
Sec[c + d*x])/d + (b^4*Sec[c + d*x]^3)/(3*d) + (2*a*b^3*Sec[c + d*x]*Tan[c
+ d*x])/d
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a^4\csc^3(c+dx)+4a^3b\csc^2(c+dx)\sec(c+dx)+6a^2b^2\csc(c+dx)\sec^2(c+$$

$$= a^4\int \csc^3(c+dx)\,dx+\left(4a^3b\right)\int \csc^2(c+dx)\sec(c+dx)\,dx+\left(6a^2b^2\right)\int \csc(c$$

$$= -\frac{a^4\cot(c+dx)\csc(c+dx)}{2d}+\frac{2ab^3\sec(c+dx)\tan(c+dx)}{d}+\frac{1}{2}a^4\int \csc(c+dx$$

$$= -\frac{a^4\tanh^{-1}(\cos(c+dx))}{2d}+\frac{2ab^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{4a^3b\csc(c+dx)}{d}-\frac{a^4\,c}{}$$

$$= -\frac{a^4\tanh^{-1}(\cos(c+dx))}{2d}-\frac{6a^2b^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+d}{d}$$

Mathematica [B] time = 6.19325, size = 1128, normalized size = 7.01

$$-\frac{2a^3b\cos^4(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4}+\frac{b^2\left(36a^2+b^2\right)\cos^4(c+dx)(a+b\tan(c+dx))^4}{6d(a\cos(c+dx)+b\sin(c+dx))^4}-\frac{a^4\cos^4(c+}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (b^2*(36*a^2 + b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(6*d*(a*Cos[c +
d*x] + b*Sin[c + d*x])^4) - (2*a^3*b*Cos[c + d*x]^4*Cot[(c + d*x)/2]*(a + b
*Tan[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (a^4*Cos[c + d*
x]^4*Csc[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Si
n[c + d*x])^4) + ((-a^4 - 12*a^2*b^2)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2]]*
(a + b*Tan[c + d*x])^4)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (2*(2*a
^3*b + a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a +
b*Tan[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((a^4 + 12*a^2
*b^2)*Cos[c + d*x]^4*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(2*d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*(2*a^3*b + a*b^3)*Cos[c + d*x]^4*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(d*(a*Cos[c +
 d*x] + b*Sin[c + d*x])^4) + (a^4*Cos[c + d*x]^4*Sec[(c + d*x)/2]^2*(a + b*
Tan[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((12*a*b^3 + b
^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] - Sin[(c
```

```
 + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*Si
n[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*Sin[(c
+ d*x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((-12*a*b^3 + b^4)*Cos[c + d*x]^
4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(-36*a^2*b^2*Sin[(c + d
*x)/2] - b^4*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]
^4*(36*a^2*b^2*Sin[(c + d*x)/2] + b^4*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x]
)^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c +
 d*x])^4) - (2*a^3*b*Cos[c + d*x]^4*Tan[(c + d*x)/2]*(a + b*Tan[c + d*x])^4
)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

Maple [A] time = 0.075, size = 192, normalized size = 1.2

$$\frac{b^4}{3\,d\,(\cos(dx+c))^3}+2\,\frac{b^3a\sec(dx+c)\tan(dx+c)}{d}+2\,\frac{b^3a\ln(\sec(dx+c)+\tan(dx+c))}{d}+6\,\frac{a^2b^2}{d\cos(dx+c)}+6\,\frac{a^2b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x)
```

```
[Out] 1/3/d*b^4/cos(d*x+c)^3+2*a*b^3*sec(d*x+c)*tan(d*x+c)/d+2/d*b^3*a*ln(sec(d*x
+c)+tan(d*x+c))+6/d*a^2*b^2/cos(d*x+c)+6/d*a^2*b^2*ln(csc(d*x+c)-cot(d*x+c)
)-4/d*b*a^3/sin(d*x+c)+4/d*b*a^3*ln(sec(d*x+c)+tan(d*x+c))-1/2*a^4*cot(d*x+
c)*csc(d*x+c)/d+1/2/d*a^4*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.17389, size = 254, normalized size = 1.58

$$\frac{3\,a^4\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-12\,ab^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^4*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) +
log(cos(d*x + c) - 1)) - 12*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - lo
```

```
g(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 36*a^2*b^2*(2/cos(d*x + c) -
 log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 24*a^3*b*(2/sin(d*x + c)
- log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 4*b^4/cos(d*x + c)^3)/d
```

Fricas [B] time = 3.03541, size = 818, normalized size = 5.08

$$\frac{6\left(a^{4}+12\,a^{2}b^{2}\right)\cos\left(dx+c\right)^{4}-4\,b^{4}-4\left(18\,a^{2}b^{2}-b^{4}\right)\cos\left(dx+c\right)^{2}-3\left(\left(a^{4}+12\,a^{2}b^{2}\right)\cos\left(dx+c\right)^{5}-\left(a^{4}+12\,a^{2}b^{2}\right)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^4 + 12*a^2*b^2)*cos(d*x + c)^4 - 4*b^4 - 4*(18*a^2*b^2 - b^4)*co
s(d*x + c)^2 - 3*((a^4 + 12*a^2*b^2)*cos(d*x + c)^5 - (a^4 + 12*a^2*b^2)*co
s(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2) + 3*((a^4 + 12*a^2*b^2)*cos(d*x +
 c)^5 - (a^4 + 12*a^2*b^2)*cos(d*x + c)^3)*log(-1/2*cos(d*x + c) + 1/2) + 1
2*((2*a^3*b + a*b^3)*cos(d*x + c)^5 - (2*a^3*b + a*b^3)*cos(d*x + c)^3)*log
(sin(d*x + c) + 1) - 12*((2*a^3*b + a*b^3)*cos(d*x + c)^5 - (2*a^3*b + a*b^
3)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 24*(a*b^3*cos(d*x + c) - (2*a^3
*b + a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^5 - d*cos(d*x + c
)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.76962, size = 405, normalized size = 2.52

$$3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-48\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+48\left(2\,a^3b+ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-48\left(2\,a^3b+ab^3\right)\log\left(\left|\mathrm{t}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^4*tan(1/2*d*x + 1/2*c)^2 - 48*a^3*b*tan(1/2*d*x + 1/2*c) + 48*(2*
a^3*b + a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 48*(2*a^3*b + a*b^3)*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 12*(a^4 + 12*a^2*b^2)*log(abs(tan(1/2*d*
x + 1/2*c))) - 3*(6*a^4*tan(1/2*d*x + 1/2*c)^2 + 72*a^2*b^2*tan(1/2*d*x + 1
/2*c)^2 + 16*a^3*b*tan(1/2*d*x + 1/2*c) + a^4)/tan(1/2*d*x + 1/2*c)^2 + 16*
(6*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 3*b^4
*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a*b^3*tan(1
/2*d*x + 1/2*c) - 18*a^2*b^2 - b^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.47 $\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=137

$$\frac{b^2\left(6a^2+b^2\right)\tan(c+dx)}{d}-\frac{a^2\left(a^2+6b^2\right)\cot(c+dx)}{d}+\frac{4ab\left(a^2+b^2\right)\log(\tan(c+dx))}{d}-\frac{2a^3b\cot^2(c+dx)}{d}-\frac{a^4\cot^3(c\cdots}{3d}$$

```
[Out] -((a^2*(a^2 + 6*b^2)*Cot[c + d*x])/d) - (2*a^3*b*Cot[c + d*x]^2)/d - (a^4*C
ot[c + d*x]^3)/(3*d) + (4*a*b*(a^2 + b^2)*Log[Tan[c + d*x]])/d + (b^2*(6*a^
2 + b^2)*Tan[c + d*x])/d + (2*a*b^3*Tan[c + d*x]^2)/d + (b^4*Tan[c + d*x]^3
)/(3*d)
```

Rubi [A] time = 0.104227, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 894}

$$\frac{b^2\left(6a^2+b^2\right)\tan(c+dx)}{d}-\frac{a^2\left(a^2+6b^2\right)\cot(c+dx)}{d}+\frac{4ab\left(a^2+b^2\right)\log(\tan(c+dx))}{d}-\frac{2a^3b\cot^2(c+dx)}{d}-\frac{a^4\cot^3(c\cdots}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -((a^2*(a^2 + 6*b^2)*Cot[c + d*x])/d) - (2*a^3*b*Cot[c + d*x]^2)/d - (a^4*C
ot[c + d*x]^3)/(3*d) + (4*a*b*(a^2 + b^2)*Log[Tan[c + d*x]])/d + (b^2*(6*a^
2 + b^2)*Tan[c + d*x])/d + (2*a*b^3*Tan[c + d*x]^2)/d + (b^4*Tan[c + d*x]^3
)/(3*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^4\left(b^2+x^2\right)}{x^4}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \left(6a^2\left(1+\frac{b^2}{6a^2}\right)+\frac{a^4b^2}{x^4}+\frac{4a^3b^2}{x^3}+\frac{a^4+6a^2b^2}{x^2}+\frac{4a\left(a^2+b^2\right)}{x}+4ax+x^2\right)dx,\right.}{d}$$
$$= -\frac{a^2\left(a^2+6b^2\right)\cot(c+dx)}{d}-\frac{2a^3b\cot^2(c+dx)}{d}-\frac{a^4\cot^3(c+dx)}{3d}+\frac{4ab\left(a^2+\right.}{}$$

Mathematica [A] time = 3.67706, size = 188, normalized size = 1.37

$$-\frac{\sin(c+dx)\tan^3(c+dx)(a\cot(c+dx)+b)^4\left(-2b^2\left(9a^2+b^2\right)\sin(c+dx)\cos^2(c+dx)+\cos(c+dx)\left(6a^3b\cot^2(c+dx\right.\right.}{3d(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -((b + a*Cot[c + d*x])^4*Sin[c + d*x]*(Cos[c + d*x]*(-6*a*b^3 + 6*a^3*b*Cot
[c + d*x]^2 + a^4*Cot[c + d*x]^3) + 2*a*Cos[c + d*x]^3*(a*(a^2 + 9*b^2)*Cot
[c + d*x] + 6*b*(a^2 + b^2)*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])) - b^4*
Sin[c + d*x] - 2*b^2*(9*a^2 + b^2)*Cos[c + d*x]^2*Sin[c + d*x])*Tan[c + d*x
]^3)/(3*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

Maple [A] time = 0.077, size = 184, normalized size = 1.3

$$\frac{2\,b^4\tan(dx+c)}{3\,d}+\frac{b^4\tan(dx+c)\left(\sec(dx+c)\right)^2}{3\,d}+2\,\frac{b^3a}{d\left(\cos(dx+c)\right)^2}+4\,\frac{b^3a\ln(\tan(dx+c))}{d}+6\,\frac{a^2b^2}{d\sin(dx+c)\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x)
```

```
[Out] 2/3/d*b^4*tan(d*x+c)+1/3/d*b^4*tan(d*x+c)*sec(d*x+c)^2+2/d*b^3*a/cos(d*x+c)
^2+4/d*b^3*a*ln(tan(d*x+c))+6/d*a^2*b^2/sin(d*x+c)/cos(d*x+c)-12/d*a^2*b^2*
cot(d*x+c)-2/d*b*a^3/sin(d*x+c)^2+4*a^3*b*ln(tan(d*x+c))/d-2/3*a^4*cot(d*x+
```

```
c)/d-1/3/d*a^4*cot(d*x+c)*csc(d*x+c)^2
```

Maxima [A] time = 1.07015, size = 162, normalized size = 1.18

$$\frac{b^4 \tan(dx+c)^3 + 6\,ab^3 \tan(dx+c)^2 + 12\left(a^3b + ab^3\right)\log(\tan(dx+c)) + 3\left(6\,a^2b^2 + b^4\right)\tan(dx+c) - \frac{6\,a^3b\tan(dx+c)+a^4+\cdots}{\tan\cdots}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 12*(a^3*b + a*b^3)*log(t
an(d*x + c)) + 3*(6*a^2*b^2 + b^4)*tan(d*x + c) - (6*a^3*b*tan(d*x + c) + a
^4 + 3*(a^4 + 6*a^2*b^2)*tan(d*x + c)^2)/tan(d*x + c)^3)/d
```

Fricas [A] time = 2.19935, size = 633, normalized size = 4.62

$$-\frac{2\left(a^4 + 18\,a^2b^2 + b^4\right)\cos(dx+c)^6 + 18\,a^2b^2\cos(dx+c)^2 - 3\left(a^4 + 18\,a^2b^2 + b^4\right)\cos(dx+c)^4 + b^4 + 6\left(\left(a^3b + ab^3\right)\cos\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^4 + 18*a^2*b^2 + b^4)*cos(d*x + c)^6 + 18*a^2*b^2*cos(d*x + c)^2
 - 3*(a^4 + 18*a^2*b^2 + b^4)*cos(d*x + c)^4 + b^4 + 6*((a^3*b + a*b^3)*cos
(d*x + c)^5 - (a^3*b + a*b^3)*cos(d*x + c)^3)*log(cos(d*x + c)^2)*sin(d*x +
 c) - 6*((a^3*b + a*b^3)*cos(d*x + c)^5 - (a^3*b + a*b^3)*cos(d*x + c)^3)*l
og(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) + 6*(a*b^3*cos(d*x + c) - (a^3*b
 + a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/((d*cos(d*x + c)^5 - d*cos(d*x + c)
^3)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.64509, size = 217, normalized size = 1.58

$$\frac{b^4\tan(dx+c)^3+6\,ab^3\tan(dx+c)^2+18\,a^2b^2\tan(dx+c)+3\,b^4\tan(dx+c)+12\left(a^3b+ab^3\right)\log(|\tan(dx+c)|)-\cdots}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 18*a^2*b^2*tan(d*x + c)
+ 3*b^4*tan(d*x + c) + 12*(a^3*b + a*b^3)*log(abs(tan(d*x + c))) - (22*a^3*
b*tan(d*x + c)^3 + 22*a*b^3*tan(d*x + c)^3 + 3*a^4*tan(d*x + c)^2 + 18*a^2*
b^2*tan(d*x + c)^2 + 6*a^3*b*tan(d*x + c) + a^4)/tan(d*x + c)^3)/d
```

3.48 $\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=274

$$\frac{9a^2b^2\sec(c+dx)}{d}-\frac{9a^2b^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{3a^2b^2\csc^2(c+dx)\sec(c+dx)}{d}-\frac{4a^3b\csc^3(c+dx)}{3d}-\frac{4a^3b\csc(c+dx)}{d}$$

```
[Out] (-3*a^4*ArcTanh[Cos[c + d*x]])/(8*d) - (9*a^2*b^2*ArcTanh[Cos[c + d*x]])/d
- (b^4*ArcTanh[Cos[c + d*x]])/d + (4*a^3*b*ArcTanh[Sin[c + d*x]])/d + (6*a*
b^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*b*Csc[c + d*x])/d - (6*a*b^3*Csc[c +
d*x])/d - (3*a^4*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (4*a^3*b*Csc[c + d*x]^3
)/(3*d) - (a^4*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d) + (9*a^2*b^2*Sec[c + d*x]
)/d + (b^4*Sec[c + d*x])/d - (3*a^2*b^2*Csc[c + d*x]^2*Sec[c + d*x])/d + (2
*a*b^3*Csc[c + d*x]*Sec[c + d*x]^2)/d + (b^4*Sec[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.240883, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3517, 3768, 3770, 2621, 302, 207, 2622, 288, 321}

$$\frac{9a^2b^2\sec(c+dx)}{d}-\frac{9a^2b^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{3a^2b^2\csc^2(c+dx)\sec(c+dx)}{d}-\frac{4a^3b\csc^3(c+dx)}{3d}-\frac{4a^3b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^4,x]
```


```
[Out] (-3*a^4*ArcTanh[Cos[c + d*x]])/(8*d) - (9*a^2*b^2*ArcTanh[Cos[c + d*x]])/d
- (b^4*ArcTanh[Cos[c + d*x]])/d + (4*a^3*b*ArcTanh[Sin[c + d*x]])/d + (6*a*
b^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*b*Csc[c + d*x])/d - (6*a*b^3*Csc[c +
d*x])/d - (3*a^4*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (4*a^3*b*Csc[c + d*x]^3
)/(3*d) - (a^4*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d) + (9*a^2*b^2*Sec[c + d*x]
)/d + (b^4*Sec[c + d*x])/d - (3*a^2*b^2*Csc[c + d*x]^2*Sec[c + d*x])/d + (2
*a*b^3*Csc[c + d*x]*Sec[c + d*x]^2)/d + (b^4*Sec[c + d*x]^3)/(3*d)
```


Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a^4\csc^5(c+dx)+4a^3b\csc^4(c+dx)\sec(c+dx)+6a^2b^2\csc^3(c+dx)\sec^2(c+\right.$$

$$= a^4\int \csc^5(c+dx)\,dx+\left(4a^3b\right)\int \csc^4(c+dx)\sec(c+dx)\,dx+\left(6a^2b^2\right)\int \csc^3($$

$$= -\frac{a^4\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{1}{4}\left(3a^4\right)\int \csc^3(c+dx)\,dx-\frac{\left(4a^3b\right)\text{Subst}\left(\int \frac{}{-1}\right.}{}$$

$$= -\frac{3a^4\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^4\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^2b^2\csc^2(c+dx)}{d}$$

$$= -\frac{3a^4\tanh^{-1}(\cos(c+dx))}{8d}-\frac{4a^3b\csc(c+dx)}{d}-\frac{6ab^3\csc(c+dx)}{d}-\frac{3a^4\cot(c+}{}$$

$$= -\frac{3a^4\tanh^{-1}(\cos(c+dx))}{8d}-\frac{9a^2b^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{b^4\tanh^{-1}(\cos(c+dx}{d}$$

Mathematica [B] time = 6.2872, size = 1491, normalized size = 5.44

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (b^2*(36*a^2 + 7*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(6*d*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^4) + ((-7*a^3*b*Cos[(c + d*x)/2] - 6*a*b^3*Cos[(c
+ d*x)/2])*Cos[c + d*x]^4*Csc[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(3*d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) - (3*(a^4 + 8*a^2*b^2)*Cos[c + d*x]^4*Csc
[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^4)/(32*d*(a*Cos[c + d*x] + b*Sin[c + d
*x])^4) - (a^3*b*Cos[c + d*x]^4*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(a + b*
Tan[c + d*x])^4)/(6*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (a^4*Cos[c + d
*x]^4*Csc[(c + d*x)/2]^4*(a + b*Tan[c + d*x])^4)/(64*d*(a*Cos[c + d*x] + b*
```

```
Sin[c + d*x])^4) + ((-3*a^4 - 72*a^2*b^2 - 8*b^4)*Cos[c + d*x]^4*Log[Cos[(c
 + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^
4) - (2*(2*a^3*b + 3*a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]]*(a + b*Tan[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) +
((3*a^4 + 72*a^2*b^2 + 8*b^4)*Cos[c + d*x]^4*Log[Sin[(c + d*x)/2]]*(a + b*T
an[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*(2*a^3*b + 3
*a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[
c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (3*(a^4 + 8*a^2*b^2)
*Cos[c + d*x]^4*Sec[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^4)/(32*d*(a*Cos[c +
 d*x] + b*Sin[c + d*x])^4) + (a^4*Cos[c + d*x]^4*Sec[(c + d*x)/2]^4*(a + b*
Tan[c + d*x])^4)/(64*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((12*a*b^3 +
b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*S
in[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*Sin[(c
 + d*x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((-12*a*b^3 + b^4)*Cos[c + d*x]
^4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a
*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*Sec[(c + d*x)/2]*(-7*a
^3*b*Sin[(c + d*x)/2] - 6*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(
3*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(-36*a^2*b^2*Sin
[(c + d*x)/2] - 7*b^4*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos
[c + d*x]^4*(36*a^2*b^2*Sin[(c + d*x)/2] + 7*b^4*Sin[(c + d*x)/2])*(a + b*T
an[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x]
+ b*Sin[c + d*x])^4) - (a^3*b*Cos[c + d*x]^4*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

Maple [A] time = 0.082, size = 317, normalized size = 1.2

$$\frac{b^4}{3\,d\left(\cos\left(dx+c\right)\right)^3}+\frac{b^4}{d\cos\left(dx+c\right)}+\frac{b^4\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}+2\,\frac{b^3a}{d\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^2}-6\,\frac{b^3a}{d\sin\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x)
```

```
[Out] 1/3/d*b^4/cos(d*x+c)^3+1/d*b^4/cos(d*x+c)+1/d*b^4*ln(csc(d*x+c)-cot(d*x+c))
+2/d*b^3*a/sin(d*x+c)/cos(d*x+c)^2-6/d*b^3*a/sin(d*x+c)+6/d*b^3*a*ln(sec(d*
x+c)+tan(d*x+c))-3/d*a^2*b^2/sin(d*x+c)^2/cos(d*x+c)+9/d*a^2*b^2/cos(d*x+c)
+9/d*a^2*b^2*ln(csc(d*x+c)-cot(d*x+c))-4/3/d*b*a^3/sin(d*x+c)^3-4/d*b*a^3/s
in(d*x+c)+4/d*b*a^3*ln(sec(d*x+c)+tan(d*x+c))-1/4*a^4*cot(d*x+c)*csc(d*x+c)
```

```
^3/d-3/8*a^4*cot(d*x+c)*csc(d*x+c)/d+3/8/d*a^4*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.17311, size = 410, normalized size = 1.5

$$\frac{3\,a^4\left(\frac{2\left(3\,\cos(dx+c)^3-5\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1} - 3\,\log\left(\cos\left(dx+c\right)+1\right) + 3\,\log\left(\cos\left(dx+c\right)-1\right)\right) + 72\,a^2b^2\left(\frac{2\left(3\,\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)} - 3\,\log\left(c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*a^4*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(
d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 72*a
^2*b^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(co
s(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 48*a*b^3*(2*(3*sin(d*x + c)^2
- 2)/(sin(d*x + c)^3 - sin(d*x + c)) - 3*log(sin(d*x + c) + 1) + 3*log(sin(
d*x + c) - 1)) + 8*b^4*(2*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)^3 - 3*log(cos
(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 32*a^3*b*(2*(3*sin(d*x + c)^2 +
 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Fricas [B] time = 3.98277, size = 1289, normalized size = 4.7

$$\frac{6\left(3\,a^4+72\,a^2b^2+8\,b^4\right)\cos\left(dx+c\right)^6 - 10\left(3\,a^4+72\,a^2b^2+8\,b^4\right)\cos\left(dx+c\right)^4 + 16\,b^4 + 16\left(18\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/48*(6*(3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^6 - 10*(3*a^4 + 72*a^2*b^
2 + 8*b^4)*cos(d*x + c)^4 + 16*b^4 + 16*(18*a^2*b^2 + b^4)*cos(d*x + c)^2 -
 3*((3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 - 2*(3*a^4 + 72*a^2*b^2 + 8
*b^4)*cos(d*x + c)^5 + (3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^3)*log(1/2
*cos(d*x + c) + 1/2) + 3*((3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 - 2*(
3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^5 + (3*a^4 + 72*a^2*b^2 + 8*b^4)*c
os(d*x + c)^3)*log(-1/2*cos(d*x + c) + 1/2) + 48*((2*a^3*b + 3*a*b^3)*cos(d
*x + c)^7 - 2*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^5 + (2*a^3*b + 3*a*b^3)*cos(
d*x + c)^3)*log(sin(d*x + c) + 1) - 48*((2*a^3*b + 3*a*b^3)*cos(d*x + c)^7
```

```
- 2*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^5 + (2*a^3*b + 3*a*b^3)*cos(d*x + c)^3
)*log(-sin(d*x + c) + 1) + 32*(3*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^5 + 3*a*b
^3*cos(d*x + c) - 4*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*co
s(d*x + c)^7 - 2*d*cos(d*x + c)^5 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.68359, size = 647, normalized size = 2.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^4*tan(1/2*d*x + 1/2*c)^4 - 32*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 24*
a^4*tan(1/2*d*x + 1/2*c)^2 + 144*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 480*a^3*b
*tan(1/2*d*x + 1/2*c) - 384*a*b^3*tan(1/2*d*x + 1/2*c) + 384*(2*a^3*b + 3*a
*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 384*(2*a^3*b + 3*a*b^3)*log(abs(
tan(1/2*d*x + 1/2*c) - 1)) + 24*(3*a^4 + 72*a^2*b^2 + 8*b^4)*log(abs(tan(1/
2*d*x + 1/2*c))) + 256*(3*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^2*b^2*tan(1/2*
d*x + 1/2*c)^4 - 3*b^4*tan(1/2*d*x + 1/2*c)^4 + 18*a^2*b^2*tan(1/2*d*x + 1/
2*c)^2 + 3*b^4*tan(1/2*d*x + 1/2*c)^2 - 3*a*b^3*tan(1/2*d*x + 1/2*c) - 9*a^
2*b^2 - 2*b^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3 - (150*a^4*tan(1/2*d*x + 1/2*
c)^4 + 3600*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 400*b^4*tan(1/2*d*x + 1/2*c)^4
 + 480*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 384*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 24
*a^4*tan(1/2*d*x + 1/2*c)^2 + 144*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 32*a^3*b
*tan(1/2*d*x + 1/2*c) + 3*a^4)/tan(1/2*d*x + 1/2*c)^4)/d
```

3.49 $\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=194

$$\frac{2b^2\left(3a^2+b^2\right)\tan(c+dx)}{d}-\frac{2a^2\left(a^2+3b^2\right)\cot^3(c+dx)}{3d}-\frac{2ab\left(2a^2+b^2\right)\cot^2(c+dx)}{d}-\frac{\left(12a^2b^2+a^4+b^4\right)\cot(c+dx)}{d}$$

```
[Out] -(((a^4 + 12*a^2*b^2 + b^4)*Cot[c + d*x])/d) - (2*a*b*(2*a^2 + b^2)*Cot[c +
 d*x]^2)/d - (2*a^2*(a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*d) - (a^3*b*Cot[c + d*
x]^4)/d - (a^4*Cot[c + d*x]^5)/(5*d) + (4*a*b*(a^2 + 2*b^2)*Log[Tan[c + d*x
]])/d + (2*b^2*(3*a^2 + b^2)*Tan[c + d*x])/d + (2*a*b^3*Tan[c + d*x]^2)/d +
 (b^4*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.157077, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 948}

$$\frac{2b^2\left(3a^2+b^2\right)\tan(c+dx)}{d}-\frac{2a^2\left(a^2+3b^2\right)\cot^3(c+dx)}{3d}-\frac{2ab\left(2a^2+b^2\right)\cot^2(c+dx)}{d}-\frac{\left(12a^2b^2+a^4+b^4\right)\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(((a^4 + 12*a^2*b^2 + b^4)*Cot[c + d*x])/d) - (2*a*b*(2*a^2 + b^2)*Cot[c +
 d*x]^2)/d - (2*a^2*(a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*d) - (a^3*b*Cot[c + d*
x]^4)/d - (a^4*Cot[c + d*x]^5)/(5*d) + (4*a*b*(a^2 + 2*b^2)*Log[Tan[c + d*x
]])/d + (2*b^2*(3*a^2 + b^2)*Tan[c + d*x])/d + (2*a*b^3*Tan[c + d*x]^2)/d +
 (b^4*Tan[c + d*x]^3)/(3*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
```

```
& EqQ[d, 0]))
```

Rubi steps

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^4\left(b^2+x^2\right)^2}{x^6}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\,\text{Subst}\left(\int \left(2\left(3a^2+b^2\right)+\frac{a^4b^4}{x^6}+\frac{4a^3b^4}{x^5}+\frac{2a^2b^2\left(a^2+3b^2\right)}{x^4}+\frac{4ab^2\left(2a^2+b^2\right)}{x^3}+\frac{a^4+12a^2b^2}{x^2}\right.\right.}{d}$$

$$= -\frac{\left(a^4+12a^2b^2+b^4\right)\cot(c+dx)}{d}-\frac{2ab\left(2a^2+b^2\right)\cot^2(c+dx)}{d}-\frac{2a^2\left(a^2+3b\right.}{}$$

Mathematica [A] time = 3.91889, size = 233, normalized size = 1.2

$$-\frac{(a+b\tan(c+dx))^4\left(-5b^2\left(18a^2+5b^2\right)\sin(c+dx)\cos^3(c+dx)+2a\cos^2(c+dx)\left(15b\left(a^2+b^2\right)\cot^2(c+dx)+a\left(2a\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -((15*a^3*b*Cot[c + d*x]^4 + 3*a^4*Cot[c + d*x]^5 + 2*a*Cos[c + d*x]^2*(-15
*b^3 + 15*b*(a^2 + b^2)*Cot[c + d*x]^2 + a*(2*a^2 + 15*b^2)*Cot[c + d*x]^3)
 + Cos[c + d*x]^4*((8*a^4 + 150*a^2*b^2 + 15*b^4)*Cot[c + d*x] + 60*a*b*(a^
2 + 2*b^2)*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])) - 5*b^2*(18*a^2 + 5*b^2
)*Cos[c + d*x]^3*Sin[c + d*x] - (5*b^4*Sin[2*(c + d*x)])/2)*(a + b*Tan[c +
d*x])^4)/(15*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

Maple [A] time = 0.082, size = 301, normalized size = 1.6

$$\frac{b^4}{3d\sin(dx+c)(\cos(dx+c))^3}+\frac{4b^4}{3d\sin(dx+c)\cos(dx+c)}-\frac{8b^4\cot(dx+c)}{3d}+2\frac{b^3a}{d(\sin(dx+c))^2(\cos(dx+c))^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x)
```

```
[Out] 1/3/d*b^4/sin(d*x+c)/cos(d*x+c)^3+4/3/d*b^4/sin(d*x+c)/cos(d*x+c)-8/3/d*b^4
*cot(d*x+c)+2/d*b^3*a/sin(d*x+c)^2/cos(d*x+c)^2-4/d*b^3*a/sin(d*x+c)^2+8/d*
b^3*a*ln(tan(d*x+c))-2/d*a^2*b^2/sin(d*x+c)^3/cos(d*x+c)+8/d*a^2*b^2/sin(d*
x+c)/cos(d*x+c)-16/d*a^2*b^2*cot(d*x+c)-1/d*b*a^3/sin(d*x+c)^4-2/d*b*a^3/si
n(d*x+c)^2+4*a^3*b*ln(tan(d*x+c))/d-8/15*a^4*cot(d*x+c)/d-1/5/d*a^4*cot(d*x
+c)*csc(d*x+c)^4-4/15/d*a^4*cot(d*x+c)*csc(d*x+c)^2
```

Maxima [A] time = 1.06929, size = 231, normalized size = 1.19

$$\frac{5\,b^4\tan\left(dx+c\right)^3+30\,ab^3\tan\left(dx+c\right)^2+60\left(a^3b+2\,ab^3\right)\log\left(\tan\left(dx+c\right)\right)+30\left(3\,a^2b^2+b^4\right)\tan\left(dx+c\right)-\frac{15\,a^3b\tan(d\cdots}{\cdots}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/15*(5*b^4*tan(d*x + c)^3 + 30*a*b^3*tan(d*x + c)^2 + 60*(a^3*b + 2*a*b^3)
*log(tan(d*x + c)) + 30*(3*a^2*b^2 + b^4)*tan(d*x + c) - (15*a^3*b*tan(d*x
+ c) + 15*(a^4 + 12*a^2*b^2 + b^4)*tan(d*x + c)^4 + 3*a^4 + 30*(2*a^3*b + a
*b^3)*tan(d*x + c)^3 + 10*(a^4 + 3*a^2*b^2)*tan(d*x + c)^2)/tan(d*x + c)^5)
/d
```

Fricas [B] time = 2.63441, size = 922, normalized size = 4.75

$$-\frac{8\left(a^4+30\,a^2b^2+5\,b^4\right)\cos\left(dx+c\right)^8-20\left(a^4+30\,a^2b^2+5\,b^4\right)\cos\left(dx+c\right)^6+15\left(a^4+30\,a^2b^2+5\,b^4\right)\cos\left(dx+c\right)^4-5\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^4 + 30*a^2*b^2 + 5*b^4)*cos(d*x + c)^8 - 20*(a^4 + 30*a^2*b^2 +
 5*b^4)*cos(d*x + c)^6 + 15*(a^4 + 30*a^2*b^2 + 5*b^4)*cos(d*x + c)^4 - 5*b
^4 - 10*(9*a^2*b^2 + b^4)*cos(d*x + c)^2 + 30*((a^3*b + 2*a*b^3)*cos(d*x +
c)^7 - 2*(a^3*b + 2*a*b^3)*cos(d*x + c)^5 + (a^3*b + 2*a*b^3)*cos(d*x + c)^
3)*log(cos(d*x + c)^2)*sin(d*x + c) - 30*((a^3*b + 2*a*b^3)*cos(d*x + c)^7
- 2*(a^3*b + 2*a*b^3)*cos(d*x + c)^5 + (a^3*b + 2*a*b^3)*cos(d*x + c)^3)*lo
g(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 15*(2*(a^3*b + 2*a*b^3)*cos(d*x
```

```
 + c)^5 + 2*a*b^3*cos(d*x + c) - 3*(a^3*b + 2*a*b^3)*cos(d*x + c)^3)*sin(d*
x + c))/((d*cos(d*x + c)^7 - 2*d*cos(d*x + c)^5 + d*cos(d*x + c)^3)*sin(d*x
 + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.65297, size = 317, normalized size = 1.63

$$\frac{5\,b^4\tan\left(dx+c\right)^3+30\,ab^3\tan\left(dx+c\right)^2+90\,a^2b^2\tan\left(dx+c\right)+30\,b^4\tan\left(dx+c\right)+60\left(a^3b+2\,ab^3\right)\log\left(\left|\tan\left(dx+c\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/15*(5*b^4*tan(d*x + c)^3 + 30*a*b^3*tan(d*x + c)^2 + 90*a^2*b^2*tan(d*x +
 c) + 30*b^4*tan(d*x + c) + 60*(a^3*b + 2*a*b^3)*log(abs(tan(d*x + c))) - (
137*a^3*b*tan(d*x + c)^5 + 274*a*b^3*tan(d*x + c)^5 + 15*a^4*tan(d*x + c)^4
 + 180*a^2*b^2*tan(d*x + c)^4 + 15*b^4*tan(d*x + c)^4 + 60*a^3*b*tan(d*x +
c)^3 + 30*a*b^3*tan(d*x + c)^3 + 10*a^4*tan(d*x + c)^2 + 30*a^2*b^2*tan(d*x
 + c)^2 + 15*a^3*b*tan(d*x + c) + 3*a^4)/tan(d*x + c)^5)/d
```

3.50 $\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=402

$$\frac{45a^2b^2\sec(c+dx)}{4d}-\frac{45a^2b^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{3a^2b^2\csc^4(c+dx)\sec(c+dx)}{2d}-\frac{15a^2b^2\csc^2(c+dx)\sec(c+dx)}{4d}-$$

```
[Out] (-5*a^4*ArcTanh[Cos[c + d*x]])/(16*d) - (45*a^2*b^2*ArcTanh[Cos[c + d*x]])/
(4*d) - (5*b^4*ArcTanh[Cos[c + d*x]])/(2*d) + (4*a^3*b*ArcTanh[Sin[c + d*x]
])/d + (10*a*b^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*b*Csc[c + d*x])/d - (10*
a*b^3*Csc[c + d*x])/d - (5*a^4*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (4*a^3*b
*Csc[c + d*x]^3)/(3*d) - (10*a*b^3*Csc[c + d*x]^3)/(3*d) - (5*a^4*Cot[c + d
*x]*Csc[c + d*x]^3)/(24*d) - (4*a^3*b*Csc[c + d*x]^5)/(5*d) - (a^4*Cot[c +
d*x]*Csc[c + d*x]^5)/(6*d) + (45*a^2*b^2*Sec[c + d*x])/(4*d) + (5*b^4*Sec[c
 + d*x])/(2*d) - (15*a^2*b^2*Csc[c + d*x]^2*Sec[c + d*x])/(4*d) - (3*a^2*b^
2*Csc[c + d*x]^4*Sec[c + d*x])/(2*d) + (2*a*b^3*Csc[c + d*x]^3*Sec[c + d*x]
^2)/d + (5*b^4*Sec[c + d*x]^3)/(6*d) - (b^4*Csc[c + d*x]^2*Sec[c + d*x]^3)/
(2*d)
```

Rubi [A] time = 0.305542, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3517, 3768, 3770, 2621, 302, 207, 2622, 288, 321}

$$\frac{45a^2b^2\sec(c+dx)}{4d}-\frac{45a^2b^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{3a^2b^2\csc^4(c+dx)\sec(c+dx)}{2d}-\frac{15a^2b^2\csc^2(c+dx)\sec(c+dx)}{4d}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^7*(a + b*Tan[c + d*x])^4,x]
```


```
[Out] (-5*a^4*ArcTanh[Cos[c + d*x]])/(16*d) - (45*a^2*b^2*ArcTanh[Cos[c + d*x]])/
(4*d) - (5*b^4*ArcTanh[Cos[c + d*x]])/(2*d) + (4*a^3*b*ArcTanh[Sin[c + d*x]
])/d + (10*a*b^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*b*Csc[c + d*x])/d - (10*
a*b^3*Csc[c + d*x])/d - (5*a^4*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (4*a^3*b
*Csc[c + d*x]^3)/(3*d) - (10*a*b^3*Csc[c + d*x]^3)/(3*d) - (5*a^4*Cot[c + d
*x]*Csc[c + d*x]^3)/(24*d) - (4*a^3*b*Csc[c + d*x]^5)/(5*d) - (a^4*Cot[c +
d*x]*Csc[c + d*x]^5)/(6*d) + (45*a^2*b^2*Sec[c + d*x])/(4*d) + (5*b^4*Sec[c
 + d*x])/(2*d) - (15*a^2*b^2*Csc[c + d*x]^2*Sec[c + d*x])/(4*d) - (3*a^2*b^
2*Csc[c + d*x]^4*Sec[c + d*x])/(2*d) + (2*a*b^3*Csc[c + d*x]^3*Sec[c + d*x]
^2)/d + (5*b^4*Sec[c + d*x]^3)/(6*d) - (b^4*Csc[c + d*x]^2*Sec[c + d*x]^3)/
(2*d)
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a^4\csc^7(c+dx)+4a^3b\csc^6(c+dx)\sec(c+dx)+6a^2b^2\csc^5(c+dx)\sec^2(c+\right.$$

$$= a^4\int \csc^7(c+dx)\,dx+\left(4a^3b\right)\int \csc^6(c+dx)\sec(c+dx)\,dx+\left(6a^2b^2\right)\int \csc^5($$

$$= -\frac{a^4\cot(c+dx)\csc^5(c+dx)}{6d}+\frac{1}{6}\left(5a^4\right)\int \csc^5(c+dx)\,dx-\frac{\left(4a^3b\right)\text{Subst}\left(\int \frac{}{-1}\right.}{}$$

$$= -\frac{5a^4\cot(c+dx)\csc^3(c+dx)}{24d}-\frac{a^4\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{3a^2b^2\csc^4(c+dx}{2d}$$

$$= -\frac{4a^3b\csc(c+dx)}{d}-\frac{5a^4\cot(c+dx)\csc(c+dx)}{16d}-\frac{4a^3b\csc^3(c+dx)}{3d}-\frac{5a^4\cot(}{}$$

$$= -\frac{5a^4\tanh^{-1}(\cos(c+dx))}{16d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{4a^3b\csc(c+dx)}{d}-\frac{10}{}$$

$$= -\frac{5a^4\tanh^{-1}(\cos(c+dx))}{16d}-\frac{45a^2b^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{5b^4\tanh^{-1}(\cos(c+}{2d}$$

Mathematica [A] time = 6.25887, size = 660, normalized size = 1.64

$$-\frac{5\left(36a^2b^2+a^4+8b^4\right)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{16d(a\cos(c+dx)+b\sin(c+dx))^4}-\frac{2\left(2a^3b+5ab^3\right)\cos^4(c+dx)(a+b\tan(}{d(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^7*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (-5*(a^4 + 36*a^2*b^2 + 8*b^4)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2]]*(a + b*
Tan[c + d*x])^4)/(16*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (2*(2*a^3*b +
 5*a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Ta
n[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (5*(a^4 + 36*a^2*b
^2 + 8*b^4)*Cos[c + d*x]^4*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(1
6*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*(2*a^3*b + 5*a*b^3)*Cos[c + d
*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(d*(
a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cot[c + d*x]*Csc[c + d*x]^5*(-2545*a
^4 + 540*a^2*b^2 + 5240*b^4 - 2760*a^4*Cos[2*(c + d*x)] - 7200*a^2*b^2*Cos[
2*(c + d*x)] - 6720*b^4*Cos[2*(c + d*x)] + 60*a^4*Cos[4*(c + d*x)] + 2160*a
^2*b^2*Cos[4*(c + d*x)] + 480*b^4*Cos[4*(c + d*x)] + 200*a^4*Cos[6*(c + d*x
)] + 7200*a^2*b^2*Cos[6*(c + d*x)] + 1600*b^4*Cos[6*(c + d*x)] - 75*a^4*Cos
[8*(c + d*x)] - 2700*a^2*b^2*Cos[8*(c + d*x)] - 600*b^4*Cos[8*(c + d*x)] -
15744*a^3*b*Sin[2*(c + d*x)] - 8640*a*b^3*Sin[2*(c + d*x)] - 1152*a^3*b*Sin
[4*(c + d*x)] - 2880*a*b^3*Sin[4*(c + d*x)] + 3200*a^3*b*Sin[6*(c + d*x)] +
 8000*a*b^3*Sin[6*(c + d*x)] - 960*a^3*b*Sin[8*(c + d*x)] - 2400*a*b^3*Sin[
8*(c + d*x)])*(a + b*Tan[c + d*x])^4)/(30720*d*(a*Cos[c + d*x] + b*Sin[c +
d*x])^4)
```

Maple [A] time = 0.127, size = 442, normalized size = 1.1

$$\frac{b^4}{3\,d\,(\sin(dx+c))^2\,(\cos(dx+c))^3} - \frac{5\,b^4}{6\,d\,(\sin(dx+c))^2\cos(dx+c)} + \frac{5\,b^4}{2\,d\cos(dx+c)} + \frac{5\,b^4\ln(\csc(dx+c)-\cot(dx+}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x)
```

```
[Out] 1/3/d*b^4/sin(d*x+c)^2/cos(d*x+c)^3-5/6/d*b^4/sin(d*x+c)^2/cos(d*x+c)+5/2/d
*b^4/cos(d*x+c)+5/2/d*b^4*ln(csc(d*x+c)-cot(d*x+c))-4/3/d*b^3*a/sin(d*x+c)^
3/cos(d*x+c)^2+10/3/d*b^3*a/sin(d*x+c)/cos(d*x+c)^2-10/d*b^3*a/sin(d*x+c)+1
0/d*b^3*a*ln(sec(d*x+c)+tan(d*x+c))-3/2/d*a^2*b^2/sin(d*x+c)^4/cos(d*x+c)-1
5/4/d*a^2*b^2/sin(d*x+c)^2/cos(d*x+c)+45/4/d*a^2*b^2/cos(d*x+c)+45/4/d*a^2*
b^2*ln(csc(d*x+c)-cot(d*x+c))-4/5/d*b*a^3/sin(d*x+c)^5-4/3/d*b*a^3/sin(d*x+
c)^3-4/d*b*a^3/sin(d*x+c)+4/d*b*a^3*ln(sec(d*x+c)+tan(d*x+c))-1/6*a^4*cot(d
*x+c)*csc(d*x+c)^5/d-5/24*a^4*cot(d*x+c)*csc(d*x+c)^3/d-5/16*a^4*cot(d*x+c)
*csc(d*x+c)/d+5/16/d*a^4*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.15415, size = 522, normalized size = 1.3

$$5\,a^4\left(\frac{2\left(15\,\cos(dx+c)^5-40\,\cos(dx+c)^3+33\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+40\,b^4\left(\frac{2\left(15\,\cos(dx+c)^4-\right.}{\cos(dx+c)^5-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/480*(5*a^4*(2*(15*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 33*cos(d*x + c))/(
cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 15*log(cos(d*x
+ c) + 1) + 15*log(cos(d*x + c) - 1)) + 40*b^4*(2*(15*cos(d*x + c)^4 - 10*c
os(d*x + c)^2 - 2)/(cos(d*x + c)^5 - cos(d*x + c)^3) - 15*log(cos(d*x + c)
+ 1) + 15*log(cos(d*x + c) - 1)) + 180*a^2*b^2*(2*(15*cos(d*x + c)^4 - 25*c
os(d*x + c)^2 + 8)/(cos(d*x + c)^5 - 2*cos(d*x + c)^3 + cos(d*x + c)) - 15*
log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) - 160*a*b^3*(2*(15*sin(d*
x + c)^4 - 10*sin(d*x + c)^2 - 2)/(sin(d*x + c)^5 - sin(d*x + c)^3) - 15*lo
g(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 64*a^3*b*(2*(15*sin(d*x +
 c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 15*log(sin(d*x + c) + 1) + 1
5*log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 4.56755, size = 1661, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/480*(150*(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^8 - 400*(a^4 + 36*a^2*b^
2 + 8*b^4)*cos(d*x + c)^6 + 330*(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 -
 160*b^4 - 480*(6*a^2*b^2 + b^4)*cos(d*x + c)^2 - 75*((a^4 + 36*a^2*b^2 + 8
*b^4)*cos(d*x + c)^9 - 3*(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 + 3*(a^4
 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^5 - (a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x
+ c)^3)*log(1/2*cos(d*x + c) + 1/2) + 75*((a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*
x + c)^9 - 3*(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 + 3*(a^4 + 36*a^2*b^
2 + 8*b^4)*cos(d*x + c)^5 - (a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^3)*log(
-1/2*cos(d*x + c) + 1/2) + 480*((2*a^3*b + 5*a*b^3)*cos(d*x + c)^9 - 3*(2*a
^3*b + 5*a*b^3)*cos(d*x + c)^7 + 3*(2*a^3*b + 5*a*b^3)*cos(d*x + c)^5 - (2*
a^3*b + 5*a*b^3)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 480*((2*a^3*b + 5*
a*b^3)*cos(d*x + c)^9 - 3*(2*a^3*b + 5*a*b^3)*cos(d*x + c)^7 + 3*(2*a^3*b +
```

```
 5*a*b^3)*cos(d*x + c)^5 - (2*a^3*b + 5*a*b^3)*cos(d*x + c)^3)*log(-sin(d*x
 + c) + 1) + 64*(15*(2*a^3*b + 5*a*b^3)*cos(d*x + c)^7 - 35*(2*a^3*b + 5*a*
b^3)*cos(d*x + c)^5 - 15*a*b^3*cos(d*x + c) + 23*(2*a^3*b + 5*a*b^3)*cos(d*
x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^9 - 3*d*cos(d*x + c)^7 + 3*d*cos(d*
x + c)^5 - d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**7*(a+b*tan(d*x+c))**4,x)

[Out] Timed out
```

Giac [A] time = 2.80009, size = 873, normalized size = 2.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="giac")

[Out] 1/1920*(5*a^4*tan(1/2*d*x + 1/2*c)^6 - 48*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 45
*a^4*tan(1/2*d*x + 1/2*c)^4 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 560*a^3*
b*tan(1/2*d*x + 1/2*c)^3 - 320*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 225*a^4*tan(1
/2*d*x + 1/2*c)^2 + 2880*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 240*b^4*tan(1/2*d
*x + 1/2*c)^2 - 5280*a^3*b*tan(1/2*d*x + 1/2*c) - 8640*a*b^3*tan(1/2*d*x +
1/2*c) + 3840*(2*a^3*b + 5*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3840
*(2*a^3*b + 5*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 600*(a^4 + 36*a^2
*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c))) + 1280*(6*a*b^3*tan(1/2*d*x +
1/2*c)^5 - 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 9*b^4*tan(1/2*d*x + 1/2*c)^4
 + 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^4*tan(1/2*d*x + 1/2*c)^2 - 6*a*
b^3*tan(1/2*d*x + 1/2*c) - 18*a^2*b^2 - 7*b^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)
^3 - (1470*a^4*tan(1/2*d*x + 1/2*c)^6 + 52920*a^2*b^2*tan(1/2*d*x + 1/2*c)^
6 + 11760*b^4*tan(1/2*d*x + 1/2*c)^6 + 5280*a^3*b*tan(1/2*d*x + 1/2*c)^5 +
8640*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 225*a^4*tan(1/2*d*x + 1/2*c)^4 + 2880*a
^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 240*b^4*tan(1/2*d*x + 1/2*c)^4 + 560*a^3*b*
```

```
tan(1/2*d*x + 1/2*c)^3 + 320*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 45*a^4*tan(1/2*
d*x + 1/2*c)^2 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 48*a^3*b*tan(1/2*d*x
+ 1/2*c) + 5*a^4)/tan(1/2*d*x + 1/2*c)^6)/d
```

3.51 $\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=274

$$\frac{a^2b\sin^3(c+dx)}{3d\left(a^2+b^2\right)^2}+\frac{b\sin^5(c+dx)}{5d\left(a^2+b^2\right)}+\frac{a^4b\sin(c+dx)}{d\left(a^2+b^2\right)^3}-\frac{a\cos^5(c+dx)}{5d\left(a^2+b^2\right)}+\frac{2a\cos^3(c+dx)}{3d\left(a^2+b^2\right)}-\frac{ab^2\cos^3(c+dx)}{3d\left(a^2+b^2\right)^2}+\frac{a^3b^2\cos(}{d\left(a^2+\right.}$$

```
[Out] (a^5*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(7/2)*d) + (a^3*b^2*Cos[c + d*x])/((a^2 + b^2)^3*d) + (a*b^2*Cos[c + d
*x])/((a^2 + b^2)^2*d) - (a*Cos[c + d*x])/((a^2 + b^2)*d) - (a*b^2*Cos[c +
d*x]^3)/(3*(a^2 + b^2)^2*d) + (2*a*Cos[c + d*x]^3)/(3*(a^2 + b^2)*d) - (a*C
os[c + d*x]^5)/(5*(a^2 + b^2)*d) + (a^4*b*Sin[c + d*x])/((a^2 + b^2)^3*d) +
 (a^2*b*Sin[c + d*x]^3)/(3*(a^2 + b^2)^2*d) + (b*Sin[c + d*x]^5)/(5*(a^2 +
b^2)*d)
```

Rubi [A] time = 0.353902, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3518, 3109, 2564, 30, 2633, 3099, 3074, 206, 2638}

$$\frac{a^2b\sin^3(c+dx)}{3d\left(a^2+b^2\right)^2}+\frac{b\sin^5(c+dx)}{5d\left(a^2+b^2\right)}+\frac{a^4b\sin(c+dx)}{d\left(a^2+b^2\right)^3}-\frac{a\cos^5(c+dx)}{5d\left(a^2+b^2\right)}+\frac{2a\cos^3(c+dx)}{3d\left(a^2+b^2\right)}-\frac{ab^2\cos^3(c+dx)}{3d\left(a^2+b^2\right)^2}+\frac{a^3b^2\cos(}{d\left(a^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```


```
[Out] (a^5*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(7/2)*d) + (a^3*b^2*Cos[c + d*x])/((a^2 + b^2)^3*d) + (a*b^2*Cos[c + d
*x])/((a^2 + b^2)^2*d) - (a*Cos[c + d*x])/((a^2 + b^2)*d) - (a*b^2*Cos[c +
d*x]^3)/(3*(a^2 + b^2)^2*d) + (2*a*Cos[c + d*x]^3)/(3*(a^2 + b^2)*d) - (a*C
os[c + d*x]^5)/(5*(a^2 + b^2)*d) + (a^4*b*Sin[c + d*x])/((a^2 + b^2)^3*d) +
 (a^2*b*Sin[c + d*x]^3)/(3*(a^2 + b^2)^2*d) + (b*Sin[c + d*x]^5)/(5*(a^2 +
b^2)*d)
```


Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
```

```
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cos(c+dx)\sin^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

$$= \frac{a\int \sin^5(c+dx)\,dx}{a^2+b^2} + \frac{b\int \cos(c+dx)\sin^4(c+dx)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\sin^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{a^2 b\sin^3(c+dx)}{3\left(a^2+b^2\right)^2 d} - \frac{\left(a^3 b\right)\int \frac{\sin^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{\left(a^2+b^2\right)^2} - \frac{\left(ab^2\right)\int \sin^3(c+dx)\,dx}{\left(a^2+b^2\right)^2} - \frac{a\,\mathrm{Subst}\left(\int (1\right.}{}$$

$$= -\frac{a\cos(c+dx)}{\left(a^2+b^2\right)d} + \frac{2a\cos^3(c+dx)}{3\left(a^2+b^2\right)d} - \frac{a\cos^5(c+dx)}{5\left(a^2+b^2\right)d} + \frac{a^4 b\sin(c+dx)}{\left(a^2+b^2\right)^3 d} + \frac{a^2 b\sin^3(c+dx)}{3\left(a^2+b^2\right)^2 d} + \frac{b}{5}$$

$$= \frac{a^3 b^2\cos(c+dx)}{\left(a^2+b^2\right)^3 d} + \frac{ab^2\cos(c+dx)}{\left(a^2+b^2\right)^2 d} - \frac{a\cos(c+dx)}{\left(a^2+b^2\right)d} - \frac{ab^2\cos^3(c+dx)}{3\left(a^2+b^2\right)^2 d} + \frac{2a\cos^3(c+dx)}{3\left(a^2+b^2\right)d} -$$

$$= \frac{a^5 b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2} d} + \frac{a^3 b^2\cos(c+dx)}{\left(a^2+b^2\right)^3 d} + \frac{ab^2\cos(c+dx)}{\left(a^2+b^2\right)^2 d} - \frac{a\cos(c+dx)}{\left(a^2+b^2\right)d} - \frac{ab}{3}$$

Mathematica [A] time = 3.12434, size = 289, normalized size = 1.05

$$\frac{\sqrt{a^2+b^2}\left(120a^2b^3\sin(c+dx) - 50a^2b^3\sin(3(c+dx)) + 6a^2b^3\sin(5(c+dx)) - 6a^3b^2\cos(5(c+dx)) - 30a\left(-4a^2b^2+5a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-480*a^5*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 +
 b^2]*(-30*a*(5*a^4 - 4*a^2*b^2 - b^4)*Cos[c + d*x] + 5*a*(5*a^4 + 6*a^2*b^
2 + b^4)*Cos[3*(c + d*x)] - 3*a^5*Cos[5*(c + d*x)] - 6*a^3*b^2*Cos[5*(c + d
*x)] - 3*a*b^4*Cos[5*(c + d*x)] + 330*a^4*b*Sin[c + d*x] + 120*a^2*b^3*Sin[
c + d*x] + 30*b^5*Sin[c + d*x] - 35*a^4*b*Sin[3*(c + d*x)] - 50*a^2*b^3*Sin
[3*(c + d*x)] - 15*b^5*Sin[3*(c + d*x)] + 3*a^4*b*Sin[5*(c + d*x)] + 6*a^2*
b^3*Sin[5*(c + d*x)] + 3*b^5*Sin[5*(c + d*x)]))/(240*(a^2 + b^2)^(7/2)*d)
```

Maple [A] time = 0.079, size = 361, normalized size = 1.3

$$\frac{1}{d}\left(-2\,\frac{1}{\left(a^4+2\,a^2b^2+b^4\right)\left(a^2+b^2\right)\left(1+(\tan(1/2\,dx+c/2))^2\right)^5}\left(-a^4b\,(\tan(1/2\,dx+c/2))^9-a^3b^2\,(\tan(1/2\,dx+c/2))^8+\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d*(-2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*(-a^4*b*tan(1/2*d*x+1/2*c)^9-a^3*b^2*
tan(1/2*d*x+1/2*c)^8+(-16/3*a^4*b-4/3*a^2*b^3)*tan(1/2*d*x+1/2*c)^7+(-6*a^3
*b^2-2*a*b^4)*tan(1/2*d*x+1/2*c)^6+(-178/15*a^4*b-136/15*a^2*b^3-16/5*b^5)*
tan(1/2*d*x+1/2*c)^5+(16/3*a^5+2/3*a*b^4)*tan(1/2*d*x+1/2*c)^4+(-16/3*a^4*b
-4/3*a^2*b^3)*tan(1/2*d*x+1/2*c)^3+(-2*a^3*b^2+8/3*a^5-2/3*a*b^4)*tan(1/2*d
*x+1/2*c)^2-a^4*b*tan(1/2*d*x+1/2*c)+8/15*a^5-3/5*a^3*b^2-2/15*a*b^4)/(1+ta
n(1/2*d*x+1/2*c)^2)^5-64*a^5*b/(32*a^6+96*a^4*b^2+96*a^2*b^4+32*b^6)/(a^2+b
^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.3272, size = 840, normalized size = 3.07

$$15\sqrt{a^2+b^2}a^5b\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)-6\left(a^7+3\,a^5b^2+3\,a^3b^4+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*sqrt(a^2 + b^2)*a^5*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2
- b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a
*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)
^2 + b^2)) - 6*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^5 + 10*(2
*a^7 + 5*a^5*b^2 + 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 30*(a^7 + a^5*b^2)*c
os(d*x + c) + 2*(23*a^6*b + 34*a^4*b^3 + 14*a^2*b^5 + 3*b^7 + 3*(a^6*b + 3*
a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^4 - (11*a^6*b + 28*a^4*b^3 + 23*a^2
*b^5 + 6*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34169, size = 626, normalized size = 2.28

$$\frac{15\,a^5b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{2\left(15\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+15\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+80\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+20\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+90\,a^3b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/15*(15*a^5*b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/
abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2*(15*a^4*b*tan(1/2*d*x + 1/2*c)^9 +
15*a^3*b^2*tan(1/2*d*x + 1/2*c)^8 + 80*a^4*b*tan(1/2*d*x + 1/2*c)^7 + 20*a^
2*b^3*tan(1/2*d*x + 1/2*c)^7 + 90*a^3*b^2*tan(1/2*d*x + 1/2*c)^6 + 30*a*b^4
*tan(1/2*d*x + 1/2*c)^6 + 178*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 136*a^2*b^3*ta
n(1/2*d*x + 1/2*c)^5 + 48*b^5*tan(1/2*d*x + 1/2*c)^5 - 80*a^5*tan(1/2*d*x +
 1/2*c)^4 - 10*a*b^4*tan(1/2*d*x + 1/2*c)^4 + 80*a^4*b*tan(1/2*d*x + 1/2*c)
^3 + 20*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 40*a^5*tan(1/2*d*x + 1/2*c)^2 + 30
*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 10*a*b^4*tan(1/2*d*x + 1/2*c)^2 + 15*a^4*
b*tan(1/2*d*x + 1/2*c) - 8*a^5 + 9*a^3*b^2 + 2*a*b^4)/((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*(tan(1/2*d*x + 1/2*c)^2 + 1)^5))/d
```

3.52 $\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=158

$$\frac{\cos^4(c+dx)(a\tan(c+dx)+b)}{4d\left(a^2+b^2\right)}-\frac{\cos^2(c+dx)\left(a\left(5a^2+b^2\right)\tan(c+dx)+4b\left(2a^2+b^2\right)\right)}{8d\left(a^2+b^2\right)^2}+\frac{a^4b\log(a\cos(c+dx)+b\sin}{d\left(a^2+b^2\right)^3}$$

```
[Out] (a*(3*a^4 - 6*a^2*b^2 - b^4)*x)/(8*(a^2 + b^2)^3) + (a^4*b*Log[a*Cos[c + d*
x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d) + (Cos[c + d*x]^4*(b + a*Tan[c + d*
x]))/(4*(a^2 + b^2)*d) - (Cos[c + d*x]^2*(4*b*(2*a^2 + b^2) + a*(5*a^2 + b^
2)*Tan[c + d*x]))/(8*(a^2 + b^2)^2*d)
```

Rubi [A] time = 0.338949, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1647, 801, 635, 203, 260}

$$\frac{\cos^4(c+dx)(a\tan(c+dx)+b)}{4d\left(a^2+b^2\right)}-\frac{\cos^2(c+dx)\left(a\left(5a^2+b^2\right)\tan(c+dx)+4b\left(2a^2+b^2\right)\right)}{8d\left(a^2+b^2\right)^2}+\frac{a^4b\log(a\cos(c+dx)+b\sin}{d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*(3*a^4 - 6*a^2*b^2 - b^4)*x)/(8*(a^2 + b^2)^3) + (a^4*b*Log[a*Cos[c + d*
x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d) + (Cos[c + d*x]^4*(b + a*Tan[c + d*
x]))/(4*(a^2 + b^2)*d) - (Cos[c + d*x]^2*(4*b*(2*a^2 + b^2) + a*(5*a^2 + b^
2)*Tan[c + d*x]))/(8*(a^2 + b^2)^2*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
```

```
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^4}{(a+x)\left(b^2+x^2\right)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^4}{a^2+b^2}-\frac{3ab^4x}{a^2+b^2}-4b^2x^2}{(a+x)\left(b^2+x^2\right)^2}\,dx, x, b\tan(c+dx)\right)}{4bd}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d} - \frac{\cos^2(c+dx)\left(4b\left(2a^2+b^2\right)+a\left(5a^2+b^2\right)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^2d} +$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d} - \frac{\cos^2(c+dx)\left(4b\left(2a^2+b^2\right)+a\left(5a^2+b^2\right)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^2d} +$$

$$= \frac{a^4b\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3d} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d} - \frac{\cos^2(c+dx)\left(4b\left(2a^2+b^2\right)+\right.}{8\left(a^2+\right.}$$

$$= \frac{a^4b\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3d} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d} - \frac{\cos^2(c+dx)\left(4b\left(2a^2+b^2\right)+\right.}{8\left(a^2+\right.}$$

$$= \frac{a\left(3a^4-6a^2b^2-b^4\right)x}{8\left(a^2+b^2\right)^3} + \frac{a^4b\log(\cos(c+dx))}{\left(a^2+b^2\right)^3d} + \frac{a^4b\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3d} + \frac{\cos^4(c+dx)(b}{4\left(a^2\right.}$$

<u>Mathematica</u> [A] time = 2.59212, size = 249, normalized size = 1.58

$$-\frac{a\left(6a^2b^3+5a^4b+b^5\right)\sin(2(c+dx))-4b^2\left(a^2+b^2\right)^2\cos^4(c+dx)+8b^2\left(3a^2b^2+2a^4+b^4\right)\cos^2(c+dx)+2ab\left(6a^2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(2*a*b*(5*a^4 + 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]] + 8*b^2*(2*a^4 + 3*a
^2*b^2 + b^4)*Cos[c + d*x]^2 - 4*b^2*(a^2 + b^2)^2*Cos[c + d*x]^4 + 8*a^4*(
(b^2 + a*Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*b^2*Log[a + b*Tan
```

```
[c + d*x]] + (b^2 - a*Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]]) - 4*a*b
*(a^2 + b^2)^2*Cos[c + d*x]^3*Sin[c + d*x] + a*(5*a^4*b + 6*a^2*b^3 + b^5)*
Sin[2*(c + d*x)])/(16*b*(a^2 + b^2)^3*d)
```

Maple [B] time = 0.068, size = 565, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*tan(d*x+c)),x)
```

```
[Out] -5/8/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*a^5-3/4/d/(a^2+b^2)^3/(1
+tan(d*x+c)^2)^2*tan(d*x+c)^3*a^3*b^2-1/8/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*
tan(d*x+c)^3*a*b^4-1/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*tan(d*x+c)^2*a^4*b-3/
2/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*tan(d*x+c)^2*a^2*b^3-1/2/d/(a^2+b^2)^3/(
1+tan(d*x+c)^2)^2*tan(d*x+c)^2*b^5-3/8/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*tan
(d*x+c)*a^5-1/4/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^3*b^2+1/8/d/(
a^2+b^2)^3/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a*b^4-3/4/d/(a^2+b^2)^3/(1+tan(d*x
+c)^2)^2*a^4*b-1/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*a^2*b^3-1/4/d/(a^2+b^2)^3
/(1+tan(d*x+c)^2)^2*b^5-1/2/d/(a^2+b^2)^3*ln(1+tan(d*x+c)^2)*a^4*b+3/8/d/(a
^2+b^2)^3*arctan(tan(d*x+c))*a^5-3/4/d/(a^2+b^2)^3*arctan(tan(d*x+c))*a^3*b
^2-1/8/d/(a^2+b^2)^3*arctan(tan(d*x+c))*a*b^4+1/d*b/(a^2+b^2)^3*a^4*ln(a+b*
tan(d*x+c))
```

Maxima [A] time = 1.65528, size = 378, normalized size = 2.39

$$\frac{\frac{8\,a^4b\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{4\,a^4b\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{\left(3\,a^5-6\,a^3b^2-ab^4\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{\left(5\,a^3+ab^2\right)\tan(dx+c)^3+6\,a^2b+2\,b^3+4\left(2\,a^2b+b^3\right)\tan(dx+c)^2+\left(3\,a^3-ab^2\right.}{\left(a^4+2\,a^2b^2+b^4\right)\tan(dx+c)^4+a^4+2\,a^2b^2+b^4+2\left(a^4+2\,a^2b^2+b^4\right)\tan(}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(8*a^4*b*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
4*a^4*b*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (3*a^
5 - 6*a^3*b^2 - a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((5*
a^3 + a*b^2)*tan(d*x + c)^3 + 6*a^2*b + 2*b^3 + 4*(2*a^2*b + b^3)*tan(d*x +
 c)^2 + (3*a^3 - a*b^2)*tan(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)
```

```
^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)^2))/d
```

Fricas [A] time = 2.28896, size = 481, normalized size = 3.04

$$\frac{4\,a^4b\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+2\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4+\left(3\,a^5-6\,a\right.}{8\left(a^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*a^4*b*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)
^2 + b^2) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^4 + (3*a^5 - 6*a^3*b^2
 - a*b^4)*d*x - 4*(2*a^4*b + 3*a^2*b^3 + b^5)*cos(d*x + c)^2 + (2*(a^5 + 2*
a^3*b^2 + a*b^4)*cos(d*x + c)^3 - (5*a^5 + 6*a^3*b^2 + a*b^4)*cos(d*x + c))
*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.19573, size = 451, normalized size = 2.85

$$\frac{\frac{8\,a^4b^2\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{4\,a^4b\log\left(\tan\left(dx+c\right)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(3\,a^5-6\,a^3b^2-ab^4\right)\left(dx+c\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{6\,a^4b\tan\left(dx+c\right)^4-5\,a^5\tan\left(dx+c\right)^3-6\,a^3b^2\tan\left(dx+c\right)^3-ab^4\tan\left(dx+\right.}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(8*a^4*b^2*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5
+ b^7) - 4*a^4*b*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
) + (3*a^5 - 6*a^3*b^2 - a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6) + (6*a^4*b*tan(d*x + c)^4 - 5*a^5*tan(d*x + c)^3 - 6*a^3*b^2*tan(d*x + c
)^3 - a*b^4*tan(d*x + c)^3 + 4*a^4*b*tan(d*x + c)^2 - 12*a^2*b^3*tan(d*x +
c)^2 - 4*b^5*tan(d*x + c)^2 - 3*a^5*tan(d*x + c) - 2*a^3*b^2*tan(d*x + c) +
 a*b^4*tan(d*x + c) - 8*a^2*b^3 - 2*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)*(tan(d*x + c)^2 + 1)^2))/d
```

3.53 $\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=168

$$\frac{b\sin^3(c+dx)}{3d\left(a^2+b^2\right)}+\frac{a^2b\sin(c+dx)}{d\left(a^2+b^2\right)^2}+\frac{a\cos^3(c+dx)}{3d\left(a^2+b^2\right)}-\frac{a\cos(c+dx)}{d\left(a^2+b^2\right)}+\frac{ab^2\cos(c+dx)}{d\left(a^2+b^2\right)^2}+\frac{a^3b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{5/2}}$$

```
[Out] (a^3*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(5/2)*d) + (a*b^2*Cos[c + d*x])/((a^2 + b^2)^2*d) - (a*Cos[c + d*x])/(
(a^2 + b^2)*d) + (a*Cos[c + d*x]^3)/(3*(a^2 + b^2)*d) + (a^2*b*Sin[c + d*x]
)/((a^2 + b^2)^2*d) + (b*Sin[c + d*x]^3)/(3*(a^2 + b^2)*d)
```

Rubi [A] time = 0.22299, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3518, 3109, 2564, 30, 2633, 3099, 3074, 206, 2638}

$$\frac{b\sin^3(c+dx)}{3d\left(a^2+b^2\right)}+\frac{a^2b\sin(c+dx)}{d\left(a^2+b^2\right)^2}+\frac{a\cos^3(c+dx)}{3d\left(a^2+b^2\right)}-\frac{a\cos(c+dx)}{d\left(a^2+b^2\right)}+\frac{ab^2\cos(c+dx)}{d\left(a^2+b^2\right)^2}+\frac{a^3b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a^3*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(5/2)*d) + (a*b^2*Cos[c + d*x])/((a^2 + b^2)^2*d) - (a*Cos[c + d*x])/(
(a^2 + b^2)*d) + (a*Cos[c + d*x]^3)/(3*(a^2 + b^2)*d) + (a^2*b*Sin[c + d*x]
)/((a^2 + b^2)^2*d) + (b*Sin[c + d*x]^3)/(3*(a^2 + b^2)*d)
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
```

```
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cos(c+dx)\sin^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$= \frac{a\int \sin^3(c+dx)\,dx}{a^2+b^2} + \frac{b\int \cos(c+dx)\sin^2(c+dx)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\sin^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}$$
$$= \frac{a^2b\sin(c+dx)}{\left(a^2+b^2\right)^2 d} - \frac{\left(a^3b\right)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{\left(a^2+b^2\right)^2} - \frac{\left(ab^2\right)\int \sin(c+dx)\,dx}{\left(a^2+b^2\right)^2} - \frac{a\,\mathrm{Subst}\left(\int \left(1-\right.\right.}{(a}$$
$$= \frac{ab^2\cos(c+dx)}{\left(a^2+b^2\right)^2 d} - \frac{a\cos(c+dx)}{\left(a^2+b^2\right)d} + \frac{a\cos^3(c+dx)}{3\left(a^2+b^2\right)d} + \frac{a^2b\sin(c+dx)}{\left(a^2+b^2\right)^2 d} + \frac{b\sin^3(c+dx)}{3\left(a^2+b^2\right)d} + \frac{\left(a^3b\right)}{}$$
$$= \frac{a^3b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2} d} + \frac{ab^2\cos(c+dx)}{\left(a^2+b^2\right)^2 d} - \frac{a\cos(c+dx)}{\left(a^2+b^2\right)d} + \frac{a\cos^3(c+dx)}{3\left(a^2+b^2\right)d} + \frac{a^2b\,\mathrm{s}}{(a^2}$$

Mathematica [A] time = 1.37492, size = 139, normalized size = 0.83

$$\frac{\sqrt{a^2+b^2}\left(\left(3ab^2-9a^3\right)\cos(c+dx)+a\left(a^2+b^2\right)\cos(3(c+dx))-2b\sin(c+dx)\left(\left(a^2+b^2\right)\cos(2(c+dx))-7a^2-b^2\right)\right)}{12d\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-24*a^3*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 +
b^2]*((-9*a^3 + 3*a*b^2)*Cos[c + d*x] + a*(a^2 + b^2)*Cos[3*(c + d*x)] - 2*
b*(-7*a^2 - b^2 + (a^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*(a^2 + b
^2)^(5/2)*d)
```

Maple [A] time = 0.065, size = 205, normalized size = 1.2

$$\frac{1}{d}\left(-2\,\frac{-ba^2\left(\tan\left(1/2\,dx+c/2\right)\right)^5-ab^2\left(\tan\left(1/2\,dx+c/2\right)\right)^4+\left(-10/3\,ba^2-4/3\,b^3\right)\left(\tan\left(1/2\,dx+c/2\right)\right)^3+2\,a^3\left(\tan\left(1/2\,dx\right.\right.}{\left(a^4+2\,a^2b^2+b^4\right)\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^2\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d*(-2/(a^4+2*a^2*b^2+b^4)*(-b*a^2*tan(1/2*d*x+1/2*c)^5-a*b^2*tan(1/2*d*x+
1/2*c)^4+(-10/3*b*a^2-4/3*b^3)*tan(1/2*d*x+1/2*c)^3+2*a^3*tan(1/2*d*x+1/2*c
)^2-b*a^2*tan(1/2*d*x+1/2*c)+2/3*a^3-1/3*a*b^2)/(1+tan(1/2*d*x+1/2*c)^2)^3-
16*b*a^3/(8*a^4+16*a^2*b^2+8*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*
d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27774, size = 597, normalized size = 3.55

$$\frac{3\,\sqrt{a^2+b^2}a^3b\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)+2\left(a^5+2\,a^3b^2+ab^4\right)\cos\left(d\right.}{6\left(a^6+3\,a^4b^2+3\,a^2b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + b^2)*a^3*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*s
in(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
```

```
 + b^2)) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)^3 - 6*(a^5 + a^3*b^2)*c
os(d*x + c) + 2*(4*a^4*b + 5*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*cos(
d*x + c)^2)*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38606, size = 325, normalized size = 1.93

$$\frac{\frac{3\,a^{3}b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}+\frac{2\left(3\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+10\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+4\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-6\,a^{3}\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3*(3*a^3*b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/ab
s(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 +
b^4)*sqrt(a^2 + b^2)) + 2*(3*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 3*a*b^2*tan(1/2
*d*x + 1/2*c)^4 + 10*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 4*b^3*tan(1/2*d*x + 1/2
*c)^3 - 6*a^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^2*b*tan(1/2*d*x + 1/2*c) - 2*a^3
 + a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 + 1)^3))/d
```

3.54 $\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=94

$$-\frac{\cos^2(c+dx)(a\tan(c+dx)+b)}{2d\left(a^2+b^2\right)}+\frac{a^2b\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{ax\left(a^2-b^2\right)}{2\left(a^2+b^2\right)^2}$$

```
[Out] (a*(a^2 - b^2)*x)/(2*(a^2 + b^2)^2) + (a^2*b*Log[a*Cos[c + d*x] + b*Sin[c +
 d*x]])/((a^2 + b^2)^2*d) - (Cos[c + d*x]^2*(b + a*Tan[c + d*x]))/(2*(a^2 +
 b^2)*d)
```

Rubi [A] time = 0.163218, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1647, 801, 635, 203, 260}

$$-\frac{\cos^2(c+dx)(a\tan(c+dx)+b)}{2d\left(a^2+b^2\right)}+\frac{a^2b\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{ax\left(a^2-b^2\right)}{2\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```


```
[Out] (a*(a^2 - b^2)*x)/(2*(a^2 + b^2)^2) + (a^2*b*Log[a*Cos[c + d*x] + b*Sin[c +
 d*x]])/((a^2 + b^2)^2*d) - (Cos[c + d*x]^2*(b + a*Tan[c + d*x]))/(2*(a^2 +
 b^2)*d)
```


Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
```

```
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^2}{(a+x)\left(b^2+x^2\right)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^2}{a^2+b^2}+\frac{ab^2x}{a^2+b^2}}{(a+x)\left(b^2+x^2\right)}\,dx, x, b\tan(c+dx)\right)}{2bd}$$

$$= -\frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} - \frac{\text{Subst}\left(\int \left(-\frac{2a^2b^2}{\left(a^2+b^2\right)^2(a+x)} - \frac{ab^2\left(a^2-b^2-2ax\right)}{\left(a^2+b^2\right)^2\left(b^2+x^2\right)}\right)dx, x, b\tan(c+\right.}{2bd}$$

$$= \frac{a^2b\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2d} - \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} + \frac{(ab)\,\text{Subst}\left(\int \frac{a^2-b^2-2ax}{b^2+x^2}\,dx, x, b\right.}{2\left(a^2+b^2\right)^2d}$$

$$= \frac{a^2b\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2d} - \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} - \frac{\left(a^2b\right)\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\,\text{ta}\right.}{\left(a^2+b^2\right)^2d}$$

$$= \frac{a\left(a^2-b^2\right)x}{2\left(a^2+b^2\right)^2} + \frac{a^2b\log(\cos(c+dx))}{\left(a^2+b^2\right)^2d} + \frac{a^2b\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2d} - \frac{\cos^2(c+dx)(b+a\tan(c-}{2\left(a^2+b^2\right)d}$$

Mathematica [A] time = 0.722595, size = 170, normalized size = 1.81

$$-\frac{2b^2\left(a^2+b^2\right)\cos^2(c+dx)+2ab\left(a^2+b^2\right)\tan^{-1}(\tan(c+dx))+a\left(b\left(a^2+b^2\right)\sin(2(c+dx))+2a\left(-2b^2\log(a+b\tan(c\right.\right.}{4bd\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(2*a*b*(a^2 + b^2)*ArcTan[Tan[c + d*x]] + 2*b^2*(a^2 + b^2)*Cos[c + d*x]^2
 + a*(2*a*((b^2 + a*Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*b^2*Lo
g[a + b*Tan[c + d*x]] + (b^2 - a*Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x
]]) + b*(a^2 + b^2)*Sin[2*(c + d*x)]))/(4*b*(a^2 + b^2)^2*d)
```

Maple [B] time = 0.059, size = 238, normalized size = 2.5

$$-\frac{a^3\tan(dx+c)}{2d\left(a^2+b^2\right)^2\left(1+(\tan(dx+c))^2\right)}-\frac{a\tan(dx+c)\,b^2}{2d\left(a^2+b^2\right)^2\left(1+(\tan(dx+c))^2\right)}-\frac{ba^2}{2d\left(a^2+b^2\right)^2\left(1+(\tan(dx+c))^2\right)}-\frac{}{2d(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*tan(d*x+c)),x)
```

```
[Out] -1/2/d/(a^2+b^2)^2/(1+tan(d*x+c)^2)*tan(d*x+c)*a^3-1/2/d/(a^2+b^2)^2/(1+tan
(d*x+c)^2)*tan(d*x+c)*a*b^2-1/2/d/(a^2+b^2)^2/(1+tan(d*x+c)^2)*b*a^2-1/2/d/
(a^2+b^2)^2/(1+tan(d*x+c)^2)*b^3-1/2/d/(a^2+b^2)^2*ln(1+tan(d*x+c)^2)*a^2*b
+1/2/d/(a^2+b^2)^2*arctan(tan(d*x+c))*a^3-1/2/d/(a^2+b^2)^2*arctan(tan(d*x+
c))*a*b^2+1/d*b*a^2/(a^2+b^2)^2*ln(a+b*tan(d*x+c))
```

Maxima [A] time = 1.5721, size = 194, normalized size = 2.06

$$\frac{\frac{2\,a^2b\log(b\tan(dx+c)+a)}{a^4+2\,a^2b^2+b^4}-\frac{a^2b\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^3-ab^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{a\tan(dx+c)+b}{\left(a^2+b^2\right)\tan(dx+c)^2+a^2+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2*b*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) - a^2*b*log(ta
n(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 - a*b^2)*(d*x + c)/(a^4 +
2*a^2*b^2 + b^4) - (a*tan(d*x + c) + b)/((a^2 + b^2)*tan(d*x + c)^2 + a^2 +
 b^2))/d
```

Fricas [A] time = 2.12276, size = 278, normalized size = 2.96

$$\frac{a^2b\log\left(2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2\right)+\left(a^3-ab^2\right)dx-\left(a^2b+b^3\right)\cos(dx+c)^2-\left(a^3+a\right.}{2\left(a^4+2\,a^2b^2+b^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2) + (a^3 - a*b^2)*d*x - (a^2*b + b^3)*cos(d*x + c)^2 - (a^3 + a*b^2)*
cos(d*x + c)*sin(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.14829, size = 248, normalized size = 2.64

$$\frac{\frac{2\,a^2b^2\log\left(|b\tan(dx+c)+a|\right)}{a^4b+2\,a^2b^3+b^5}-\frac{a^2b\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^3-ab^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{a^2b\tan(dx+c)^2-a^3\tan(dx+c)-ab^2\tan(dx+c)-b^3}{\left(a^4+2\,a^2b^2+b^4\right)\left(\tan(dx+c)^2+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*b^2*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - a^2
*b*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 - a*b^2)*(d*x + c
)/(a^4 + 2*a^2*b^2 + b^4) + (a^2*b*tan(d*x + c)^2 - a^3*tan(d*x + c) - a*b^
2*tan(d*x + c) - b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(d*x + c)^2 + 1)))/d
```

3.55 $\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=90

$$\frac{b\sin(c+dx)}{d\left(a^2+b^2\right)}-\frac{a\cos(c+dx)}{d\left(a^2+b^2\right)}+\frac{ab\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

```
[Out] (a*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 + b^
2)^(3/2)*d) - (a*Cos[c + d*x])/((a^2 + b^2)*d) + (b*Sin[c + d*x])/((a^2 + b
^2)*d)
```

Rubi [A] time = 0.105716, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3518, 3109, 2637, 2638, 3074, 206}

$$\frac{b\sin(c+dx)}{d\left(a^2+b^2\right)}-\frac{a\cos(c+dx)}{d\left(a^2+b^2\right)}+\frac{ab\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 + b^
2)^(3/2)*d) - (a*Cos[c + d*x])/((a^2 + b^2)*d) + (b*Sin[c + d*x])/((a^2 + b
^2)*d)
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
```

```
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx &= \int \frac{\cos(c+dx)\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx \\
&= \frac{a\int \sin(c+dx)\,dx}{a^2+b^2}+\frac{b\int \cos(c+dx)\,dx}{a^2+b^2}-\frac{(ab)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2} \\
&= -\frac{a\cos(c+dx)}{\left(a^2+b^2\right)d}+\frac{b\sin(c+dx)}{\left(a^2+b^2\right)d}+\frac{(ab)\,\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(c+dx)-a\sin(c+dx)\right)}{\left(a^2+b^2\right)d} \\
&= \frac{ab\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d}-\frac{a\cos(c+dx)}{\left(a^2+b^2\right)d}+\frac{b\sin(c+dx)}{\left(a^2+b^2\right)d}
\end{aligned}$$

Mathematica [A] time = 0.336073, size = 79, normalized size = 0.88

$$\frac{\sqrt{a^2+b^2}(b\sin(c+dx)-a\cos(c+dx))-2ab\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-2*a*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2
]*(-(a*Cos[c + d*x]) + b*Sin[c + d*x]))/((a^2 + b^2)^(3/2)*d)
```

Maple [A] time = 0.05, size = 100, normalized size = 1.1

$$\frac{1}{d}\left(-2\,\frac{-\tan(1/2\,dx+c/2)\,b+a}{\left(a^2+b^2\right)\left(1+(\tan(1/2\,dx+c/2))^2\right)}-4\,\frac{ab}{\left(2\,a^2+2\,b^2\right)\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\,\frac{2\,a\tan(1/2\,dx+c/2)-2\,b}{\sqrt{a^2+b^2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d*(-2/(a^2+b^2)*(-tan(1/2*d*x+1/2*c)*b+a)/(1+tan(1/2*d*x+1/2*c)^2)-4*a*b/
(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2
+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.0465, size = 435, normalized size = 4.83

$$\frac{\sqrt{a^2+b^2}ab\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)-2\left(a^3+ab^2\right)\cos\left(dx+c\right)+2\left(\right.}{2\left(a^4+2\,a^2b^2+b^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*a*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)
*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d
*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b
^2)) - 2*(a^3 + a*b^2)*cos(d*x + c) + 2*(a^2*b + b^3)*sin(d*x + c))/((a^4 +
 2*a^2*b^2 + b^4)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*tan(c + d*x)), x)
```

Giac [A] time = 1.33428, size = 159, normalized size = 1.77

$$\frac{\frac{ab\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}{\left(a^2+b^2\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*ta
n(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(b*tan
(1/2*d*x + 1/2*c) - a)/((a^2 + b^2)*(tan(1/2*d*x + 1/2*c)^2 + 1)))/d
```

3.56 $\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=66

$$\frac{b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + (b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x
])/Sqrt[a^2 + b^2]])/(a*Sqrt[a^2 + b^2]*d)
```

Rubi [A] time = 0.128764, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3518, 3110, 3770, 3074, 206}

$$\frac{b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + (b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x
])/Sqrt[a^2 + b^2]])/(a*Sqrt[a^2 + b^2]*d)
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx &= \int \frac{\cot(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx \\
&= \int \left(\frac{\csc(c+dx)}{a}-\frac{b}{a(a\cos(c+dx)+b\sin(c+dx))}\right)dx \\
&= \frac{\int \csc(c+dx)\,dx}{a}-\frac{b\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{b\,\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(c+dx)-a\sin(c+dx)\right)}{ad} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}
\end{aligned}$$

Mathematica [A] time = 0.105158, size = 75, normalized size = 1.14

$$\frac{-\frac{2b\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((-2*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
- Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]])/(a*d)
```

Maple [A] time = 0.056, size = 65, normalized size = 1.

$$-2\,{\frac{b}{ad\sqrt{a^{2}+b^{2}}}}{\rm Artanh} \left(1/2\,{\frac{2\,a\tan \left(1/2\,dx+c/2 \right) -2\,b}{\sqrt{a^{2}+b^{2}}}} \right) +{\frac{1}{ad}}\ln \left(\tan \left({\frac{dx}{2}}+{\frac{c}{2}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] -2/d*b/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)
^(1/2))+1/d/a*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.37691, size = 450, normalized size = 6.82

$$\frac{\sqrt{a^2+b^2}b\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)-\left(a^2+b^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\right.}{2\left(a^3+ab^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*c
os(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x
```

```
 + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2
)) - (a^2 + b^2)*log(1/2*cos(d*x + c) + 1/2) + (a^2 + b^2)*log(-1/2*cos(d*x
 + c) + 1/2))/((a^3 + a*b^2)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*tan(c + d*x)), x)
```

Giac [A] time = 1.43594, size = 127, normalized size = 1.92

$$\frac{\frac{b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(
1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) + log(abs(
tan(1/2*d*x + 1/2*c)))/a)/d
```

3.57 $\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=50

$$-\frac{b\log(\tan(c+dx))}{a^2d}+\frac{b\log(a+b\tan(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

```
[Out] -(Cot[c + d*x]/(a*d)) - (b*Log[Tan[c + d*x]])/(a^2*d) + (b*Log[a + b*Tan[c
+ d*x]])/(a^2*d)
```

Rubi [A] time = 0.0602185, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 44}

$$-\frac{b\log(\tan(c+dx))}{a^2d}+\frac{b\log(a+b\tan(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(Cot[c + d*x]/(a*d)) - (b*Log[Tan[c + d*x]])/(a^2*d) + (b*Log[a + b*Tan[c
+ d*x]])/(a^2*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{1}{x^2(a+x)}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{b\,\mathrm{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{1}{a^2x}+\frac{1}{a^2(a+x)}\right)dx, x, b\tan(c+dx)\right)}{d}$$
$$= -\frac{\cot(c+dx)}{ad}-\frac{b\log(\tan(c+dx))}{a^2d}+\frac{b\log(a+b\tan(c+dx))}{a^2d}$$

Mathematica [A] time = 0.130935, size = 47, normalized size = 0.94

$$\frac{b(\log(a\cos(c+dx)+b\sin(c+dx))-\log(\sin(c+dx)))-a\cot(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-(a*Cot[c + d*x]) + b*(-Log[Sin[c + d*x]] + Log[a*Cos[c + d*x] + b*Sin[c +
 d*x]]))/(a^2*d)
```

Maple [A] time = 0.062, size = 53, normalized size = 1.1

$$-\frac{1}{ad\tan(dx+c)}-\frac{b\ln(\tan(dx+c))}{a^2d}+\frac{b\ln(a+b\tan(dx+c))}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*tan(d*x+c)),x)
```

```
[Out] -1/d/a/tan(d*x+c)-b*ln(tan(d*x+c))/a^2/d+b*ln(a+b*tan(d*x+c))/a^2/d
```

Maxima [A] time = 1.06775, size = 63, normalized size = 1.26

$$\frac{\frac{b\log(b\tan(dx+c)+a)}{a^2}-\frac{b\log(\tan(dx+c))}{a^2}-\frac{1}{a\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b*log(b*tan(d*x + c) + a)/a^2 - b*log(tan(d*x + c))/a^2 - 1/(a*tan(d*x + c
)))/d
```

Fricas [A] time = 2.05537, size = 246, normalized size = 4.92

$$\frac{b\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)\sin\left(dx+c\right)-b\log\left(-\frac{1}{4}\,\cos\left(dx+c\right)^2+\frac{1}{4}\right)\sin\left(dx+c\right)}{2\,a^2d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b
^2)*sin(d*x + c) - b*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 2*a*cos(
d*x + c))/(a^2*d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*tan(c + d*x)), x)
```

Giac [A] time = 1.20827, size = 81, normalized size = 1.62

$$\frac{\frac{b\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^2}-\frac{b\log\left(|\tan\left(dx+c\right)|\right)}{a^2}+\frac{b\tan\left(dx+c\right)-a}{a^2\tan\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (b*log(abs(b*tan(d*x + c) + a))/a^2 - b*log(abs(tan(d*x + c)))/a^2 + (b*tan
(d*x + c) - a)/(a^2*tan(d*x + c)))/d
```

3.58 $\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=122

$$-\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot(c+dx}{2}$$

```
[Out] -ArcTanh[Cos[c + d*x]]/(2*a*d) - (b^2*ArcTanh[Cos[c + d*x]])/(a^3*d) + (b*S
qrt[a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/
(a^3*d) + (b*Csc[c + d*x])/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.307234, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3518, 3110, 3768, 3770, 2621, 321, 207, 2622, 3104, 3074, 206}

$$-\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot(c+dx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(2*a*d) - (b^2*ArcTanh[Cos[c + d*x]])/(a^3*d) + (b*S
qrt[a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/
(a^3*d) + (b*Csc[c + d*x])/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
```

```
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cot(c+dx)\csc^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

$$= \int \left(\frac{\csc^3(c+dx)}{a} - \frac{b\csc^2(c+dx)\sec(c+dx)}{a^2} + \frac{b^2\csc(c+dx)\sec^2(c+dx)}{a^3} - \frac{b^3\sec^2}{a^3(a\cos(c+dx)}\right.$$

$$= \frac{\int \csc^3(c+dx)\,dx}{a} - \frac{b\int \csc^2(c+dx)\sec(c+dx)\,dx}{a^2} + \frac{b^2\int \csc(c+dx)\sec^2(c+dx)\,dx}{a^3} - \frac{b^3\int}{}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{b^2\sec(c+dx)}{a^3d} + \frac{\int \csc(c+dx)\,dx}{2a} + \frac{b\int \sec(c+dx)\,dx}{a^2} - \frac{\left(b\left(a^2+\right.\right.}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad} + \frac{b\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{b\csc(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} + -\frac{b}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a^3d} +$$

Mathematica [A] time = 0.826587, size = 179, normalized size = 1.47

$$\frac{-16b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right) + a^2\left(-\csc^2\left(\frac{1}{2}(c+dx)\right)\right) + a^2\sec^2\left(\frac{1}{2}(c+dx)\right) + 4a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 4a^2}{8a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-16*b*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] +
 4*a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 - 4*a^2*Log[Cos[(c + d*x)/
2]] - 8*b^2*Log[Cos[(c + d*x)/2]] + 4*a^2*Log[Sin[(c + d*x)/2]] + 8*b^2*Log
[Sin[(c + d*x)/2]] + a^2*Sec[(c + d*x)/2]^2 + 4*a*b*Tan[(c + d*x)/2])/(8*a^
3*d)
```

Maple [A] time = 0.069, size = 162, normalized size = 1.3

$$\frac{1}{8\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+\frac{b}{2\,a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\,\frac{b\sqrt{a^2+b^2}}{da^3}\mathrm{Artanh}\left(1/2\,\frac{2\,a\tan(1/2\,dx+c/2)-2\,b}{\sqrt{a^2+b^2}}\right)-\frac{1}{8\,ad}\left(\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/8/d/a*tan(1/2*d*x+1/2*c)^2+1/2/d/a^2*tan(1/2*d*x+1/2*c)*b-2/d/a^3*b*(a^2+
b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-1/8/d/
a/tan(1/2*d*x+1/2*c)^2+1/2/d/a*ln(tan(1/2*d*x+1/2*c))+1/d/a^3*ln(tan(1/2*d*
x+1/2*c))*b^2+1/2/d*b/a^2/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.68281, size = 644, normalized size = 5.28

$$\frac{2\,a^2\cos(dx+c)-4\,ab\sin(dx+c)+2\left(b\cos(dx+c)^2-b\right)\sqrt{a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{}}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*cos(d*x + c) - 4*a*b*sin(d*x + c) + 2*(b*cos(d*x + c)^2 - b)*sqr
t(a^2 + b^2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c
)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2
*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - ((a^2
 + 2*b^2)*cos(d*x + c)^2 - a^2 - 2*b^2)*log(1/2*cos(d*x + c) + 1/2) + ((a^2
 + 2*b^2)*cos(d*x + c)^2 - a^2 - 2*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(a^3*
d*cos(d*x + c)^2 - a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*tan(c + d*x)), x)
```

Giac [A] time = 1.35144, size = 282, normalized size = 2.31

$$\frac{\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}+\frac{4\left(a^2+2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{8\left(a^2b+b^3\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^3}-\frac{6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,b^2\,\mathrm{t}}{a^3\,\mathrm{t}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*((a*tan(1/2*d*x + 1/2*c)^2 + 4*b*tan(1/2*d*x + 1/2*c))/a^2 + 4*(a^2 + 2
*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 8*(a^2*b + b^3)*log(abs(2*a*tan(
1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) -
2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3) - (6*a^2*tan(1/2*d*x + 1/2*
c)^2 + 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/(a
```

```
^3*tan(1/2*d*x + 1/2*c)^2))/d
```

3.59 $\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=108

$$-\frac{(a^2+b^2)\cot(c+dx)}{a^3d}-\frac{b(a^2+b^2)\log(\tan(c+dx))}{a^4d}+\frac{b(a^2+b^2)\log(a+b\tan(c+dx))}{a^4d}+\frac{b\cot^2(c+dx)}{2a^2d}-\frac{\cot^3(c+a}{3ad}$$

```
[Out] -(((a^2 + b^2)*Cot[c + d*x])/(a^3*d)) + (b*Cot[c + d*x]^2)/(2*a^2*d) - Cot[
c + d*x]^3/(3*a*d) - (b*(a^2 + b^2)*Log[Tan[c + d*x]])/(a^4*d) + (b*(a^2 +
b^2)*Log[a + b*Tan[c + d*x]])/(a^4*d)
```

Rubi [A] time = 0.104453, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 894}

$$-\frac{(a^2+b^2)\cot(c+dx)}{a^3d}-\frac{b(a^2+b^2)\log(\tan(c+dx))}{a^4d}+\frac{b(a^2+b^2)\log(a+b\tan(c+dx))}{a^4d}+\frac{b\cot^2(c+dx)}{2a^2d}-\frac{\cot^3(c+a}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(((a^2 + b^2)*Cot[c + d*x])/(a^3*d)) + (b*Cot[c + d*x]^2)/(2*a^2*d) - Cot[
c + d*x]^3/(3*a*d) - (b*(a^2 + b^2)*Log[Tan[c + d*x]])/(a^4*d) + (b*(a^2 +
b^2)*Log[a + b*Tan[c + d*x]])/(a^4*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{b^2+x^2}{x^4(a+x)}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \left(\frac{b^2}{ax^4}-\frac{b^2}{a^2x^3}+\frac{a^2+b^2}{a^3x^2}+\frac{-a^2-b^2}{a^4x}+\frac{a^2+b^2}{a^4(a+x)}\right) dx, x, b\tan(c+dx)\right)}{d}$$
$$= -\frac{(a^2+b^2)\cot(c+dx)}{a^3d}+\frac{b\cot^2(c+dx)}{2a^2d}-\frac{\cot^3(c+dx)}{3ad}-\frac{b(a^2+b^2)\log(\tan(c+dx))}{a^4d}+\frac{b(}{}$$

Mathematica [A] time = 0.455689, size = 95, normalized size = 0.88

$$\frac{-2\cot(c+dx)\left(a^3\csc^2(c+dx)+2a^3+3ab^2\right)-6b\left(a^2+b^2\right)(\log(\sin(c+dx))-\log(a\cos(c+dx)+b\sin(c+dx)))+3a}{6a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*a^2*b*Csc[c + d*x]^2 - 2*Cot[c + d*x]*(2*a^3 + 3*a*b^2 + a^3*Csc[c + d*x
]^2) - 6*b*(a^2 + b^2)*(Log[Sin[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin[c +
d*x]]))/(6*a^4*d)
```

Maple [A] time = 0.076, size = 144, normalized size = 1.3

$$-\frac{1}{3\,ad\,(\tan(dx+c))^3}-\frac{1}{ad\tan(dx+c)}-\frac{b^2}{da^3\tan(dx+c)}+\frac{b}{2\,a^2d\,(\tan(dx+c))^2}-\frac{b\ln(\tan(dx+c))}{a^2d}-\frac{b^3\ln(\tan(dx}{da^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*tan(d*x+c)),x)
```

```
[Out] -1/3/d/a/tan(d*x+c)^3-1/d/a/tan(d*x+c)-1/d/a^3/tan(d*x+c)*b^2+1/2/d*b/a^2/t
an(d*x+c)^2-b*ln(tan(d*x+c))/a^2/d-1/d/a^4*b^3*ln(tan(d*x+c))+b*ln(a+b*tan(
d*x+c))/a^2/d+1/d/a^4*b^3*ln(a+b*tan(d*x+c))
```

Maxima [A] time = 1.06377, size = 131, normalized size = 1.21

$$\frac{\frac{6\left(a^2b+b^3\right)\log(b\tan(dx+c)+a)}{a^4}-\frac{6\left(a^2b+b^3\right)\log(\tan(dx+c))}{a^4}+\frac{3\,ab\tan(dx+c)-6\left(a^2+b^2\right)\tan(dx+c)^2-2\,a^2}{a^3\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(a^2*b + b^3)*log(b*tan(d*x + c) + a)/a^4 - 6*(a^2*b + b^3)*log(tan(
d*x + c))/a^4 + (3*a*b*tan(d*x + c) - 6*(a^2 + b^2)*tan(d*x + c)^2 - 2*a^2)
/(a^3*tan(d*x + c)^3))/d
```

Fricas [B] time = 2.15412, size = 500, normalized size = 4.63

$$-\frac{2\left(2\,a^3+3\,ab^2\right)\cos\left(dx+c\right)^3+3\,a^2b\sin\left(dx+c\right)+3\left(a^2b+b^3-\left(a^2b+b^3\right)\cos\left(dx+c\right)^2\right)\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx\right.\right.}{6\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3 + 3*a^2*b*sin(d*x + c) + 3*(a^2*b
+ b^3 - (a^2*b + b^3)*cos(d*x + c)^2)*log(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c) - 3*(a^2*b + b^3 - (a^2*b +
 b^3)*cos(d*x + c)^2)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 6*(a^3
+ a*b^2)*cos(d*x + c))/((a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18105, size = 194, normalized size = 1.8

$$-\frac{\frac{6\left(a^{2}b+b^{3}\right)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{a^{4}}-\frac{6\left(a^{2}b^{2}+b^{4}\right)\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^{4}b}-\frac{11\,a^{2}b\tan\left(dx+c\right)^{3}+11\,b^{3}\tan\left(dx+c\right)^{3}-6\,a^{3}\tan\left(dx+c\right)^{2}-6\,ab^{2}\tan\left(dx+c\right)^{2}+3\,a^{2}b\tan\left(dx\right.}{a^{4}\tan\left(dx+c\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(a^2*b + b^3)*log(abs(tan(d*x + c)))/a^4 - 6*(a^2*b^2 + b^4)*log(ab
s(b*tan(d*x + c) + a))/(a^4*b) - (11*a^2*b*tan(d*x + c)^3 + 11*b^3*tan(d*x
+ c)^3 - 6*a^3*tan(d*x + c)^2 - 6*a*b^2*tan(d*x + c)^2 + 3*a^2*b*tan(d*x +
c) - 2*a^3)/(a^4*tan(d*x + c)^3))/d
```

3.60 $\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=169

$$-\frac{\left(2a^2+b^2\right)\cot^3(c+dx)}{3a^3d}+\frac{b\left(2a^2+b^2\right)\cot^2(c+dx)}{2a^4d}-\frac{\left(a^2+b^2\right)^2\cot(c+dx)}{a^5d}-\frac{b\left(a^2+b^2\right)^2\log(\tan(c+dx))}{a^6d}+\frac{b\left(a^2+\right.}{}$$

```
[Out] -(((a^2 + b^2)^2*Cot[c + d*x])/(a^5*d)) + (b*(2*a^2 + b^2)*Cot[c + d*x]^2)/
(2*a^4*d) - ((2*a^2 + b^2)*Cot[c + d*x]^3)/(3*a^3*d) + (b*Cot[c + d*x]^4)/(
4*a^2*d) - Cot[c + d*x]^5/(5*a*d) - (b*(a^2 + b^2)^2*Log[Tan[c + d*x]])/(a^
6*d) + (b*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]])/(a^6*d)
```

Rubi [A] time = 0.150783, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 894}

$$-\frac{\left(2a^2+b^2\right)\cot^3(c+dx)}{3a^3d}+\frac{b\left(2a^2+b^2\right)\cot^2(c+dx)}{2a^4d}-\frac{\left(a^2+b^2\right)^2\cot(c+dx)}{a^5d}-\frac{b\left(a^2+b^2\right)^2\log(\tan(c+dx))}{a^6d}+\frac{b\left(a^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```


```
[Out] -(((a^2 + b^2)^2*Cot[c + d*x])/(a^5*d)) + (b*(2*a^2 + b^2)*Cot[c + d*x]^2)/
(2*a^4*d) - ((2*a^2 + b^2)*Cot[c + d*x]^3)/(3*a^3*d) + (b*Cot[c + d*x]^4)/(
4*a^2*d) - Cot[c + d*x]^5/(5*a*d) - (b*(a^2 + b^2)^2*Log[Tan[c + d*x]])/(a^
6*d) + (b*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]])/(a^6*d)
```


Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
```

```
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{\left(b^2+x^2\right)^2}{x^6(a+x)}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\,\text{Subst}\left(\int \left(\frac{b^4}{ax^6}-\frac{b^4}{a^2x^5}+\frac{2a^2b^2+b^4}{a^3x^4}+\frac{b^2\left(-2a^2-b^2\right)}{a^4x^3}+\frac{\left(a^2+b^2\right)^2}{a^5x^2}-\frac{\left(a^2+b^2\right)^2}{a^6x}+\frac{\left(a^2+b^2\right)^2}{a^6(a+x)}\right)dx, x, b\tan(c+$$

$$= -\frac{\left(a^2+b^2\right)^2\cot(c+dx)}{a^5d}+\frac{b\left(2a^2+b^2\right)\cot^2(c+dx)}{2a^4d}-\frac{\left(2a^2+b^2\right)\cot^3(c+dx)}{3a^3d}+\frac{b\cot^4(c+}{4a^2d}$$

Mathematica [A] time = 2.16387, size = 150, normalized size = 0.89

$$\frac{15b\left(2a^2\left(a^2+b^2\right)\csc^2(c+dx)-4\left(a^2+b^2\right)^2\left(\log(\sin(c+dx))-\log(a\cos(c+dx)+b\sin(c+dx))\right)+a^4\csc^4(c+dx)\right)}{60a^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-4*Cot[c + d*x]*(8*a^5 + 25*a^3*b^2 + 15*a*b^4 + a^3*(4*a^2 + 5*b^2)*Csc[c
 + d*x]^2 + 3*a^5*Csc[c + d*x]^4) + 15*b*(2*a^2*(a^2 + b^2)*Csc[c + d*x]^2
+ a^4*Csc[c + d*x]^4 - 4*(a^2 + b^2)^2*(Log[Sin[c + d*x]] - Log[a*Cos[c + d
*x] + b*Sin[c + d*x]])))/(60*a^6*d)
```

Maple [A] time = 0.083, size = 273, normalized size = 1.6

$$-\frac{1}{5\,ad\left(\tan\left(dx+c\right)\right)^5}-\frac{2}{3\,ad\left(\tan\left(dx+c\right)\right)^3}-\frac{b^2}{3\,da^3\left(\tan\left(dx+c\right)\right)^3}-\frac{1}{ad\tan\left(dx+c\right)}-2\,\frac{b^2}{da^3\tan\left(dx+c\right)}-\frac{b^4}{da^5\tan\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+b*tan(d*x+c)),x)
```

```
[Out] -1/5/d/a/tan(d*x+c)^5-2/3/d/a/tan(d*x+c)^3-1/3/d/a^3/tan(d*x+c)^3*b^2-1/d/a
/tan(d*x+c)-2/d/a^3/tan(d*x+c)*b^2-1/d/a^5/tan(d*x+c)*b^4+1/4/d*b/a^2/tan(d
```

```
*x+c)^4+1/d*b/a^2/tan(d*x+c)^2+1/2/d/a^4*b^3/tan(d*x+c)^2-b*ln(tan(d*x+c))/
a^2/d-2/d/a^4*b^3*ln(tan(d*x+c))-1/d/a^6*b^5*ln(tan(d*x+c))+b*ln(a+b*tan(d*
x+c))/a^2/d+2/d/a^4*b^3*ln(a+b*tan(d*x+c))+1/d/a^6*b^5*ln(a+b*tan(d*x+c))
```

Maxima [A] time = 1.09497, size = 227, normalized size = 1.34

$$\frac{\frac{60\left(a^4b+2\,a^2b^3+b^5\right)\log(b\tan(dx+c)+a)}{a^6}-\frac{60\left(a^4b+2\,a^2b^3+b^5\right)\log(\tan(dx+c))}{a^6}+\frac{15\,a^3b\tan(dx+c)-60\left(a^4+2\,a^2b^2+b^4\right)\tan(dx+c)^4-12\,a^4+30\left(2\,a^3b+ab^3\right)\tan}{a^5\tan(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*(a^4*b + 2*a^2*b^3 + b^5)*log(b*tan(d*x + c) + a)/a^6 - 60*(a^4*b
+ 2*a^2*b^3 + b^5)*log(tan(d*x + c))/a^6 + (15*a^3*b*tan(d*x + c) - 60*(a^4
 + 2*a^2*b^2 + b^4)*tan(d*x + c)^4 - 12*a^4 + 30*(2*a^3*b + a*b^3)*tan(d*x
+ c)^3 - 20*(2*a^4 + a^2*b^2)*tan(d*x + c)^2)/(a^5*tan(d*x + c)^5))/d
```

Fricas [B] time = 2.41224, size = 902, normalized size = 5.34

$$-\frac{4\left(8\,a^5+25\,a^3b^2+15\,ab^4\right)\cos(dx+c)^5-20\left(4\,a^5+11\,a^3b^2+6\,ab^4\right)\cos(dx+c)^3-30\left(a^4b+2\,a^2b^3+b^5+\left(a^4b+2\,a^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(4*(8*a^5 + 25*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 20*(4*a^5 + 11*a^
3*b^2 + 6*a*b^4)*cos(d*x + c)^3 - 30*(a^4*b + 2*a^2*b^3 + b^5 + (a^4*b + 2*
a^2*b^3 + b^5)*cos(d*x + c)^4 - 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^2)
*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*si
n(d*x + c) + 30*(a^4*b + 2*a^2*b^3 + b^5 + (a^4*b + 2*a^2*b^3 + b^5)*cos(d*
x + c)^4 - 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^2)*log(-1/4*cos(d*x + c
)^2 + 1/4)*sin(d*x + c) + 60*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c) - 15*(3
*a^4*b + 2*a^2*b^3 - 2*(a^4*b + a^2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/((a^
6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 + a^6*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+b*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24413, size = 339, normalized size = 2.01

$$-\frac{\frac{60\left(a^4b+2\,a^2b^3+b^5\right)\log(|\tan(dx+c)|)}{a^6}-\frac{60\left(a^4b^2+2\,a^2b^4+b^6\right)\log(|b\tan(dx+c)+a|)}{a^6b}-\frac{137\,a^4b\tan(dx+c)^5+274\,a^2b^3\tan(dx+c)^5+137\,b^5\tan(dx+c)^5-60\,a^5\,\mathrm{t}}{}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(60*(a^4*b + 2*a^2*b^3 + b^5)*log(abs(tan(d*x + c)))/a^6 - 60*(a^4*b^
2 + 2*a^2*b^4 + b^6)*log(abs(b*tan(d*x + c) + a))/(a^6*b) - (137*a^4*b*tan(
d*x + c)^5 + 274*a^2*b^3*tan(d*x + c)^5 + 137*b^5*tan(d*x + c)^5 - 60*a^5*t
an(d*x + c)^4 - 120*a^3*b^2*tan(d*x + c)^4 - 60*a*b^4*tan(d*x + c)^4 + 60*a
^4*b*tan(d*x + c)^3 + 30*a^2*b^3*tan(d*x + c)^3 - 40*a^5*tan(d*x + c)^2 - 2
0*a^3*b^2*tan(d*x + c)^2 + 15*a^4*b*tan(d*x + c) - 12*a^5)/(a^6*tan(d*x + c
)^5))/d
```

3.61 $\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=297

$$-\frac{a^6 b}{d\left(a^2+b^2\right)^4 (a+b\tan(c+dx))}-\frac{\cos^6(c+dx)\left(\left(a^2-b^2\right)\tan(c+dx)+2ab\right)}{6d\left(a^2+b^2\right)^2}+\frac{\cos^4(c+dx)\left(\left(-18a^2b^2+13a^4-7b^4\right)\mathrm{t}\cdots\right.}{24d\left(a^2+b^2\cdots\right.}$$

```
[Out] ((5*a^8 - 80*a^6*b^2 + 50*a^4*b^4 + 8*a^2*b^6 + b^8)*x)/(16*(a^2 + b^2)^5)
+ (2*a^5*b*(a^2 - 3*b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)
^5*d) - (a^6*b)/((a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^6*(2
*a*b + (a^2 - b^2)*Tan[c + d*x]))/(6*(a^2 + b^2)^2*d) + (Cos[c + d*x]^4*(12
*a*b*(3*a^2 + b^2) + (13*a^4 - 18*a^2*b^2 - 7*b^4)*Tan[c + d*x]))/(24*(a^2
+ b^2)^3*d) - (Cos[c + d*x]^2*(48*a^5*b + (11*a^6 - 43*a^4*b^2 - 7*a^2*b^4
- b^6)*Tan[c + d*x]))/(16*(a^2 + b^2)^4*d)
```

Rubi [A] time = 0.913641, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$-\frac{a^6 b}{d\left(a^2+b^2\right)^4 (a+b\tan(c+dx))}-\frac{\cos^6(c+dx)\left(\left(a^2-b^2\right)\tan(c+dx)+2ab\right)}{6d\left(a^2+b^2\right)^2}+\frac{\cos^4(c+dx)\left(\left(-18a^2b^2+13a^4-7b^4\right)\mathrm{t}\cdots\right.}{24d\left(a^2+b^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((5*a^8 - 80*a^6*b^2 + 50*a^4*b^4 + 8*a^2*b^6 + b^8)*x)/(16*(a^2 + b^2)^5)
+ (2*a^5*b*(a^2 - 3*b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)
^5*d) - (a^6*b)/((a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^6*(2
*a*b + (a^2 - b^2)*Tan[c + d*x]))/(6*(a^2 + b^2)^2*d) + (Cos[c + d*x]^4*(12
*a*b*(3*a^2 + b^2) + (13*a^4 - 18*a^2*b^2 - 7*b^4)*Tan[c + d*x]))/(24*(a^2
+ b^2)^3*d) - (Cos[c + d*x]^2*(48*a^5*b + (11*a^6 - 43*a^4*b^2 - 7*a^2*b^4
- b^6)*Tan[c + d*x]))/(16*(a^2 + b^2)^4*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^6}{(a+x)^2\left(b^2+x^2\right)^4}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^6(c+dx)\left(2ab+\left(a^2-b^2\right)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^2 d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^6\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2}+\frac{2ab^6\left(5a^2+b^2\right)x}{\left(a^2+b^2\right)^2}+\frac{b^4\left(6a^4+17a^2b^2+b\right.}{\left(a^2+b^2\right)^2}}{(a+x)^2\left(b^2+x^2\right)^3}\right.}{6bd}$$

$$= -\frac{\cos^6(c+dx)\left(2ab+\left(a^2-b^2\right)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^2 d} + \frac{\cos^4(c+dx)\left(12ab\left(3a^2+b^2\right)+\left(13a^4-18a\right.\right.}{24\left(a^2+b^2\right)^3 d}$$

$$= -\frac{\cos^6(c+dx)\left(2ab+\left(a^2-b^2\right)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^2 d} + \frac{\cos^4(c+dx)\left(12ab\left(3a^2+b^2\right)+\left(13a^4-18a\right.\right.}{24\left(a^2+b^2\right)^3 d}$$

$$= -\frac{\cos^6(c+dx)\left(2ab+\left(a^2-b^2\right)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^2 d} + \frac{\cos^4(c+dx)\left(12ab\left(3a^2+b^2\right)+\left(13a^4-18a\right.\right.}{24\left(a^2+b^2\right)^3 d}$$

$$= \frac{2a^5b\left(a^2-3b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^6b}{\left(a^2+b^2\right)^4 d(a+b\tan(c+dx))} - \frac{\cos^6(c+dx)\left(2a\right.}{6}$$

$$= \frac{2a^5b\left(a^2-3b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^6b}{\left(a^2+b^2\right)^4 d(a+b\tan(c+dx))} - \frac{\cos^6(c+dx)\left(2a\right.}{6}$$

$$= \frac{\left(5a^8-80a^6b^2+50a^4b^4+8a^2b^6+b^8\right)x}{16\left(a^2+b^2\right)^5} + \frac{2a^5b\left(a^2-3b^2\right)\log(\cos(c+dx))}{\left(a^2+b^2\right)^5 d} + \frac{2a^5b\left(a^2-3b\right.}{}$$

Mathematica [A] time = 6.47982, size = 526, normalized size = 1.77

$$\frac{b\left(\frac{12\left(a^2+b^2\right)\left(6a^4b^2+4a^2b^4-3a^6+b^6\right)\sin(2(c+dx))}{b} - 144a^5\left(a^2+b^2\right)\cos^2(c+dx) - 16a\left(a^2+b^2\right)^3\cos^6(c+dx) + 24a\left(a^2+b^2\right)^2\left(3a^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b*((24*(a^2 + b^2)*(-3*a^6 + 6*a^4*b^2 + 4*a^2*b^4 + b^6)*ArcTan[Tan[c + d
*x]])/b - 144*a^5*(a^2 + b^2)*Cos[c + d*x]^2 + 24*a*(a^2 + b^2)^2*(3*a^2 +
b^2)*Cos[c + d*x]^4 - 16*a*(a^2 + b^2)^3*Cos[c + d*x]^6 - 24*a^5*(2*a^2 - 6
*b^2 + (-a^3 + 7*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 96*a
^5*(a^2 - 3*b^2)*Log[a + b*Tan[c + d*x]] - 24*a^5*(2*a^2 - 6*b^2 + (a^3 - 7
*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] - (12*(a^2 + b^2)^2*(-
3*a^4 + 3*a^2*b^2 + 2*b^4)*Cos[c + d*x]^3*Sin[c + d*x])/b + (8*(-a^2 + b^2)
*(a^2 + b^2)^3*Cos[c + d*x]^5*Sin[c + d*x])/b + (12*(a^2 + b^2)*(-3*a^6 + 6
*a^4*b^2 + 4*a^2*b^4 + b^6)*Sin[2*(c + d*x)])/b - (9*(a^2 + b^2)^2*(-3*a^4
+ 3*a^2*b^2 + 2*b^4)*(2*ArcTan[Tan[c + d*x]] + Sin[2*(c + d*x)]))/b + (5*(-
a^2 + b^2)*(a^2 + b^2)^3*(12*ArcTan[Tan[c + d*x]] + 8*Sin[2*(c + d*x)] + Si
n[4*(c + d*x)]))/(4*b) - (48*a^6*(a^2 + b^2))/(a + b*Tan[c + d*x])))/(48*(a
^2 + b^2)^5*d)
```

Maple [B] time = 0.103, size = 1211, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x)
```

```
[Out] -1/d/(a^2+b^2)^5*ln(1+tan(d*x+c)^2)*a^7*b+3/d/(a^2+b^2)^5*ln(1+tan(d*x+c)^2
)*a^5*b^3-5/d/(a^2+b^2)^5*arctan(tan(d*x+c))*a^6*b^2+25/8/d/(a^2+b^2)^5*arc
tan(tan(d*x+c))*a^4*b^4+1/2/d/(a^2+b^2)^5*arctan(tan(d*x+c))*a^2*b^6+2/d*b*
a^7/(a^2+b^2)^5*ln(a+b*tan(d*x+c))-6/d*b^3*a^5/(a^2+b^2)^5*ln(a+b*tan(d*x+c
))-1/2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*b^3*a^5-11/16/d/(a^2+b^2)^5/(1+tan(
d*x+c)^2)^3*tan(d*x+c)^5*a^8+2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)*
a^6*b^2+15/8/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)*a^4*b^4-1/2/d/(a^2
+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)*a^2*b^6-3/d/(a^2+b^2)^5/(1+tan(d*x+c)
^2)^3*tan(d*x+c)^4*a^7*b-3/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^4*a^
5*b^3+2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^5*a^6*b^2-1/3/d/(a^2+b^
2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^3*a^2*b^6-9/2/d/(a^2+b^2)^5/(1+tan(d*x+c
)^2)^3*tan(d*x+c)^2*a^7*b-5/2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^2
*b^3*a^5+5/2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^2*a^3*b^5+1/2/d/(a
^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^2*a*b^7+1/2/d/(a^2+b^2)^5/(1+tan(d*
x+c)^2)^3*tan(d*x+c)^5*a^2*b^6+13/3/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*
x+c)^3*a^6*b^2+5/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^3*a^4*b^4+25/8
/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^5*a^4*b^4+5/16/d/(a^2+b^2)^5*a
rctan(tan(d*x+c))*a^8+1/16/d/(a^2+b^2)^5*arctan(tan(d*x+c))*b^8-11/6/d/(a^2
```

```
+b^2)^5/(1+tan(d*x+c)^2)^3*a^7*b-a^6*b/(a^2+b^2)^4/d/(a+b*tan(d*x+c))+1/16/
d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^5*b^8-5/6/d/(a^2+b^2)^5/(1+tan(
d*x+c)^2)^3*tan(d*x+c)^3*a^8-1/6/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c
)^3*b^8-5/16/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)*a^8-1/16/d/(a^2+b^
2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)*b^8+3/2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3
*a^3*b^5+1/6/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*a*b^7
```

Maxima [B] time = 1.79554, size = 1079, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(5*a^8 - 80*a^6*b^2 + 50*a^4*b^4 + 8*a^2*b^6 + b^8)*(d*x + c)/(a^10
 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 96*(a^7*b - 3*
a^5*b^3)*log(b*tan(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^
6 + 5*a^2*b^8 + b^10) - 48*(a^7*b - 3*a^5*b^3)*log(tan(d*x + c)^2 + 1)/(a^1
0 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - (136*a^6*b -
64*a^4*b^3 - 8*a^2*b^5 + 3*(27*a^6*b - 43*a^4*b^3 - 7*a^2*b^5 - b^7)*tan(d*
x + c)^6 + 3*(11*a^7 + 5*a^5*b^2 - 7*a^3*b^4 - a*b^6)*tan(d*x + c)^5 + 8*(4
1*a^6*b - 31*a^4*b^3 + a^2*b^5 + b^7)*tan(d*x + c)^4 + 8*(5*a^7 - 4*a^5*b^2
 - 11*a^3*b^4 - 2*a*b^6)*tan(d*x + c)^3 + 3*(125*a^6*b - 69*a^4*b^3 - a^2*b
^5 + b^7)*tan(d*x + c)^2 + (15*a^7 - 23*a^5*b^2 - 43*a^3*b^4 - 5*a*b^6)*tan
(d*x + c))/(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8 + (a^8*b + 4*a^
6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*tan(d*x + c)^7 + (a^9 + 4*a^7*b^2 + 6*
a^5*b^4 + 4*a^3*b^6 + a*b^8)*tan(d*x + c)^6 + 3*(a^8*b + 4*a^6*b^3 + 6*a^4*
b^5 + 4*a^2*b^7 + b^9)*tan(d*x + c)^5 + 3*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*
a^3*b^6 + a*b^8)*tan(d*x + c)^4 + 3*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*
b^7 + b^9)*tan(d*x + c)^3 + 3*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*
b^8)*tan(d*x + c)^2 + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*tan
(d*x + c)))/d
```

Fricas [B] time = 2.89634, size = 1388, normalized size = 4.67

$$-\frac{8\left(a^{8}b+4\,a^{6}b^{3}+6\,a^{4}b^{5}+4\,a^{2}b^{7}+b^{9}\right)\cos\left(dx+c\right)^{7}-2\left(19\,a^{8}b+68\,a^{6}b^{3}+90\,a^{4}b^{5}+52\,a^{2}b^{7}+11\,b^{9}\right)\cos\left(dx+c\right)^{5}+\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/48*(8*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^7 -
 2*(19*a^8*b + 68*a^6*b^3 + 90*a^4*b^5 + 52*a^2*b^7 + 11*b^9)*cos(d*x + c)^
5 + (85*a^8*b + 224*a^6*b^3 + 210*a^4*b^5 + 88*a^2*b^7 + 17*b^9)*cos(d*x +
c)^3 - (17*a^8*b + 72*a^6*b^3 + 120*a^4*b^5 + 20*a^2*b^7 + 3*b^9 + 3*(5*a^9
 - 80*a^7*b^2 + 50*a^5*b^4 + 8*a^3*b^6 + a*b^8)*d*x)*cos(d*x + c) - 48*((a^
8*b - 3*a^6*b^3)*cos(d*x + c) + (a^7*b^2 - 3*a^5*b^4)*sin(d*x + c))*log(2*a
*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (98*a^7*
b^2 + 24*a^5*b^4 - 30*a^3*b^6 - 4*a*b^8 - 8*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 +
4*a^3*b^6 + a*b^8)*cos(d*x + c)^6 + 2*(13*a^9 + 44*a^7*b^2 + 54*a^5*b^4 + 2
8*a^3*b^6 + 5*a*b^8)*cos(d*x + c)^4 + 3*(5*a^8*b - 80*a^6*b^3 + 50*a^4*b^5
+ 8*a^2*b^7 + b^9)*d*x - 3*(11*a^9 + 16*a^7*b^2 - 2*a^5*b^4 - 8*a^3*b^6 - a
*b^8)*cos(d*x + c)^2)*sin(d*x + c))/((a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^
5*b^6 + 5*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^10*b + 5*a^8*b^3 + 10*a^6*b
^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11)*d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23314, size = 992, normalized size = 3.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^8 - 80*a^6*b^2 + 50*a^4*b^4 + 8*a^2*b^6 + b^8)*(d*x + c)/(a^10
 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - 48*(a^7*b - 3*
a^5*b^3)*log(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^
6 + 5*a^2*b^8 + b^10) + 96*(a^7*b^2 - 3*a^5*b^4)*log(abs(b*tan(d*x + c) + a
```

```
))/(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11) - 48*(
2*a^7*b^2*tan(d*x + c) - 6*a^5*b^4*tan(d*x + c) + 3*a^8*b - 5*a^6*b^3)/((a^
10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*(b*tan(d*x + c
) + a)) + (88*a^7*b*tan(d*x + c)^6 - 264*a^5*b^3*tan(d*x + c)^6 - 33*a^8*ta
n(d*x + c)^5 + 96*a^6*b^2*tan(d*x + c)^5 + 150*a^4*b^4*tan(d*x + c)^5 + 24*
a^2*b^6*tan(d*x + c)^5 + 3*b^8*tan(d*x + c)^5 + 120*a^7*b*tan(d*x + c)^4 -
936*a^5*b^3*tan(d*x + c)^4 - 40*a^8*tan(d*x + c)^3 + 208*a^6*b^2*tan(d*x +
c)^3 + 240*a^4*b^4*tan(d*x + c)^3 - 16*a^2*b^6*tan(d*x + c)^3 - 8*b^8*tan(d
*x + c)^3 + 48*a^7*b*tan(d*x + c)^2 - 912*a^5*b^3*tan(d*x + c)^2 + 120*a^3*
b^5*tan(d*x + c)^2 + 24*a*b^7*tan(d*x + c)^2 - 15*a^8*tan(d*x + c) + 96*a^6
*b^2*tan(d*x + c) + 90*a^4*b^4*tan(d*x + c) - 24*a^2*b^6*tan(d*x + c) - 3*b
^8*tan(d*x + c) - 288*a^5*b^3 + 72*a^3*b^5 + 8*a*b^7)/((a^10 + 5*a^8*b^2 +
10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*(tan(d*x + c)^2 + 1)^3))/d
```

3.62 $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=217

$$-\frac{a^4b}{d\left(a^2+b^2\right)^3(a+b\tan(c+dx))}+\frac{\cos^4(c+dx)\left(\left(a^2-b^2\right)\tan(c+dx)+2ab\right)}{4d\left(a^2+b^2\right)^2}-\frac{\cos^2(c+dx)\left(\left(-12a^2b^2+5a^4-b^4\right)\text{ta}\right.}{8d\left(a^2+b^2\right)^3}$$

```
[Out] ((3*a^6 - 33*a^4*b^2 + 13*a^2*b^4 + b^6)*x)/(8*(a^2 + b^2)^4) + (2*a^3*b*(a
^2 - 2*b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) - (a^4*
b)/((a^2 + b^2)^3*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^4*(2*a*b + (a^2 -
 b^2)*Tan[c + d*x]))/(4*(a^2 + b^2)^2*d) - (Cos[c + d*x]^2*(16*a^3*b + (5*a
^4 - 12*a^2*b^2 - b^4)*Tan[c + d*x]))/(8*(a^2 + b^2)^3*d)
```

Rubi [A] time = 0.561945, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$-\frac{a^4b}{d\left(a^2+b^2\right)^3(a+b\tan(c+dx))}+\frac{\cos^4(c+dx)\left(\left(a^2-b^2\right)\tan(c+dx)+2ab\right)}{4d\left(a^2+b^2\right)^2}-\frac{\cos^2(c+dx)\left(\left(-12a^2b^2+5a^4-b^4\right)\text{ta}\right.}{8d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```


```
[Out] ((3*a^6 - 33*a^4*b^2 + 13*a^2*b^4 + b^6)*x)/(8*(a^2 + b^2)^4) + (2*a^3*b*(a
^2 - 2*b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) - (a^4*
b)/((a^2 + b^2)^3*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^4*(2*a*b + (a^2 -
 b^2)*Tan[c + d*x]))/(4*(a^2 + b^2)^2*d) - (Cos[c + d*x]^2*(16*a^3*b + (5*a
^4 - 12*a^2*b^2 - b^4)*Tan[c + d*x]))/(8*(a^2 + b^2)^3*d)
```


Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
```

```
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{x^4}{(a+x)^2\left(b^2+x^2\right)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^4(c+dx)\left(2ab+\left(a^2-b^2\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^2 d} - \frac{\mathrm{Subst}\left(\int \frac{\frac{a^2b^4\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2} - \frac{2ab^4\left(3a^2+b^2\right)x}{\left(a^2+b^2\right)^2} - \frac{b^2\left(4a^4+11a^2b^2+b^4\right.}{\left(a^2+b^2\right)^2}}{(a+x)^2\left(b^2+x^2\right)^2}\right.}{4bd}$$

$$= \frac{\cos^4(c+dx)\left(2ab+\left(a^2-b^2\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^2 d} - \frac{\cos^2(c+dx)\left(16a^3b+\left(5a^4-12a^2b^2-b^4\right)\mathrm{ta}\right.}{8\left(a^2+b^2\right)^3 d}$$

$$= \frac{\cos^4(c+dx)\left(2ab+\left(a^2-b^2\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^2 d} - \frac{\cos^2(c+dx)\left(16a^3b+\left(5a^4-12a^2b^2-b^4\right)\mathrm{ta}\right.}{8\left(a^2+b^2\right)^3 d}$$

$$= \frac{2a^3b\left(a^2-2b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^4 d} - \frac{a^4b}{\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))} + \frac{\cos^4(c+dx)\left(2\right.}{}$$

$$= \frac{2a^3b\left(a^2-2b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^4 d} - \frac{a^4b}{\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))} + \frac{\cos^4(c+dx)\left(2\right.}{}$$

$$= \frac{\left(3a^6-33a^4b^2+13a^2b^4+b^6\right)x}{8\left(a^2+b^2\right)^4} + \frac{2a^3b\left(a^2-2b^2\right)\log(\cos(c+dx))}{\left(a^2+b^2\right)^4 d} + \frac{2a^3b\left(a^2-2b^2\right)\log(}{\left(a^2+b\right.}$$

Mathematica [A] time = 3.80693, size = 373, normalized size = 1.72

$$\frac{b\left(\frac{2\left(a^2+b^2\right)\left(3a^2b^2-2a^4+b^4\right)\sin(2(c+dx))}{b} - 16a^3\left(a^2+b^2\right)\cos^2(c+dx) + 4a\left(a^2+b^2\right)^2\cos^4(c+dx) + \frac{4\left(a^2+b^2\right)\left(3a^2b^2-2a^4+b^4\right)\tan^{-1}(\mathrm{t}}{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b*((4*(a^2 + b^2)*(-2*a^4 + 3*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]])/b - 16*
a^3*(a^2 + b^2)*Cos[c + d*x]^2 + 4*a*(a^2 + b^2)^2*Cos[c + d*x]^4 - 4*a^3*(
```

```
2*a^2 - 4*b^2 + (-a^3 + 5*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x
]] + 16*a^3*(a^2 - 2*b^2)*Log[a + b*Tan[c + d*x]] - 4*a^3*(2*a^2 - 4*b^2 +
(a^3 - 5*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (2*(a^2 - b^
2)*(a^2 + b^2)^2*Cos[c + d*x]^3*Sin[c + d*x])/b + (2*(a^2 + b^2)*(-2*a^4 +
3*a^2*b^2 + b^4)*Sin[2*(c + d*x)])/b + (3*(a^2 - b^2)*(a^2 + b^2)^2*(2*ArcT
an[Tan[c + d*x]] + Sin[2*(c + d*x)]))/(2*b) - (8*a^4*(a^2 + b^2))/(a + b*Ta
n[c + d*x])))/(8*(a^2 + b^2)^4*d)
```

Maple [B] time = 0.099, size = 724, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x)
```

```
[Out] -5/8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*a^6+7/8/d/(a^2+b^2)^4/(1
+tan(d*x+c)^2)^2*tan(d*x+c)^3*a^4*b^2+13/8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2
*tan(d*x+c)^3*a^2*b^4+1/8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*b^6
-2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)^2*a^5*b-2/d/(a^2+b^2)^4/(1+t
an(d*x+c)^2)^2*tan(d*x+c)^2*a^3*b^3-3/8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*ta
n(d*x+c)*a^6+9/8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^4*b^2+11/8/d
/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^2*b^4-1/8/d/(a^2+b^2)^4/(1+tan
(d*x+c)^2)^2*tan(d*x+c)*b^6-3/2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*a^5*b-1/d/
(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*a^3*b^3+1/2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2
*a*b^5-1/d/(a^2+b^2)^4*ln(1+tan(d*x+c)^2)*a^5*b+2/d/(a^2+b^2)^4*ln(1+tan(d*
x+c)^2)*a^3*b^3+3/8/d/(a^2+b^2)^4*arctan(tan(d*x+c))*a^6-33/8/d/(a^2+b^2)^4
*arctan(tan(d*x+c))*a^4*b^2+13/8/d/(a^2+b^2)^4*arctan(tan(d*x+c))*a^2*b^4+1
/8/d/(a^2+b^2)^4*arctan(tan(d*x+c))*b^6-a^4*b/(a^2+b^2)^3/d/(a+b*tan(d*x+c)
)+2/d*a^5*b/(a^2+b^2)^4*ln(a+b*tan(d*x+c))-4/d*a^3*b^3/(a^2+b^2)^4*ln(a+b*t
an(d*x+c))
```

Maxima [B] time = 1.53015, size = 684, normalized size = 3.15

$$\frac{\frac{(3a^6-33a^4b^2+13a^2b^4+b^6)(dx+c)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}+\frac{16(a^5b-2a^3b^3)\log(b\tan(dx+c)+a)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{8(a^5b-2a^3b^3)\log(\tan(dx+c)^2+1)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{20a}{a^7+3a^5b^2+3a^3b^4+ab^6+(a^6b+3a^4b^3+3a^2b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^6 - 33*a^4*b^2 + 13*a^2*b^4 + b^6)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6
*a^4*b^4 + 4*a^2*b^6 + b^8) + 16*(a^5*b - 2*a^3*b^3)*log(b*tan(d*x + c) + a
)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 8*(a^5*b - 2*a^3*b^3)*l
og(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - (2
0*a^4*b - 4*a^2*b^3 + (13*a^4*b - 12*a^2*b^3 - b^5)*tan(d*x + c)^4 + (5*a^5
 + 4*a^3*b^2 - a*b^4)*tan(d*x + c)^3 + (35*a^4*b - 12*a^2*b^3 + b^5)*tan(d*
x + c)^2 + 3*(a^5 - a*b^4)*tan(d*x + c))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b
^6 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)^5 + (a^7 + 3*a^5*b^
2 + 3*a^3*b^4 + a*b^6)*tan(d*x + c)^4 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
b^7)*tan(d*x + c)^3 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*tan(d*x + c)^
2 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)))/d
```

Fricas [B] time = 2.63226, size = 986, normalized size = 4.54

$$\frac{4\left(a^{6}b+3\,a^{4}b^{3}+3\,a^{2}b^{5}+b^{7}\right)\cos\left(dx+c\right)^{5}-6\left(3\,a^{6}b+7\,a^{4}b^{3}+5\,a^{2}b^{5}+b^{7}\right)\cos\left(dx+c\right)^{3}+\left(3\,a^{6}b+8\,a^{4}b^{3}+23\,a^{2}b^{5}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^5 - 6*(3*a^6*b +
 7*a^4*b^3 + 5*a^2*b^5 + b^7)*cos(d*x + c)^3 + (3*a^6*b + 8*a^4*b^3 + 23*a^
2*b^5 + 2*b^7 + 2*(3*a^7 - 33*a^5*b^2 + 13*a^3*b^4 + a*b^6)*d*x)*cos(d*x +
c) + 16*((a^6*b - 2*a^4*b^3)*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4)*sin(d*x +
 c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2
) + (29*a^5*b^2 + 10*a^3*b^4 - 3*a*b^6 + 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a
*b^6)*cos(d*x + c)^4 + 2*(3*a^6*b - 33*a^4*b^3 + 13*a^2*b^5 + b^7)*d*x - 2*
(5*a^7 + 9*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^9
 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*d*cos(d*x + c) + (a^8*b + 4*a
^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21571, size = 693, normalized size = 3.19

$$\frac{\frac{\left(3\,a^6-33\,a^4b^2+13\,a^2b^4+b^6\right)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{8\left(a^5b-2\,a^3b^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{16\left(a^5b^2-2\,a^3b^4\right)\log(|b\tan(dx+c)+a|)}{a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9}-\frac{8\left(2\,a^5b^2\tan(dx+c)-4\,a^3b^4\tan(dx+c)+3}{\left(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8\right)(b\tan(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*((3*a^6 - 33*a^4*b^2 + 13*a^2*b^4 + b^6)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6
*a^4*b^4 + 4*a^2*b^6 + b^8) - 8*(a^5*b - 2*a^3*b^3)*log(tan(d*x + c)^2 + 1)
/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 16*(a^5*b^2 - 2*a^3*b^4)
*log(abs(b*tan(d*x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 +
b^9) - 8*(2*a^5*b^2*tan(d*x + c) - 4*a^3*b^4*tan(d*x + c) + 3*a^6*b - 3*a^4
*b^3)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c) + a)
) + (12*a^5*b*tan(d*x + c)^4 - 24*a^3*b^3*tan(d*x + c)^4 - 5*a^6*tan(d*x +
c)^3 + 7*a^4*b^2*tan(d*x + c)^3 + 13*a^2*b^4*tan(d*x + c)^3 + b^6*tan(d*x +
 c)^3 + 8*a^5*b*tan(d*x + c)^2 - 64*a^3*b^3*tan(d*x + c)^2 - 3*a^6*tan(d*x
+ c) + 9*a^4*b^2*tan(d*x + c) + 11*a^2*b^4*tan(d*x + c) - b^6*tan(d*x + c)
- 32*a^3*b^3 + 4*a*b^5)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(t
an(d*x + c)^2 + 1)^2))/d
```

3.63 $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=148

$$-\frac{a^2b}{d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{\cos^2(c+dx)\left(\left(a^2-b^2\right)\tan(c+dx)+2ab\right)}{2d\left(a^2+b^2\right)^2}+\frac{2ab\left(a^2-b^2\right)\log(a\cos(c+dx)+b\sin}{d\left(a^2+b^2\right)^3}$$

```
[Out] ((a^4 - 6*a^2*b^2 + b^4)*x)/(2*(a^2 + b^2)^3) + (2*a*b*(a^2 - b^2)*Log[a*Co
s[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d) - (a^2*b)/((a^2 + b^2)^2*d*
(a + b*Tan[c + d*x])) - (Cos[c + d*x]^2*(2*a*b + (a^2 - b^2)*Tan[c + d*x]))
/(2*(a^2 + b^2)^2*d)
```

Rubi [A] time = 0.294904, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$-\frac{a^2b}{d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{\cos^2(c+dx)\left(\left(a^2-b^2\right)\tan(c+dx)+2ab\right)}{2d\left(a^2+b^2\right)^2}+\frac{2ab\left(a^2-b^2\right)\log(a\cos(c+dx)+b\sin}{d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```


```
[Out] ((a^4 - 6*a^2*b^2 + b^4)*x)/(2*(a^2 + b^2)^3) + (2*a*b*(a^2 - b^2)*Log[a*Co
s[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d) - (a^2*b)/((a^2 + b^2)^2*d*
(a + b*Tan[c + d*x])) - (Cos[c + d*x]^2*(2*a*b + (a^2 - b^2)*Tan[c + d*x]))
/(2*(a^2 + b^2)^2*d)
```


Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
```

```
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{x^2}{(a+x)^2\left(b^2+x^2\right)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^2(c+dx)\left(2ab+\left(a^2-b^2\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^2 d} - \frac{\mathrm{Subst}\left(\int \frac{-\frac{a^2b^2\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2}+\frac{2ab^2x}{a^2+b^2}+\frac{b^2\left(a^2-b^2\right)x^2}{\left(a^2+b^2\right)^2}}{(a+x)^2\left(b^2+x^2\right)}\,dx, x,}{2bd}$$

$$= -\frac{\cos^2(c+dx)\left(2ab+\left(a^2-b^2\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^2 d} - \frac{\mathrm{Subst}\left(\int\left(-\frac{2a^2b^2}{\left(a^2+b^2\right)^2(a+x)^2}+\frac{4ab^2\left(-a^2+b^2\right)}{\left(a^2+b^2\right)^3(a+x)}+\right.\right.}{}$$

$$= \frac{2ab\left(a^2-b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3 d} - \frac{a^2b}{\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\left(2ab\right.}{2\left(\right.}$$

$$= \frac{2ab\left(a^2-b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3 d} - \frac{a^2b}{\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\left(2ab\right.}{2\left(\right.}$$

$$= \frac{\left(a^4-6a^2b^2+b^4\right)x}{2\left(a^2+b^2\right)^3} + \frac{2ab\left(a^2-b^2\right)\log(\cos(c+dx))}{\left(a^2+b^2\right)^3 d} + \frac{2ab\left(a^2-b^2\right)\log(a+b\tan(c+dx)}{\left(a^2+b^2\right)^3 d}$$

Mathematica [A] time = 3.25967, size = 246, normalized size = 1.66

$$-\frac{b\left(\frac{(a-b)(a+b)\left(a^2+b^2\right)\sin(2(c+dx))}{2b}+2a\left(a^2+b^2\right)\cos^2(c+dx)+\frac{\left(a^2-b^2\right)\left(a^2+b^2\right)\tan^{-1}(\tan(c+dx))}{b}+\frac{2a^2\left(a^2+b^2\right)}{a+b\tan(c+dx)}+a\left(\frac{3ab^2-a^3}{\sqrt{-b^2}}+2a^2\right.\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(b*(((a^2 - b^2)*(a^2 + b^2)*ArcTan[Tan[c + d*x]])/b + 2*a*(a^2 + b^2)*Cos
[c + d*x]^2 + a*(2*a^2 - 2*b^2 + (-a^3 + 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2
] - b*Tan[c + d*x]] - 4*a*(a - b)*(a + b)*Log[a + b*Tan[c + d*x]] + a*(2*a^
2 - 2*b^2 + (a^3 - 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] +
((a - b)*(a + b)*(a^2 + b^2)*Sin[2*(c + d*x)])/(2*b) + (2*a^2*(a^2 + b^2))/
(a + b*Tan[c + d*x]))))/(2*(a^2 + b^2)^3*d)
```

Maple [B] time = 0.089, size = 352, normalized size = 2.4

$$-\frac{\tan(dx+c)\,a^4}{2d\left(a^2+b^2\right)^3\left(1+(\tan(dx+c))^2\right)}+\frac{\tan(dx+c)\,b^4}{2d\left(a^2+b^2\right)^3\left(1+(\tan(dx+c))^2\right)}-\frac{ba^3}{d\left(a^2+b^2\right)^3\left(1+(\tan(dx+c))^2\right)}-\frac{}{d\left(a^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x)
```

```
[Out] -1/2/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)*tan(d*x+c)*a^4+1/2/d/(a^2+b^2)^3/(1+tan
(d*x+c)^2)*tan(d*x+c)*b^4-1/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)*b*a^3-1/d/(a^2+b
^2)^3/(1+tan(d*x+c)^2)*b^3*a-1/d/(a^2+b^2)^3*ln(1+tan(d*x+c)^2)*a^3*b+1/d/(
a^2+b^2)^3*b^3*a*ln(1+tan(d*x+c)^2)-3/d/(a^2+b^2)^3*arctan(tan(d*x+c))*a^2*
b^2+1/2/d/(a^2+b^2)^3*arctan(tan(d*x+c))*b^4+1/2/d/(a^2+b^2)^3*arctan(tan(d
*x+c))*a^4-a^2*b/(a^2+b^2)^2/d/(a+b*tan(d*x+c))+2/d*b*a^3/(a^2+b^2)^3*ln(a+
b*tan(d*x+c))-2/d*b^3/(a^2+b^2)^3*a*ln(a+b*tan(d*x+c))
```

Maxima [B] time = 1.55602, size = 396, normalized size = 2.68

$$\frac{\frac{\left(a^4-6a^2b^2+b^4\right)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{4\left(a^3b-ab^3\right)\log(b\tan(dx+c)+a)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2\left(a^3b-ab^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{4a^2b+\left(3a^2b-b^3\right)\tan(dx+c)^2+\left(a^3\right.}{a^5+2a^3b^2+ab^4+\left(a^4b+2a^2b^3+b^5\right)\tan(dx+c)^3+\left(a^5+2a^3b^2+a\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
+ 4*(a^3*b - a*b^3)*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6) - 2*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6) - (4*a^2*b + (3*a^2*b - b^3)*tan(d*x + c)^2 + (a^3 + a*b^2)*tan(d
*x + c))/(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x + c)^
3 + (a^5 + 2*a^3*b^2 + a*b^4)*tan(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*ta
n(d*x + c)))/d
```

Fricas [B] time = 2.2491, size = 640, normalized size = 4.32

$$-\frac{\left(a^4b+2a^2b^3+b^5\right)\cos(dx+c)^3+\left(a^2b^3-b^5-\left(a^5-6a^3b^2+ab^4\right)dx\right)\cos(dx+c)-2\left(\left(a^4b-a^2b^3\right)\cos(dx+c)+\left(a^3b\right.\right.}{2\left(\left(a^7+3a^5b^2+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 + (a^2*b^3 - b^5 - (a^5 - 6*
a^3*b^2 + a*b^4)*d*x)*cos(d*x + c) - 2*((a^4*b - a^2*b^3)*cos(d*x + c) + (a
^3*b^2 - a*b^4)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
b^2)*cos(d*x + c)^2 + b^2) - (3*a^3*b^2 + a*b^4 + (a^4*b - 6*a^2*b^3 + b^5)
*d*x - (a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 + 3*a^
5*b^2 + 3*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5
+ b^7)*d*sin(d*x + c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.17682, size = 355, normalized size = 2.4

$$\frac{\frac{(a^4-6a^2b^2+b^4)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2(a^3b-ab^3)\log(\tan(dx+c)^2+1)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{4(a^3b^2-ab^4)\log(|b\tan(dx+c)+a|)}{a^6b+3a^4b^3+3a^2b^5+b^7}-\frac{3a^2b\tan(dx+c)^2-b^3\tan(dx+c)^2+a^3\tan(dx+c)+ab^2\tan}{(a^4+2a^2b^2+b^4)(b\tan(dx+c)^3+a\tan(dx+c)^2+b\tan(}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*((a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
- 2*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6) + 4*(a^3*b^2 - a*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3
+ 3*a^2*b^5 + b^7) - (3*a^2*b*tan(d*x + c)^2 - b^3*tan(d*x + c)^2 + a^3*tan
(d*x + c) + a*b^2*tan(d*x + c) + 4*a^2*b)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(d
*x + c)^3 + a*tan(d*x + c)^2 + b*tan(d*x + c) + a)))/d
```

3.64 $\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=72

$$-\frac{b}{a^2d(a+b\tan(c+dx))}-\frac{2b\log(\tan(c+dx))}{a^3d}+\frac{2b\log(a+b\tan(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^2d}$$

```
[Out] -(Cot[c + d*x]/(a^2*d)) - (2*b*Log[Tan[c + d*x]])/(a^3*d) + (2*b*Log[a + b*
Tan[c + d*x]])/(a^3*d) - b/(a^2*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.0654904, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 44}

$$-\frac{b}{a^2d(a+b\tan(c+dx))}-\frac{2b\log(\tan(c+dx))}{a^3d}+\frac{2b\log(a+b\tan(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(Cot[c + d*x]/(a^2*d)) - (2*b*Log[Tan[c + d*x]])/(a^3*d) + (2*b*Log[a + b*
Tan[c + d*x]])/(a^3*d) - b/(a^2*d*(a + b*Tan[c + d*x]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{1}{x^2(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{b\,\mathrm{Subst}\left(\int \left(\frac{1}{a^2x^2} - \frac{2}{a^3x} + \frac{1}{a^2(a+x)^2} + \frac{2}{a^3(a+x)}\right) dx, x, b\tan(c+dx)\right)}{d}$$
$$= -\frac{\cot(c+dx)}{a^2d} - \frac{2b\log(\tan(c+dx))}{a^3d} + \frac{2b\log(a+b\tan(c+dx))}{a^3d} - \frac{b}{a^2d(a+b\tan(c+dx))}$$

Mathematica [A] time = 0.372834, size = 109, normalized size = 1.51

$$\frac{-a^2\cot^2(c+dx) + b^2(2\log(a\cos(c+dx)+b\sin(c+dx)) - 2\log(\sin(c+dx)) + 1) - ab\cot(c+dx)(-2\log(a\cos(c+d}{a^3d(a\cot(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-(a^2*Cot[c + d*x]^2) - a*b*Cot[c + d*x]*(1 + 2*Log[Sin[c + d*x]] - 2*Log[
a*Cos[c + d*x] + b*Sin[c + d*x]]) + b^2*(1 - 2*Log[Sin[c + d*x]] + 2*Log[a*
Cos[c + d*x] + b*Sin[c + d*x]]))/(a^3*d*(b + a*Cot[c + d*x]))
```

Maple [A] time = 0.092, size = 75, normalized size = 1.

$$-\frac{1}{a^2d\tan(dx+c)} - 2\frac{b\ln(\tan(dx+c))}{a^3d} - \frac{b}{a^2d(a+b\tan(dx+c))} + 2\frac{b\ln(a+b\tan(dx+c))}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x)
```

```
[Out] -1/d/a^2/tan(d*x+c)-2*b*ln(tan(d*x+c))/a^3/d-b/a^2/d/(a+b*tan(d*x+c))+2*b*l
n(a+b*tan(d*x+c))/a^3/d
```

Maxima [A] time = 0.987045, size = 100, normalized size = 1.39

$$-\frac{\frac{2b\tan(dx+c)+a}{a^2b\tan(dx+c)^2+a^3\tan(dx+c)} - \frac{2b\log(b\tan(dx+c)+a)}{a^3} + \frac{2b\log(\tan(dx+c))}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((2*b*tan(d*x + c) + a)/(a^2*b*tan(d*x + c)^2 + a^3*tan(d*x + c)) - 2*b*lo
g(b*tan(d*x + c) + a)/a^3 + 2*b*log(tan(d*x + c))/a^3)/d
```

Fricas [B] time = 2.19866, size = 663, normalized size = 9.21

$$-\frac{a^2b^2 - \left(a^4 + 2\,a^2b^2\right)\cos\left(dx+c\right)^2 - \left(a^3b + 2\,ab^3\right)\cos\left(dx+c\right)\sin\left(dx+c\right) + \left(a^2b^2 + b^4 - \left(a^2b^2 + b^4\right)\cos\left(dx+c\right)^2 + \left(a^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b^2 - (a^4 + 2*a^2*b^2)*cos(d*x + c)^2 - (a^3*b + 2*a*b^3)*cos(d*x +
c)*sin(d*x + c) + (a^2*b^2 + b^4 - (a^2*b^2 + b^4)*cos(d*x + c)^2 + (a^3*b
+ a*b^3)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (
a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*b^2 + b^4 - (a^2*b^2 + b^4)*cos(d*x
 + c)^2 + (a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(-1/4*cos(d*x + c)^
2 + 1/4))/((a^5*b + a^3*b^3)*d*cos(d*x + c)^2 - (a^6 + a^4*b^2)*d*cos(d*x +
 c)*sin(d*x + c) - (a^5*b + a^3*b^3)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*tan(c + d*x))**2, x)
```

Giac [A] time = 1.21864, size = 100, normalized size = 1.39

$$\frac{\frac{2\,b\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^3}-\frac{2\,b\log\left(\left|\tan\left(dx+c\right)\right|\right)}{a^3}-\frac{2\,b\tan\left(dx+c\right)+a}{\left(b\tan\left(dx+c\right)^2+a\tan\left(dx+c\right)\right)a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*b*log(abs(b*tan(d*x + c) + a))/a^3 - 2*b*log(abs(tan(d*x + c)))/a^3 - (2
*b*tan(d*x + c) + a)/((b*tan(d*x + c)^2 + a*tan(d*x + c))*a^2))/d
```

3.65 $\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\, dx$

Optimal. Leaf size=140

$$-\frac{b\left(a^2+b^2\right)}{a^4d(a+b\tan(c+dx))}-\frac{\left(a^2+3b^2\right)\cot(c+dx)}{a^4d}-\frac{2b\left(a^2+2b^2\right)\log(\tan(c+dx))}{a^5d}+\frac{2b\left(a^2+2b^2\right)\log(a+b\tan(c+dx))}{a^5d}$$

```
[Out] -(((a^2 + 3*b^2)*Cot[c + d*x])/(a^4*d)) + (b*Cot[c + d*x]^2)/(a^3*d) - Cot[
c + d*x]^3/(3*a^2*d) - (2*b*(a^2 + 2*b^2)*Log[Tan[c + d*x]])/(a^5*d) + (2*b
*(a^2 + 2*b^2)*Log[a + b*Tan[c + d*x]])/(a^5*d) - (b*(a^2 + b^2))/(a^4*d*(a
 + b*Tan[c + d*x]))
```

Rubi [A] time = 0.122497, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 894}

$$-\frac{b\left(a^2+b^2\right)}{a^4d(a+b\tan(c+dx))}-\frac{\left(a^2+3b^2\right)\cot(c+dx)}{a^4d}-\frac{2b\left(a^2+2b^2\right)\log(\tan(c+dx))}{a^5d}+\frac{2b\left(a^2+2b^2\right)\log(a+b\tan(c+dx))}{a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 + 3*b^2)*Cot[c + d*x])/(a^4*d)) + (b*Cot[c + d*x]^2)/(a^3*d) - Cot[
c + d*x]^3/(3*a^2*d) - (2*b*(a^2 + 2*b^2)*Log[Tan[c + d*x]])/(a^5*d) + (2*b
*(a^2 + 2*b^2)*Log[a + b*Tan[c + d*x]])/(a^5*d) - (b*(a^2 + b^2))/(a^4*d*(a
 + b*Tan[c + d*x]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
```

```
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{b\,\text{Subst}\left(\int \frac{b^2+x^2}{x^4(a+x)^2}\,dx,x,b\tan(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \left(\frac{b^2}{a^2x^4}-\frac{2b^2}{a^3x^3}+\frac{a^2+3b^2}{a^4x^2}-\frac{2(a^2+2b^2)}{a^5x}+\frac{a^2+b^2}{a^4(a+x)^2}+\frac{2(a^2+2b^2)}{a^5(a+x)}\right)dx,x,b\tan(c+dx)\right)}{d}$$
$$= -\frac{(a^2+3b^2)\cot(c+dx)}{a^4d}+\frac{b\cot^2(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{2b\left(a^2+2b^2\right)\log(\tan(c+dx))}{a^5d}$$

Mathematica [A] time = 2.50639, size = 244, normalized size = 1.74

$$\frac{-\cot^2(c+dx)\left(9a^2b^2+a^4\csc^2(c+dx)+2a^4\right)+3b^2\left(-2\left(a^2+2b^2\right)\log(\sin(c+dx))+2a^2\log(a\cos(c+dx)+b\sin(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-(Cot[c + d*x]^2*(2*a^4 + 9*a^2*b^2 + a^4*Csc[c + d*x]^2)) + 3*b^2*(a^2 +
b^2 + a^2*Csc[c + d*x]^2 - 2*(a^2 + 2*b^2)*Log[Sin[c + d*x]] + 2*a^2*Log[a*
Cos[c + d*x] + b*Sin[c + d*x]] + 4*b^2*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]
) + a*b*Cot[c + d*x]*(-2*a^2 - 9*b^2 + 2*a^2*Csc[c + d*x]^2 - 6*(a^2 + 2*b^
2)*Log[Sin[c + d*x]] + 6*a^2*Log[a*Cos[c + d*x] + b*Sin[c + d*x]] + 12*b^2*
Log[a*Cos[c + d*x] + b*Sin[c + d*x]]))/(3*a^5*d*(b + a*Cot[c + d*x]))
```

Maple [A] time = 0.153, size = 189, normalized size = 1.4

$$-\frac{1}{3a^2d(\tan(dx+c))^3}-\frac{1}{a^2d\tan(dx+c)}-3\frac{b^2}{da^4\tan(dx+c)}+\frac{b}{da^3(\tan(dx+c))^2}-2\frac{b\ln(\tan(dx+c))}{da^3}-4\frac{b^3\ln(\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x)
```

```
[Out] -1/3/d/a^2/tan(d*x+c)^3-1/d/a^2/tan(d*x+c)-3/d/a^4/tan(d*x+c)*b^2+1/d/a^3*b
/tan(d*x+c)^2-2*b*ln(tan(d*x+c))/a^3/d-4/d*b^3/a^5*ln(tan(d*x+c))-b/a^2/d/(
a+b*tan(d*x+c))-1/d*b^3/a^4/(a+b*tan(d*x+c))+2*b*ln(a+b*tan(d*x+c))/a^3/d+4
/d*b^3/a^5*ln(a+b*tan(d*x+c))
```

Maxima [A] time = 1.09184, size = 194, normalized size = 1.39

$$\frac{\frac{2\,a^2b\tan(dx+c)-6\left(a^2b+2\,b^3\right)\tan(dx+c)^3-a^3-3\left(a^3+2\,ab^2\right)\tan(dx+c)^2}{a^4b\tan(dx+c)^4+a^5\tan(dx+c)^3}+\frac{6\left(a^2b+2\,b^3\right)\log(b\tan(dx+c)+a)}{a^5}-\frac{6\left(a^2b+2\,b^3\right)\log(\tan(dx+c))}{a^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((2*a^2*b*tan(d*x + c) - 6*(a^2*b + 2*b^3)*tan(d*x + c)^3 - a^3 - 3*(a^
3 + 2*a*b^2)*tan(d*x + c)^2)/(a^4*b*tan(d*x + c)^4 + a^5*tan(d*x + c)^3) +
6*(a^2*b + 2*b^3)*log(b*tan(d*x + c) + a)/a^5 - 6*(a^2*b + 2*b^3)*log(tan(d
*x + c))/a^5)/d
```

Fricas [B] time = 2.37406, size = 1004, normalized size = 7.17

$$2\left(a^4+6\,a^2b^2\right)\cos\left(dx+c\right)^4+6\,a^2b^2-3\left(a^4+6\,a^2b^2\right)\cos\left(dx+c\right)^2+3\left(\left(a^2b^2+2\,b^4\right)\cos\left(dx+c\right)^4+a^2b^2+2\,b^4-2\left(a^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*(a^4 + 6*a^2*b^2)*cos(d*x + c)^4 + 6*a^2*b^2 - 3*(a^4 + 6*a^2*b^2)*c
os(d*x + c)^2 + 3*((a^2*b^2 + 2*b^4)*cos(d*x + c)^4 + a^2*b^2 + 2*b^4 - 2*(
a^2*b^2 + 2*b^4)*cos(d*x + c)^2 - ((a^3*b + 2*a*b^3)*cos(d*x + c)^3 - (a^3*
b + 2*a*b^3)*cos(d*x + c))*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c
) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 3*((a^2*b^2 + 2*b^4)*cos(d*x + c)^4
 + a^2*b^2 + 2*b^4 - 2*(a^2*b^2 + 2*b^4)*cos(d*x + c)^2 - ((a^3*b + 2*a*b^3
)*cos(d*x + c)^3 - (a^3*b + 2*a*b^3)*cos(d*x + c))*sin(d*x + c))*log(-1/4*c
os(d*x + c)^2 + 1/4) - 2*(6*a*b^3*cos(d*x + c) - (a^3*b + 6*a*b^3)*cos(d*x
+ c)^3)*sin(d*x + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*b*d*cos(d*x + c)^2 +
a^5*b*d - (a^6*d*cos(d*x + c)^3 - a^6*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29818, size = 274, normalized size = 1.96

$$-\frac{\frac{6\left(a^2b+2\,b^3\right)\log(|\tan(dx+c)|)}{a^5}-\frac{6\left(a^2b^2+2\,b^4\right)\log(|b\tan(dx+c)+a|)}{a^5b}+\frac{3\left(2\,a^2b^2\tan(dx+c)+4\,b^4\tan(dx+c)+3\,a^3b+5\,ab^3\right)}{(b\tan(dx+c)+a)a^5}-\frac{11\,a^2b\tan(dx+c)^3+22\,b^3\tan}{}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(6*(a^2*b + 2*b^3)*log(abs(tan(d*x + c)))/a^5 - 6*(a^2*b^2 + 2*b^4)*lo
g(abs(b*tan(d*x + c) + a))/(a^5*b) + 3*(2*a^2*b^2*tan(d*x + c) + 4*b^4*tan(
d*x + c) + 3*a^3*b + 5*a*b^3)/((b*tan(d*x + c) + a)*a^5) - (11*a^2*b*tan(d*
x + c)^3 + 22*b^3*tan(d*x + c)^3 - 3*a^3*tan(d*x + c)^2 - 9*a*b^2*tan(d*x +
 c)^2 + 3*a^2*b*tan(d*x + c) - a^3)/(a^5*tan(d*x + c)^3))/d
```

3.66 $\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=219

$$-\frac{b\left(a^2+b^2\right)^2}{a^6d(a+b\tan(c+dx))}-\frac{\left(2a^2+3b^2\right)\cot^3(c+dx)}{3a^4d}+\frac{2b\left(a^2+b^2\right)\cot^2(c+dx)}{a^5d}-\frac{\left(a^2+b^2\right)\left(a^2+5b^2\right)\cot(c+dx)}{a^6d}-\frac{2b}{}$$

```
[Out] -(((a^2 + b^2)*(a^2 + 5*b^2)*Cot[c + d*x])/(a^6*d)) + (2*b*(a^2 + b^2)*Cot[
c + d*x]^2)/(a^5*d) - ((2*a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*a^4*d) + (b*Cot[c
 + d*x]^4)/(2*a^3*d) - Cot[c + d*x]^5/(5*a^2*d) - (2*b*(a^2 + b^2)*(a^2 + 3
*b^2)*Log[Tan[c + d*x]])/(a^7*d) + (2*b*(a^2 + b^2)*(a^2 + 3*b^2)*Log[a + b
*Tan[c + d*x]])/(a^7*d) - (b*(a^2 + b^2)^2)/(a^6*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.198334, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 894}

$$-\frac{b\left(a^2+b^2\right)^2}{a^6d(a+b\tan(c+dx))}-\frac{\left(2a^2+3b^2\right)\cot^3(c+dx)}{3a^4d}+\frac{2b\left(a^2+b^2\right)\cot^2(c+dx)}{a^5d}-\frac{\left(a^2+b^2\right)\left(a^2+5b^2\right)\cot(c+dx)}{a^6d}-\frac{2b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 + b^2)*(a^2 + 5*b^2)*Cot[c + d*x])/(a^6*d)) + (2*b*(a^2 + b^2)*Cot[
c + d*x]^2)/(a^5*d) - ((2*a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*a^4*d) + (b*Cot[c
 + d*x]^4)/(2*a^3*d) - Cot[c + d*x]^5/(5*a^2*d) - (2*b*(a^2 + b^2)*(a^2 + 3
*b^2)*Log[Tan[c + d*x]])/(a^7*d) + (2*b*(a^2 + b^2)*(a^2 + 3*b^2)*Log[a + b
*Tan[c + d*x]])/(a^7*d) - (b*(a^2 + b^2)^2)/(a^6*d*(a + b*Tan[c + d*x]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{\left(b^2+x^2\right)^2}{x^6(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\,\mathrm{Subst}\left(\int \left(\frac{b^4}{a^2x^6} - \frac{2b^4}{a^3x^5} + \frac{2a^2b^2+3b^4}{a^4x^4} - \frac{4b^2\left(a^2+b^2\right)}{a^5x^3} + \frac{a^4+6a^2b^2+5b^4}{a^6x^2} - \frac{2\left(a^4+4a^2b^2+3b^4\right)}{a^7x} + \frac{\left(a^2+b^2\right)^2}{a^6(a+x)^2} + \right.\right.}{d}$$

$$= -\frac{\left(a^2+b^2\right)\left(a^2+5b^2\right)\cot(c+dx)}{a^6d} + \frac{2b\left(a^2+b^2\right)\cot^2(c+dx)}{a^5d} - \frac{\left(2a^2+3b^2\right)\cot^3(c+dx)}{3a^4d}$$

Mathematica [B] time = 6.23138, size = 589, normalized size = 2.69

$$\frac{\sec^2(c+dx)\left(2a^2b^4\sin(c+dx)+a^4b^2\sin(c+dx)+b^6\sin(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))}{a^7d(a+b\tan(c+dx))^2} - \frac{2\left(4a^2b^3+a^4b+3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]^5*Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(5*a^2*d
*(a + b*Tan[c + d*x])^2) + ((-8*a^4*Cos[c + d*x] - 75*a^2*b^2*Cos[c + d*x]
- 75*b^4*Cos[c + d*x])*Csc[c + d*x]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(15*a^6*d*(a + b*Tan[c + d*x])^2) + (b*(a^2 + 2*b^2)*Csc[c + d
*x]^2*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(a^5*d*(a + b*Tan
[c + d*x])^2) + ((-4*a^2*Cos[c + d*x] - 15*b^2*Cos[c + d*x])*Csc[c + d*x]^3
*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(15*a^4*d*(a + b*Tan[c
 + d*x])^2) + (b*Csc[c + d*x]^4*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c +
d*x])^2)/(2*a^3*d*(a + b*Tan[c + d*x])^2) - (2*(a^4*b + 4*a^2*b^3 + 3*b^5)*
Log[Sin[c + d*x]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(a^7*
d*(a + b*Tan[c + d*x])^2) + (2*(a^4*b + 4*a^2*b^3 + 3*b^5)*Log[a*Cos[c + d*
x] + b*Sin[c + d*x]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(a
^7*d*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c +
d*x])*(a^4*b^2*Sin[c + d*x] + 2*a^2*b^4*Sin[c + d*x] + b^6*Sin[c + d*x]))/(
a^7*d*(a + b*Tan[c + d*x])^2)
```

Maple [A] time = 0.122, size = 343, normalized size = 1.6

$$-\frac{1}{5\,a^2d\left(\tan\left(dx+c\right)\right)^5}-\frac{2}{3\,a^2d\left(\tan\left(dx+c\right)\right)^3}-\frac{b^2}{da^4\left(\tan\left(dx+c\right)\right)^3}-\frac{1}{a^2d\tan\left(dx+c\right)}-6\,\frac{b^2}{da^4\tan\left(dx+c\right)}-5\,\frac{b^4}{da^6\tan(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x)
```

```
[Out] -1/5/d/a^2/tan(d*x+c)^5-2/3/d/a^2/tan(d*x+c)^3-1/d/a^4/tan(d*x+c)^3*b^2-1/d
/a^2/tan(d*x+c)-6/d/a^4/tan(d*x+c)*b^2-5/d/a^6/tan(d*x+c)*b^4+1/2/d/a^3*b/t
an(d*x+c)^4+2/d/a^3*b/tan(d*x+c)^2+2/d*b^3/a^5/tan(d*x+c)^2-2*b*ln(tan(d*x+
c))/a^3/d-8/d*b^3/a^5*ln(tan(d*x+c))-6/d*b^5/a^7*ln(tan(d*x+c))-b/a^2/d/(a+
b*tan(d*x+c))-2/d*b^3/a^4/(a+b*tan(d*x+c))-1/d*b^5/a^6/(a+b*tan(d*x+c))+2*b
*ln(a+b*tan(d*x+c))/a^3/d+8/d*b^3/a^5*ln(a+b*tan(d*x+c))+6/d*b^5/a^7*ln(a+b
*tan(d*x+c))
```

Maxima [A] time = 1.22476, size = 304, normalized size = 1.39

$$\frac{\frac{9\,a^4b\tan(dx+c)-60\left(a^4b+4\,a^2b^3+3\,b^5\right)\tan(dx+c)^5-6\,a^5-30\left(a^5+4\,a^3b^2+3\,ab^4\right)\tan(dx+c)^4+10\left(4\,a^4b+3\,a^2b^3\right)\tan(dx+c)^3-5\left(4\,a^5+3\,a^3b^2\right)\tan(dx+c)^2}{a^6b\tan(dx+c)^6+a^7\tan(dx+c)^5}+\frac{60\,(a^{\cdots}}{}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*((9*a^4*b*tan(d*x + c) - 60*(a^4*b + 4*a^2*b^3 + 3*b^5)*tan(d*x + c)^5
 - 6*a^5 - 30*(a^5 + 4*a^3*b^2 + 3*a*b^4)*tan(d*x + c)^4 + 10*(4*a^4*b + 3*
a^2*b^3)*tan(d*x + c)^3 - 5*(4*a^5 + 3*a^3*b^2)*tan(d*x + c)^2)/(a^6*b*tan(
d*x + c)^6 + a^7*tan(d*x + c)^5) + 60*(a^4*b + 4*a^2*b^3 + 3*b^5)*log(b*tan
(d*x + c) + a)/a^7 - 60*(a^4*b + 4*a^2*b^3 + 3*b^5)*log(tan(d*x + c))/a^7)/
d
```

Fricas [B] time = 2.59028, size = 1770, normalized size = 8.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(4*(4*a^6 + 45*a^4*b^2 + 45*a^2*b^4)*cos(d*x + c)^6 - 75*a^4*b^2 - 90*
a^2*b^4 - 10*(4*a^6 + 45*a^4*b^2 + 45*a^2*b^4)*cos(d*x + c)^4 + 15*(2*a^6 +
 23*a^4*b^2 + 24*a^2*b^4)*cos(d*x + c)^2 + 30*((a^4*b^2 + 4*a^2*b^4 + 3*b^6
)*cos(d*x + c)^6 - a^4*b^2 - 4*a^2*b^4 - 3*b^6 - 3*(a^4*b^2 + 4*a^2*b^4 + 3
*b^6)*cos(d*x + c)^4 + 3*(a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^2 - ((a
^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^5 - 2*(a^5*b + 4*a^3*b^3 + 3*a*b^5
)*cos(d*x + c)^3 + (a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c))*sin(d*x + c)
)*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) -
 30*((a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^6 - a^4*b^2 - 4*a^2*b^4 - 3
*b^6 - 3*(a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^4 + 3*(a^4*b^2 + 4*a^2*
b^4 + 3*b^6)*cos(d*x + c)^2 - ((a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^5
 - 2*(a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 + (a^5*b + 4*a^3*b^3 + 3*
a*b^5)*cos(d*x + c))*sin(d*x + c))*log(-1/4*cos(d*x + c)^2 + 1/4) + (4*(4*a
^5*b + 45*a^3*b^3 + 45*a*b^5)*cos(d*x + c)^5 - 10*(a^5*b + 33*a^3*b^3 + 36*
a*b^5)*cos(d*x + c)^3 - 15*(a^5*b - 10*a^3*b^3 - 12*a*b^5)*cos(d*x + c))*si
n(d*x + c))/(a^7*b*d*cos(d*x + c)^6 - 3*a^7*b*d*cos(d*x + c)^4 + 3*a^7*b*d*
cos(d*x + c)^2 - a^7*b*d - (a^8*d*cos(d*x + c)^5 - 2*a^8*d*cos(d*x + c)^3 +
 a^8*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23916, size = 448, normalized size = 2.05

$$-\frac{\frac{60\left(a^4b+4a^2b^3+3b^5\right)\log(|\tan(dx+c)|)}{a^7}-\frac{60\left(a^4b^2+4a^2b^4+3b^6\right)\log(|b\tan(dx+c)+a|)}{a^7b}+\frac{30\left(2a^4b^2\tan(dx+c)+8a^2b^4\tan(dx+c)+6b^6\tan(dx+c)+3a^5b+1\right.}{(b\tan(dx+c)+a)a^7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/30*(60*(a^4*b + 4*a^2*b^3 + 3*b^5)*log(abs(tan(d*x + c)))/a^7 - 60*(a^4*
b^2 + 4*a^2*b^4 + 3*b^6)*log(abs(b*tan(d*x + c) + a))/(a^7*b) + 30*(2*a^4*b
^2*tan(d*x + c) + 8*a^2*b^4*tan(d*x + c) + 6*b^6*tan(d*x + c) + 3*a^5*b + 1
0*a^3*b^3 + 7*a*b^5)/((b*tan(d*x + c) + a)*a^7) - (137*a^4*b*tan(d*x + c)^5
 + 548*a^2*b^3*tan(d*x + c)^5 + 411*b^5*tan(d*x + c)^5 - 30*a^5*tan(d*x + c
)^4 - 180*a^3*b^2*tan(d*x + c)^4 - 150*a*b^4*tan(d*x + c)^4 + 60*a^4*b*tan(
d*x + c)^3 + 60*a^2*b^3*tan(d*x + c)^3 - 20*a^5*tan(d*x + c)^2 - 30*a^3*b^2
*tan(d*x + c)^2 + 15*a^4*b*tan(d*x + c) - 6*a^5)/(a^7*tan(d*x + c)^5))/d
```

3.67 $\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=382

$$-\frac{a^6 b}{2d\left(a^2+b^2\right)^4 (a+b\tan(c+dx))^2}-\frac{2a^5 b\left(a^2-3b^2\right)}{d\left(a^2+b^2\right)^5 (a+b\tan(c+dx))}-\frac{a\cos^2(c+dx)\left(\left(-119a^4b^2+65a^2b^4+11a^6+3\right.\right.}{16d\left(a^2+b^2\right.}$$

```
[Out] (a*(5*a^8 - 180*a^6*b^2 + 390*a^4*b^4 - 68*a^2*b^6 - 3*b^8)*x)/(16*(a^2 + b
^2)^6) + (a^4*b*(3*a^4 - 22*a^2*b^2 + 15*b^4)*Log[a*Cos[c + d*x] + b*Sin[c
+ d*x]])/((a^2 + b^2)^6*d) - (a^6*b)/(2*(a^2 + b^2)^4*d*(a + b*Tan[c + d*x]
)^2) - (2*a^5*b*(a^2 - 3*b^2))/((a^2 + b^2)^5*d*(a + b*Tan[c + d*x])) - (Co
s[c + d*x]^6*(b*(3*a^2 - b^2) + a*(a^2 - 3*b^2)*Tan[c + d*x]))/(6*(a^2 + b^
2)^3*d) + (Cos[c + d*x]^4*(6*b*(9*a^4 - 4*a^2*b^2 - b^4) + a*(13*a^4 - 62*a
^2*b^2 - 3*b^4)*Tan[c + d*x]))/(24*(a^2 + b^2)^4*d) - (a*Cos[c + d*x]^2*(24
*a^3*b*(3*a^2 - 5*b^2) + (11*a^6 - 119*a^4*b^2 + 65*a^2*b^4 + 3*b^6)*Tan[c
+ d*x]))/(16*(a^2 + b^2)^5*d)
```

Rubi [A] time = 1.43182, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$-\frac{a^6 b}{2d\left(a^2+b^2\right)^4 (a+b\tan(c+dx))^2}-\frac{2a^5 b\left(a^2-3b^2\right)}{d\left(a^2+b^2\right)^5 (a+b\tan(c+dx))}-\frac{a\cos^2(c+dx)\left(\left(-119a^4b^2+65a^2b^4+11a^6+3\right.\right.}{16d\left(a^2+b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```


```
[Out] (a*(5*a^8 - 180*a^6*b^2 + 390*a^4*b^4 - 68*a^2*b^6 - 3*b^8)*x)/(16*(a^2 + b
^2)^6) + (a^4*b*(3*a^4 - 22*a^2*b^2 + 15*b^4)*Log[a*Cos[c + d*x] + b*Sin[c
+ d*x]])/((a^2 + b^2)^6*d) - (a^6*b)/(2*(a^2 + b^2)^4*d*(a + b*Tan[c + d*x]
)^2) - (2*a^5*b*(a^2 - 3*b^2))/((a^2 + b^2)^5*d*(a + b*Tan[c + d*x])) - (Co
s[c + d*x]^6*(b*(3*a^2 - b^2) + a*(a^2 - 3*b^2)*Tan[c + d*x]))/(6*(a^2 + b^
2)^3*d) + (Cos[c + d*x]^4*(6*b*(9*a^4 - 4*a^2*b^2 - b^4) + a*(13*a^4 - 62*a
^2*b^2 - 3*b^4)*Tan[c + d*x]))/(24*(a^2 + b^2)^4*d) - (a*Cos[c + d*x]^2*(24
*a^3*b*(3*a^2 - 5*b^2) + (11*a^6 - 119*a^4*b^2 + 65*a^2*b^4 + 3*b^6)*Tan[c
+ d*x]))/(16*(a^2 + b^2)^5*d)
```


Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{x^6}{(a+x)^3\left(b^2+x^2\right)^4}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^6(c+dx)\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^3 d} - \frac{\mathrm{Subst}\left(\int \frac{-\frac{a^4b^6\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^3}+\frac{3a^3b^6\left(5a^2+b^2\right.}{\left(a^2+b^2\right)^3}}{}\right.}{}$$

$$= -\frac{\cos^6(c+dx)\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^3 d} + \frac{\cos^4(c+dx)\left(6b\left(9a^4-4a^2b^2-\right.\right.}{2}$$

$$= -\frac{\cos^6(c+dx)\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^3 d} + \frac{\cos^4(c+dx)\left(6b\left(9a^4-4a^2b^2-\right.\right.}{2}$$

$$= -\frac{\cos^6(c+dx)\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^3 d} + \frac{\cos^4(c+dx)\left(6b\left(9a^4-4a^2b^2-\right.\right.}{2}$$

$$= \frac{a^4b\left(3a^4-22a^2b^2+15b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^6 d} - \frac{a^6b}{2\left(a^2+b^2\right)^4 d(a+b\tan(c+dx))^2} - \cdots$$

$$= \frac{a^4b\left(3a^4-22a^2b^2+15b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^6 d} - \frac{a^6b}{2\left(a^2+b^2\right)^4 d(a+b\tan(c+dx))^2} - \cdots$$

$$= \frac{a\left(5a^8-180a^6b^2+390a^4b^4-68a^2b^6-3b^8\right)x}{16\left(a^2+b^2\right)^6} + \frac{a^4b\left(3a^4-22a^2b^2+15b^4\right)\log(\cos(c+d\right.}{\left(a^2+b^2\right)^6 d}$$

Mathematica [A] time = 6.60369, size = 683, normalized size = 1.79

$$\frac{b\left(-\frac{3a^4\left(3a^2-5b^2\right)\cos^2(c+dx)}{2\left(a^2+b^2\right)^5}-\frac{\left(3a^2-b^2\right)\cos^6(c+dx)}{6\left(a^2+b^2\right)^3}+\frac{\left(-4a^2b^2+9a^4-b^4\right)\cos^4(c+dx)}{4\left(a^2+b^2\right)^4}-\frac{3a^5\left(a^2-7b^2\right)\tan^{-1}(\tan(c+dx))}{2b\left(a^2+b^2\right)^5}-\frac{a^6}{2\left(a^2+b^2\right)^4(a+b\tan(c+dx))^2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b*((-3*a^5*(a^2 - 7*b^2)*ArcTan[Tan[c + d*x]])/(2*b*(a^2 + b^2)^5) - (3*a^
4*(3*a^2 - 5*b^2)*Cos[c + d*x]^2)/(2*(a^2 + b^2)^5) + ((9*a^4 - 4*a^2*b^2 -
 b^4)*Cos[c + d*x]^4)/(4*(a^2 + b^2)^4) - ((3*a^2 - b^2)*Cos[c + d*x]^6)/(6
*(a^2 + b^2)^3) - (a^4*(3*a^4 - 22*a^2*b^2 + 15*b^4 - (a^5 - 18*a^3*b^2 + 2
1*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(2*(a^2 + b^2)^6) +
(a^4*(3*a^4 - 22*a^2*b^2 + 15*b^4)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^6 -
 (a^4*(3*a^4 - 22*a^2*b^2 + 15*b^4 + (a^5 - 18*a^3*b^2 + 21*a*b^4)/Sqrt[-b^
2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^6) - (3*a^5*(a^2 - 7*b
^2)*Cos[c + d*x]*Sin[c + d*x])/(2*b*(a^2 + b^2)^5) + (3*a*(a^4 - 4*a^2*b^2
- b^4)*Cos[c + d*x]^3*Sin[c + d*x])/(4*b*(a^2 + b^2)^4) - (a*(a^2 - 3*b^2)*
Cos[c + d*x]^5*Sin[c + d*x])/(6*b*(a^2 + b^2)^3) + (9*a*(a^4 - 4*a^2*b^2 -
b^4)*(ArcTan[Tan[c + d*x]]/b + (Cos[c + d*x]*Sin[c + d*x])/b))/(8*(a^2 + b^
2)^4) - (5*a*(a^2 - 3*b^2)*((2*Cos[c + d*x]^3*Sin[c + d*x])/b + 3*b^2*(ArcT
an[Tan[c + d*x]]/b^3 + (Cos[c + d*x]*Sin[c + d*x])/b^3)))/(48*(a^2 + b^2)^3
) - a^6/(2*(a^2 + b^2)^4*(a + b*Tan[c + d*x])^2) - (2*a^5*(a^2 - 3*b^2))/((
a^2 + b^2)^5*(a + b*Tan[c + d*x]))))/d
```

Maple [B] time = 0.118, size = 1449, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x)
```

```
[Out] -15/2/d/(a^2+b^2)^6*ln(1+tan(d*x+c)^2)*a^4*b^5+13/2/d/(a^2+b^2)^6/(1+tan(d*
x+c)^2)^3*a^4*b^5-11/4/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*a^8*b+31/6/d/(a^2+b
^2)^6/(1+tan(d*x+c)^2)^3*a^6*b^3-3/2/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*a^2*b
^7-11/16/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^5*a^9-5/6/d/(a^2+b^2)^
6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^3*a^9-1/4/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*
tan(d*x+c)^2*b^9-5/16/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)*a^9+3/d*a
^8*b/(a^2+b^2)^6*ln(a+b*tan(d*x+c))-22/d*a^6*b^3/(a^2+b^2)^6*ln(a+b*tan(d*x
+c))+15/d*a^4*b^5/(a^2+b^2)^6*ln(a+b*tan(d*x+c))-2/d*b*a^7/(a^2+b^2)^5/(a+b
*tan(d*x+c))+6/d*b^3*a^5/(a^2+b^2)^5/(a+b*tan(d*x+c))-45/4/d/(a^2+b^2)^6*ar
ctan(tan(d*x+c))*a^7*b^2+195/8/d/(a^2+b^2)^6*arctan(tan(d*x+c))*a^5*b^4-17/
4/d/(a^2+b^2)^6*arctan(tan(d*x+c))*a^3*b^6-3/16/d/(a^2+b^2)^6*arctan(tan(d*
x+c))*a*b^8-3/2/d/(a^2+b^2)^6*ln(1+tan(d*x+c)^2)*a^8*b+11/d/(a^2+b^2)^6*ln(
1+tan(d*x+c)^2)*a^6*b^3-1/12/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*b^9+5/16/d/(a
^2+b^2)^6*arctan(tan(d*x+c))*a^9+27/4/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(
d*x+c)^5*a^7*b^2+27/8/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^5*a^5*b^4
```

```
-17/4/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^5*a^3*b^6-3/16/d/(a^2+b^2
)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^5*a*b^8-9/2/d/(a^2+b^2)^6/(1+tan(d*x+c)^2
)^3*tan(d*x+c)^4*a^8*b+3/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^4*a^6*
b^3+15/2/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^4*a^4*b^5+12/d/(a^2+b^
2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^3*a^7*b^2+2/d/(a^2+b^2)^6/(1+tan(d*x+c)^
2)^3*tan(d*x+c)^3*a^5*b^4-34/3/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^
3*a^3*b^6-1/2/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^3*a*b^8-1/2*a^6*b
/(a^2+b^2)^4/d/(a+b*tan(d*x+c))^2-27/4/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan
(d*x+c)^2*a^8*b+19/2/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^2*a^6*b^3+
15/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^2*a^4*b^5-3/2/d/(a^2+b^2)^6/
(1+tan(d*x+c)^2)^3*tan(d*x+c)^2*a^2*b^7+21/4/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)
^3*tan(d*x+c)*a^7*b^2-3/8/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)*a^5*b
^4-23/4/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)*a^3*b^6+3/16/d/(a^2+b^2
)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)*a*b^8
```

Maxima [B] time = 1.93332, size = 1469, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(5*a^9 - 180*a^7*b^2 + 390*a^5*b^4 - 68*a^3*b^6 - 3*a*b^8)*(d*x + c
)/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
b^12) + 48*(3*a^8*b - 22*a^6*b^3 + 15*a^4*b^5)*log(b*tan(d*x + c) + a)/(a^1
2 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)
- 24*(3*a^8*b - 22*a^6*b^3 + 15*a^4*b^5)*log(tan(d*x + c)^2 + 1)/(a^12 + 6*
a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12) - (252
*a^8*b - 644*a^6*b^3 + 68*a^4*b^5 + 4*a^2*b^7 + 3*(43*a^7*b^2 - 215*a^5*b^4
 + 65*a^3*b^6 + 3*a*b^8)*tan(d*x + c)^7 + 6*(31*a^8*b - 127*a^6*b^3 + 5*a^4
*b^5 + 3*a^2*b^7)*tan(d*x + c)^6 + (33*a^9 + 403*a^7*b^2 - 2005*a^5*b^4 + 5
29*a^3*b^6 + 24*a*b^8)*tan(d*x + c)^5 + 4*(164*a^8*b - 515*a^6*b^3 + 65*a^4
*b^5 + 27*a^2*b^7 + 3*b^9)*tan(d*x + c)^4 + (40*a^9 + 335*a^7*b^2 - 2171*a^
5*b^4 + 429*a^3*b^6 + 15*a*b^8)*tan(d*x + c)^3 + 2*(357*a^8*b - 987*a^6*b^3
 + 125*a^4*b^5 + 31*a^2*b^7 + 2*b^9)*tan(d*x + c)^2 + (15*a^9 + 93*a^7*b^2
- 763*a^5*b^4 + 127*a^3*b^6 + 8*a*b^8)*tan(d*x + c))/(a^12 + 5*a^10*b^2 + 1
0*a^8*b^4 + 10*a^6*b^6 + 5*a^4*b^8 + a^2*b^10 + (a^10*b^2 + 5*a^8*b^4 + 10*
a^6*b^6 + 10*a^4*b^8 + 5*a^2*b^10 + b^12)*tan(d*x + c)^8 + 2*(a^11*b + 5*a^
9*b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9 + a*b^11)*tan(d*x + c)^7 + (a^1
2 + 8*a^10*b^2 + 25*a^8*b^4 + 40*a^6*b^6 + 35*a^4*b^8 + 16*a^2*b^10 + 3*b^1
2)*tan(d*x + c)^6 + 6*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3
```

```
*b^9 + a*b^11)*tan(d*x + c)^5 + 3*(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*
b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*tan(d*x + c)^4 + 6*(a^11*b + 5*a^9*b^
3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9 + a*b^11)*tan(d*x + c)^3 + (3*a^12
+ 16*a^10*b^2 + 35*a^8*b^4 + 40*a^6*b^6 + 25*a^4*b^8 + 8*a^2*b^10 + b^12)*t
an(d*x + c)^2 + 2*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9
 + a*b^11)*tan(d*x + c)))/d
```

Fricas [B] time = 3.58986, size = 2137, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(195*a^8*b^3 - 427*a^6*b^5 - 165*a^4*b^7 + 27*a^2*b^9 + 2*b^11 - 8*(a^
10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11)*cos(d*x + c)
^8 + 20*(2*a^10*b + 9*a^8*b^3 + 16*a^6*b^5 + 14*a^4*b^7 + 6*a^2*b^9 + b^11)
*cos(d*x + c)^6 - 2*(49*a^10*b + 162*a^8*b^3 + 198*a^6*b^5 + 112*a^4*b^7 +
33*a^2*b^9 + 6*b^11)*cos(d*x + c)^4 + 3*(5*a^9*b^2 - 180*a^7*b^4 + 390*a^5*
b^6 - 68*a^3*b^8 - 3*a*b^10)*d*x + (9*a^10*b - 46*a^8*b^3 + 994*a^6*b^5 + 1
44*a^4*b^7 - 43*a^2*b^9 - 2*b^11 + 3*(5*a^11 - 185*a^9*b^2 + 570*a^7*b^4 -
458*a^5*b^6 + 65*a^3*b^8 + 3*a*b^10)*d*x)*cos(d*x + c)^2 + 24*(3*a^8*b^3 -
22*a^6*b^5 + 15*a^4*b^7 + (3*a^10*b - 25*a^8*b^3 + 37*a^6*b^5 - 15*a^4*b^7)
*cos(d*x + c)^2 + 2*(3*a^9*b^2 - 22*a^7*b^4 + 15*a^5*b^6)*cos(d*x + c)*sin(
d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2) - (8*(a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*b^6 + 5*a^3*b^8 + a*b^1
0)*cos(d*x + c)^7 - 2*(13*a^11 + 55*a^9*b^2 + 90*a^7*b^4 + 70*a^5*b^6 + 25*
a^3*b^8 + 3*a*b^10)*cos(d*x + c)^5 + (33*a^11 + 49*a^9*b^2 - 54*a^7*b^4 - 1
26*a^5*b^6 - 59*a^3*b^8 - 3*a*b^10)*cos(d*x + c)^3 - (261*a^9*b^2 - 338*a^7
*b^4 + 120*a^5*b^6 - 150*a^3*b^8 - 5*a*b^10 + 6*(5*a^10*b - 180*a^8*b^3 + 3
90*a^6*b^5 - 68*a^4*b^7 - 3*a^2*b^9)*d*x)*cos(d*x + c))*sin(d*x + c))/((a^1
4 + 5*a^12*b^2 + 9*a^10*b^4 + 5*a^8*b^6 - 5*a^6*b^8 - 9*a^4*b^10 - 5*a^2*b^
12 - b^14)*d*cos(d*x + c)^2 + 2*(a^13*b + 6*a^11*b^3 + 15*a^9*b^5 + 20*a^7*
b^7 + 15*a^5*b^9 + 6*a^3*b^11 + a*b^13)*d*cos(d*x + c)*sin(d*x + c) + (a^12
*b^2 + 6*a^10*b^4 + 15*a^8*b^6 + 20*a^6*b^8 + 15*a^4*b^10 + 6*a^2*b^12 + b^
14)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28551, size = 1246, normalized size = 3.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^9 - 180*a^7*b^2 + 390*a^5*b^4 - 68*a^3*b^6 - 3*a*b^8)*(d*x + c
)/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
b^12) - 24*(3*a^8*b - 22*a^6*b^3 + 15*a^4*b^5)*log(tan(d*x + c)^2 + 1)/(a^1
2 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)
+ 48*(3*a^8*b^2 - 22*a^6*b^4 + 15*a^4*b^6)*log(abs(b*tan(d*x + c) + a))/(a^
12*b + 6*a^10*b^3 + 15*a^8*b^5 + 20*a^6*b^7 + 15*a^4*b^9 + 6*a^2*b^11 + b^1
3) - 24*(9*a^8*b^3*tan(d*x + c)^2 - 66*a^6*b^5*tan(d*x + c)^2 + 45*a^4*b^7*
tan(d*x + c)^2 + 22*a^9*b^2*tan(d*x + c) - 140*a^7*b^4*tan(d*x + c) + 78*a^
5*b^6*tan(d*x + c) + 14*a^10*b - 72*a^8*b^3 + 34*a^6*b^5)/((a^12 + 6*a^10*b
^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*(b*tan(d*x +
 c) + a)^2) + (132*a^8*b*tan(d*x + c)^6 - 968*a^6*b^3*tan(d*x + c)^6 + 660*
a^4*b^5*tan(d*x + c)^6 - 33*a^9*tan(d*x + c)^5 + 324*a^7*b^2*tan(d*x + c)^5
 + 162*a^5*b^4*tan(d*x + c)^5 - 204*a^3*b^6*tan(d*x + c)^5 - 9*a*b^8*tan(d*
x + c)^5 + 180*a^8*b*tan(d*x + c)^4 - 2760*a^6*b^3*tan(d*x + c)^4 + 2340*a^
4*b^5*tan(d*x + c)^4 - 40*a^9*tan(d*x + c)^3 + 576*a^7*b^2*tan(d*x + c)^3 +
 96*a^5*b^4*tan(d*x + c)^3 - 544*a^3*b^6*tan(d*x + c)^3 - 24*a*b^8*tan(d*x
+ c)^3 + 72*a^8*b*tan(d*x + c)^2 - 2448*a^6*b^3*tan(d*x + c)^2 + 2700*a^4*b
^5*tan(d*x + c)^2 - 72*a^2*b^7*tan(d*x + c)^2 - 12*b^9*tan(d*x + c)^2 - 15*
a^9*tan(d*x + c) + 252*a^7*b^2*tan(d*x + c) - 18*a^5*b^4*tan(d*x + c) - 276
*a^3*b^6*tan(d*x + c) + 9*a*b^8*tan(d*x + c) - 720*a^6*b^3 + 972*a^4*b^5 -
72*a^2*b^7 - 4*b^9)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*
b^8 + 6*a^2*b^10 + b^12)*(tan(d*x + c)^2 + 1)^3))/d
```

3.68 $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3} dx$

Optimal. Leaf size=285

$$-\frac{a^4b}{2d\left(a^2+b^2\right)^3(a+b\tan(c+dx))^2}-\frac{2a^3b\left(a^2-2b^2\right)}{d\left(a^2+b^2\right)^4(a+b\tan(c+dx))}-\frac{a\cos^2(c+dx)\left(\left(-34a^2b^2+5a^4+9b^4\right)\tan(c+d\right.}{8d\left(a^2+b^2\right)^4}$$

```
[Out] (3*a*(a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*x)/(8*(a^2 + b^2)^5) + (3*a^2*
b*(a^4 - 5*a^2*b^2 + 2*b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b
^2)^5*d) - (a^4*b)/(2*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])^2) - (2*a^3*b*(a
^2 - 2*b^2))/((a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^4*(b*(3
*a^2 - b^2) + a*(a^2 - 3*b^2)*Tan[c + d*x]))/(4*(a^2 + b^2)^3*d) - (a*Cos[c
 + d*x]^2*(24*a*b*(a^2 - b^2) + (5*a^4 - 34*a^2*b^2 + 9*b^4)*Tan[c + d*x]))
/(8*(a^2 + b^2)^4*d)
```

Rubi [A] time = 0.849748, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$-\frac{a^4b}{2d\left(a^2+b^2\right)^3(a+b\tan(c+dx))^2}-\frac{2a^3b\left(a^2-2b^2\right)}{d\left(a^2+b^2\right)^4(a+b\tan(c+dx))}-\frac{a\cos^2(c+dx)\left(\left(-34a^2b^2+5a^4+9b^4\right)\tan(c+d\right.}{8d\left(a^2+b^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*(a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*x)/(8*(a^2 + b^2)^5) + (3*a^2*
b*(a^4 - 5*a^2*b^2 + 2*b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b
^2)^5*d) - (a^4*b)/(2*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])^2) - (2*a^3*b*(a
^2 - 2*b^2))/((a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^4*(b*(3
*a^2 - b^2) + a*(a^2 - 3*b^2)*Tan[c + d*x]))/(4*(a^2 + b^2)^3*d) - (a*Cos[c
 + d*x]^2*(24*a*b*(a^2 - b^2) + (5*a^4 - 34*a^2*b^2 + 9*b^4)*Tan[c + d*x]))
/(8*(a^2 + b^2)^4*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^4}{(a+x)^3\left(b^2+x^2\right)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^4(c+dx)\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^3 d} - \frac{\text{Subst}\left(\int \frac{\frac{a^4b^4\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^3}-\frac{a^3b^4\left(9a^2+5b^2\right)x}{\left(a^2+b^2\right)^3}-\frac{a^2}{}}{(a+x)}\right.}{}$$

$$= \frac{\cos^4(c+dx)\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^3 d} - \frac{a\cos^2(c+dx)\left(24ab\left(a^2-b^2\right)+(5\right.}{8\left(a^2+\right.}$$

$$= \frac{\cos^4(c+dx)\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^3 d} - \frac{a\cos^2(c+dx)\left(24ab\left(a^2-b^2\right)+(5\right.}{8\left(a^2+\right.}$$

$$= \frac{3a^2b\left(a^4-5a^2b^2+2b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^4b}{2\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^2} - \frac{}{\left(a^2+\right.}$$

$$= \frac{3a^2b\left(a^4-5a^2b^2+2b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^4b}{2\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^2} - \frac{}{\left(a^2+\right.}$$

$$= \frac{3a\left(a^6-25a^4b^2+35a^2b^4-3b^6\right)x}{8\left(a^2+b^2\right)^5} + \frac{3a^2b\left(a^4-5a^2b^2+2b^4\right)\log(\cos(c+dx))}{\left(a^2+b^2\right)^5 d} + \frac{3a^2b\left(a^4-\right.}{}$$

Mathematica [A] time = 6.43835, size = 501, normalized size = 1.76

$$\frac{b\left(-\frac{3a^2(a-b)(a+b)\cos^2(c+dx)}{\left(a^2+b^2\right)^4}+\frac{\left(3a^2-b^2\right)\cos^4(c+dx)}{4\left(a^2+b^2\right)^3}-\frac{a^3\left(a^2-5b^2\right)\tan^{-1}(\tan(c+dx))}{b\left(a^2+b^2\right)^4}-\frac{a^4}{2\left(a^2+b^2\right)^3(a+b\tan(c+dx))^2}-\frac{2a^3\left(a^2-2b^2\right)}{\left(a^2+b^2\right)^4(a+b\tan(c+dx))}-\frac{a^2(}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b*(-((a^3*(a^2 - 5*b^2)*ArcTan[Tan[c + d*x]])/(b*(a^2 + b^2)^4)) - (3*a^2*
(a - b)*(a + b)*Cos[c + d*x]^2)/(a^2 + b^2)^4 + ((3*a^2 - b^2)*Cos[c + d*x]
```

```
^4)/(4*(a^2 + b^2)^3) - (a^2*(3*a^4 - 15*a^2*b^2 + 6*b^4 - (a^5 - 13*a^3*b^
2 + 10*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(2*(a^2 + b^2)^
5) + (3*a^2*(a^4 - 5*a^2*b^2 + 2*b^4)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^
5 - (a^2*(3*a^4 - 15*a^2*b^2 + 6*b^4 + (a^5 - 13*a^3*b^2 + 10*a*b^4)/Sqrt[-
b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^5) - (a^3*(a^2 - 5*b
^2)*Cos[c + d*x]*Sin[c + d*x])/(b*(a^2 + b^2)^4) + (a*(a^2 - 3*b^2)*Cos[c +
 d*x]^3*Sin[c + d*x])/(4*b*(a^2 + b^2)^3) + (3*a*(a^2 - 3*b^2)*(ArcTan[Tan[
c + d*x]]/b + (Cos[c + d*x]*Sin[c + d*x])/b))/(8*(a^2 + b^2)^3) - a^4/(2*(a
^2 + b^2)^3*(a + b*Tan[c + d*x])^2) - (2*a^3*(a^2 - 2*b^2))/((a^2 + b^2)^4*
(a + b*Tan[c + d*x]))))/d
```

Maple [B] time = 0.109, size = 882, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x)
```

```
[Out] 3/8/d/(a^2+b^2)^5*arctan(tan(d*x+c))*a^7+3/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2
*tan(d*x+c)^2*a^2*b^5+5/4/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*a^4*b^3+13/4/d/(
a^2+b^2)^5/(1+tan(d*x+c)^2)^2*a^2*b^5-3/d/(a^2+b^2)^5*ln(1+tan(d*x+c)^2)*a^
2*b^5-5/8/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*a^7-3/8/d/(a^2+b^2)
^5/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^7-9/8/d/(a^2+b^2)^5*arctan(tan(d*x+c))*a
*b^6-75/8/d/(a^2+b^2)^5*arctan(tan(d*x+c))*a^5*b^2+6/d*b^5/(a^2+b^2)^5*ln(a
+b*tan(d*x+c))*a^2-9/4/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*a^6*b+105/8/d/(a^2+
b^2)^5*arctan(tan(d*x+c))*a^3*b^4-3/2/d/(a^2+b^2)^5*ln(1+tan(d*x+c)^2)*a^6*
b+15/2/d/(a^2+b^2)^5*ln(1+tan(d*x+c)^2)*a^4*b^3-2/d*a^5*b/(a^2+b^2)^4/(a+b*
tan(d*x+c))+4/d*a^3*b^3/(a^2+b^2)^4/(a+b*tan(d*x+c))+3/d*b*a^6/(a^2+b^2)^5*
ln(a+b*tan(d*x+c))-15/d*b^3*a^4/(a^2+b^2)^5*ln(a+b*tan(d*x+c))-3/d/(a^2+b^2
)^5/(1+tan(d*x+c)^2)^2*tan(d*x+c)^2*a^6*b+29/8/d/(a^2+b^2)^5/(1+tan(d*x+c)^
2)^2*tan(d*x+c)^3*a^5*b^2-1/2*a^4*b/(a^2+b^2)^3/d/(a+b*tan(d*x+c))^2+27/8/d
/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^5*b^2+25/8/d/(a^2+b^2)^5/(1+ta
n(d*x+c)^2)^2*tan(d*x+c)^3*a^3*b^4-9/8/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*tan
(d*x+c)^3*a*b^6-1/4/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*b^7-15/8/d/(a^2+b^2)^5
/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a*b^6+15/8/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*
tan(d*x+c)*a^3*b^4
```

Maxima [B] time = 1.81091, size = 1004, normalized size = 3.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(a^7 - 25*a^5*b^2 + 35*a^3*b^4 - 3*a*b^6)*(d*x + c)/(a^10 + 5*a^8*b^
2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 24*(a^6*b - 5*a^4*b^3 + 2
*a^2*b^5)*log(b*tan(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b
^6 + 5*a^2*b^8 + b^10) - 12*(a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*log(tan(d*x + c
)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) -
(38*a^6*b - 56*a^4*b^3 + 2*a^2*b^5 + 3*(7*a^5*b^2 - 22*a^3*b^4 + 3*a*b^6)*t
an(d*x + c)^5 + 6*(5*a^6*b - 12*a^4*b^3 - a^2*b^5)*tan(d*x + c)^4 + (5*a^7
+ 49*a^5*b^2 - 133*a^3*b^4 + 15*a*b^6)*tan(d*x + c)^3 + 2*(35*a^6*b - 61*a^
4*b^3 + a^2*b^5 + b^7)*tan(d*x + c)^2 + (3*a^7 + 22*a^5*b^2 - 73*a^3*b^4 +
4*a*b^6)*tan(d*x + c))/(a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^8
+ (a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*tan(d*x + c)^6 + 2*(
a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)^5 + (a^10 +
 6*a^8*b^2 + 14*a^6*b^4 + 16*a^4*b^6 + 9*a^2*b^8 + 2*b^10)*tan(d*x + c)^4 +
 4*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)^3 + (2*
a^10 + 9*a^8*b^2 + 16*a^6*b^4 + 14*a^4*b^6 + 6*a^2*b^8 + b^10)*tan(d*x + c)
^2 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)))/d
```

Fricas [B] time = 3.09478, size = 1569, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(119*a^6*b^3 - 159*a^4*b^5 - 51*a^2*b^7 + 3*b^9 + 8*(a^8*b + 4*a^6*b^3
 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^6 - 8*(5*a^8*b + 16*a^6*b^3 +
18*a^4*b^5 + 8*a^2*b^7 + b^9)*cos(d*x + c)^4 + 12*(a^7*b^2 - 25*a^5*b^4 + 3
5*a^3*b^6 - 3*a*b^8)*d*x - (a^8*b + 110*a^6*b^3 - 420*a^4*b^5 - 78*a^2*b^7
+ 3*b^9 - 12*(a^9 - 26*a^7*b^2 + 60*a^5*b^4 - 38*a^3*b^6 + 3*a*b^8)*d*x)*co
s(d*x + c)^2 + 48*(a^6*b^3 - 5*a^4*b^5 + 2*a^2*b^7 + (a^8*b - 6*a^6*b^3 + 7
*a^4*b^5 - 2*a^2*b^7)*cos(d*x + c)^2 + 2*(a^7*b^2 - 5*a^5*b^4 + 2*a^3*b^6)*
cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2
)*cos(d*x + c)^2 + b^2) + 2*(4*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a
*b^8)*cos(d*x + c)^5 - 2*(5*a^9 + 12*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 - 3*a*
b^8)*cos(d*x + c)^3 + (77*a^7*b^2 - 69*a^5*b^4 + 63*a^3*b^6 - 15*a*b^8 + 12
*(a^8*b - 25*a^6*b^3 + 35*a^4*b^5 - 3*a^2*b^7)*d*x)*cos(d*x + c))*sin(d*x +
 c))/((a^12 + 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 - 4*a^2*b^10 - b^12)*d*cos
```

```
(d*x + c)^2 + 2*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9 +
 a*b^11)*d*cos(d*x + c)*sin(d*x + c) + (a^10*b^2 + 5*a^8*b^4 + 10*a^6*b^6 +
 10*a^4*b^8 + 5*a^2*b^10 + b^12)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27008, size = 794, normalized size = 2.79

$$\frac{\frac{3\left(a^7-25a^5b^2+35a^3b^4-3ab^6\right)(dx+c)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}-\frac{12\left(a^6b-5a^4b^3+2a^2b^5\right)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}+\frac{24\left(a^6b^2-5a^4b^4+2a^2b^6\right)\log(|b\tan(dx+c)+a|)}{a^{10}b+5a^8b^3+10a^6b^5+10a^4b^7+5a^2b^9+b^{11}}-\frac{21a^5b^2\tan(dx}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^7 - 25*a^5*b^2 + 35*a^3*b^4 - 3*a*b^6)*(d*x + c)/(a^10 + 5*a^8*b^
2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - 12*(a^6*b - 5*a^4*b^3 + 2
*a^2*b^5)*log(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b
^6 + 5*a^2*b^8 + b^10) + 24*(a^6*b^2 - 5*a^4*b^4 + 2*a^2*b^6)*log(abs(b*tan
(d*x + c) + a))/(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 +
 b^11) - (21*a^5*b^2*tan(d*x + c)^5 - 66*a^3*b^4*tan(d*x + c)^5 + 9*a*b^6*t
an(d*x + c)^5 + 30*a^6*b*tan(d*x + c)^4 - 72*a^4*b^3*tan(d*x + c)^4 - 6*a^2
*b^5*tan(d*x + c)^4 + 5*a^7*tan(d*x + c)^3 + 49*a^5*b^2*tan(d*x + c)^3 - 13
3*a^3*b^4*tan(d*x + c)^3 + 15*a*b^6*tan(d*x + c)^3 + 70*a^6*b*tan(d*x + c)^
2 - 122*a^4*b^3*tan(d*x + c)^2 + 2*a^2*b^5*tan(d*x + c)^2 + 2*b^7*tan(d*x +
 c)^2 + 3*a^7*tan(d*x + c) + 22*a^5*b^2*tan(d*x + c) - 73*a^3*b^4*tan(d*x +
 c) + 4*a*b^6*tan(d*x + c) + 38*a^6*b - 56*a^4*b^3 + 2*a^2*b^5)/((a^8 + 4*a
^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c)^3 + a*tan(d*x + c)^2
+ b*tan(d*x + c) + a)^2))/d
```

3.69 $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=206

$$-\frac{a^2b}{2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}-\frac{2ab\left(a^2-b^2\right)}{d\left(a^2+b^2\right)^3(a+b\tan(c+dx))}-\frac{\cos^2(c+dx)\left(a\left(a^2-3b^2\right)\tan(c+dx)+b\left(3a^2-\right.\right.}{2d\left(a^2+b^2\right)^3}$$

```
[Out] (a*(a^4 - 14*a^2*b^2 + 9*b^4)*x)/(2*(a^2 + b^2)^4) + (b*(3*a^4 - 8*a^2*b^2
+ b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) - (a^2*b)/(2
*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) - (2*a*b*(a^2 - b^2))/((a^2 + b^2)
^3*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^2*(b*(3*a^2 - b^2) + a*(a^2 - 3*
b^2)*Tan[c + d*x]))/(2*(a^2 + b^2)^3*d)
```

Rubi [A] time = 0.396246, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$-\frac{a^2b}{2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}-\frac{2ab\left(a^2-b^2\right)}{d\left(a^2+b^2\right)^3(a+b\tan(c+dx))}-\frac{\cos^2(c+dx)\left(a\left(a^2-3b^2\right)\tan(c+dx)+b\left(3a^2-\right.\right.}{2d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(a^4 - 14*a^2*b^2 + 9*b^4)*x)/(2*(a^2 + b^2)^4) + (b*(3*a^4 - 8*a^2*b^2
+ b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) - (a^2*b)/(2
*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) - (2*a*b*(a^2 - b^2))/((a^2 + b^2)
^3*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^2*(b*(3*a^2 - b^2) + a*(a^2 - 3*
b^2)*Tan[c + d*x]))/(2*(a^2 + b^2)^3*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
```

```
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^2}{(a+x)^3\left(b^2+x^2\right)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^2(c+dx)\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^3 d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^4b^2\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^3}+\frac{a^3b^2\left(3a^2+7b^2\right)x}{\left(a^2+b^2\right)^3} \cdots}{(a+x \cdots} \cdots\right.}{\cdots}$$

$$= -\frac{\cos^2(c+dx)\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^3 d} - \frac{\text{Subst}\left(\int\left(-\frac{2a^2b^2}{\left(a^2+b^2\right)^2(a+x)^3}+\frac{4ab^2(\cdots}{\left(a^2+b^2 \cdots\right.}\cdots\right.\right.}{\cdots}$$

$$= \frac{b\left(3a^4-8a^2b^2+b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^4 d} - \frac{a^2b}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} - \frac{\cdots}{\left(a^2+b^2\right)\cdots}$$

$$= \frac{b\left(3a^4-8a^2b^2+b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^4 d} - \frac{a^2b}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} - \frac{\cdots}{\left(a^2+b^2\right)\cdots}$$

$$= \frac{a\left(a^4-14a^2b^2+9b^4\right)x}{2\left(a^2+b^2\right)^4} + \frac{b\left(3a^4-8a^2b^2+b^4\right)\log(\cos(c+dx))}{\left(a^2+b^2\right)^4 d} + \frac{b\left(3a^4-8a^2b^2+b^4\right)\log(\cdots}{\left(a^2+b^2\right)\cdots}$$

Mathematica [A] time = 3.95863, size = 316, normalized size = 1.53

$$-\frac{b\left(\frac{a\left(a^2-3b^2\right)\left(a^2+b^2\right)\sin(2(c+dx))}{2b}+\left(3a^2-b^2\right)\left(a^2+b^2\right)\cos^2(c+dx)+\frac{a\left(a^2-3b^2\right)\left(a^2+b^2\right)\tan^{-1}(\tan(c+dx))}{b}+\frac{a^2\left(a^2+b^2\right)^2}{(a+b\tan(c+dx))^2}+\frac{4\left(a^5-\cdots\right.}{a+b\tan\cdots}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(b*((a*(a^2 - 3*b^2)*(a^2 + b^2)*ArcTan[Tan[c + d*x]])/b + (3*a^2 - b^2)*(
a^2 + b^2)*Cos[c + d*x]^2 + (3*a^4 - 8*a^2*b^2 + b^4 - (a^5 - 8*a^3*b^2 + 3
*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*(3*a^4 - 8*a^2*b^2
 + b^4)*Log[a + b*Tan[c + d*x]] + (3*a^4 - 8*a^2*b^2 + b^4 + (a^5 - 8*a^3*b
^2 + 3*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (a*(a^2 - 3*b^
2)*(a^2 + b^2)*Sin[2*(c + d*x)])/(2*b) + (a^2*(a^2 + b^2)^2)/(a + b*Tan[c +
```

```
d*x])^2 + (4*(a^5 - a*b^4))/(a + b*Tan[c + d*x])))/(2*(a^2 + b^2)^4*d)
```

Maple [B] time = 0.115, size = 542, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x)
```

```
[Out] -1/2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)*tan(d*x+c)*a^5+1/d/(a^2+b^2)^4/(1+tan(d
*x+c)^2)*tan(d*x+c)*a^3*b^2+3/2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)*tan(d*x+c)*a
*b^4-3/2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)*a^4*b-1/d/(a^2+b^2)^4/(1+tan(d*x+c)
^2)*a^2*b^3+1/2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)*b^5-3/2/d/(a^2+b^2)^4*ln(1+t
an(d*x+c)^2)*a^4*b+4/d/(a^2+b^2)^4*ln(1+tan(d*x+c)^2)*a^2*b^3-1/2/d/(a^2+b^
2)^4*ln(1+tan(d*x+c)^2)*b^5-7/d/(a^2+b^2)^4*arctan(tan(d*x+c))*a^3*b^2+9/2/
d/(a^2+b^2)^4*arctan(tan(d*x+c))*a*b^4+1/2/d/(a^2+b^2)^4*arctan(tan(d*x+c))
*a^5-1/2*a^2*b/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2+3/d*b/(a^2+b^2)^4*ln(a+b*ta
n(d*x+c))*a^4-8/d*b^3/(a^2+b^2)^4*ln(a+b*tan(d*x+c))*a^2+1/d*b^5/(a^2+b^2)^
4*ln(a+b*tan(d*x+c))-2/d*b*a^3/(a^2+b^2)^3/(a+b*tan(d*x+c))+2/d*b^3/(a^2+b^
2)^3*a/(a+b*tan(d*x+c))
```

Maxima [B] time = 1.84901, size = 625, normalized size = 3.03

$$\frac{\frac{\left(a^5-14a^3b^2+9ab^4\right)(dx+c)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}+\frac{2\left(3a^4b-8a^2b^3+b^5\right)\log(b\tan(dx+c)+a)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{\left(3a^4b-8a^2b^3+b^5\right)\log\left(\tan(dx+c)^2+1\right)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{}{a^8+3a^6b^2+3a^4b^4+a^2b^6+(a^6b^2+3a^4b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((a^5 - 14*a^3*b^2 + 9*a*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
4*a^2*b^6 + b^8) + 2*(3*a^4*b - 8*a^2*b^3 + b^5)*log(b*tan(d*x + c) + a)/(a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - (3*a^4*b - 8*a^2*b^3 + b^5)
*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) -
(8*a^4*b - 4*a^2*b^3 + (5*a^3*b^2 - 7*a*b^4)*tan(d*x + c)^3 + (7*a^4*b - 6*
a^2*b^3 - b^5)*tan(d*x + c)^2 + (a^5 + 7*a^3*b^2 - 6*a*b^4)*tan(d*x + c))/(
a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 +
b^8)*tan(d*x + c)^4 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c
```

```
)^3 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*tan(d*x + c)^2 + 2*(a
^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c)))/d
```

Fricas [B] time = 2.48319, size = 1152, normalized size = 5.59

$$\frac{13\,a^4b^3 - 8\,a^2b^5 - b^7 - 2\left(a^6b + 3\,a^4b^3 + 3\,a^2b^5 + b^7\right)\cos\left(dx+c\right)^4 + 2\left(a^5b^2 - 14\,a^3b^4 + 9\,ab^6\right)dx - \left(a^6b + 23\,a^4b^3 - 21\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(13*a^4*b^3 - 8*a^2*b^5 - b^7 - 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)
*cos(d*x + c)^4 + 2*(a^5*b^2 - 14*a^3*b^4 + 9*a*b^6)*d*x - (a^6*b + 23*a^4*
b^3 - 21*a^2*b^5 - 3*b^7 - 2*(a^7 - 15*a^5*b^2 + 23*a^3*b^4 - 9*a*b^6)*d*x)
*cos(d*x + c)^2 + 2*(3*a^4*b^3 - 8*a^2*b^5 + b^7 + (3*a^6*b - 11*a^4*b^3 +
9*a^2*b^5 - b^7)*cos(d*x + c)^2 + 2*(3*a^5*b^2 - 8*a^3*b^4 + a*b^6)*cos(d*x
 + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d
*x + c)^2 + b^2) - 2*((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^3
- 2*(4*a^5*b^2 - 3*a^3*b^4 + 3*a*b^6 + (a^6*b - 14*a^4*b^3 + 9*a^2*b^5)*d*x
)*cos(d*x + c))*sin(d*x + c))/((a^10 + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 -
3*a^2*b^8 - b^10)*d*cos(d*x + c)^2 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a
^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) + (a^8*b^2 + 4*a^6*b^4 + 6*a^4*
b^6 + 4*a^2*b^8 + b^10)*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.23699, size = 651, normalized size = 3.16

$$\frac{\frac{\left(a^5-14\,a^3b^2+9\,ab^4\right)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{\left(3\,a^4b-8\,a^2b^3+b^5\right)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{2\left(3\,a^4b^2-8\,a^2b^4+b^6\right)\log(|b\tan(dx+c)+a|)}{a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9}+\frac{3\,a^4b\tan(dx+c)^2-8\,a^2b^3\tan(dx+c)}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*((a^5 - 14*a^3*b^2 + 9*a*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
4*a^2*b^6 + b^8) - (3*a^4*b - 8*a^2*b^3 + b^5)*log(tan(d*x + c)^2 + 1)/(a^8
 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 2*(3*a^4*b^2 - 8*a^2*b^4 + b^
6)*log(abs(b*tan(d*x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7
+ b^9) + (3*a^4*b*tan(d*x + c)^2 - 8*a^2*b^3*tan(d*x + c)^2 + b^5*tan(d*x +
 c)^2 - a^5*tan(d*x + c) + 2*a^3*b^2*tan(d*x + c) + 3*a*b^4*tan(d*x + c) -
10*a^2*b^3 + 2*b^5)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(tan(d
*x + c)^2 + 1)) - (9*a^4*b^3*tan(d*x + c)^2 - 24*a^2*b^5*tan(d*x + c)^2 + 3
*b^7*tan(d*x + c)^2 + 22*a^5*b^2*tan(d*x + c) - 48*a^3*b^4*tan(d*x + c) + 2
*a*b^6*tan(d*x + c) + 14*a^6*b - 22*a^4*b^3)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8)*(b*tan(d*x + c) + a)^2))/d
```

3.70 $\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=95

$$-\frac{2b}{a^3d(a+b\tan(c+dx))}-\frac{b}{2a^2d(a+b\tan(c+dx))^2}-\frac{3b\log(\tan(c+dx))}{a^4d}+\frac{3b\log(a+b\tan(c+dx))}{a^4d}-\frac{\cot(c+dx)}{a^3d}$$

```
[Out] -(Cot[c + d*x]/(a^3*d)) - (3*b*Log[Tan[c + d*x]])/(a^4*d) + (3*b*Log[a + b*
Tan[c + d*x]])/(a^4*d) - b/(2*a^2*d*(a + b*Tan[c + d*x])^2) - (2*b)/(a^3*d*
(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.0774468, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 44}

$$-\frac{2b}{a^3d(a+b\tan(c+dx))}-\frac{b}{2a^2d(a+b\tan(c+dx))^2}-\frac{3b\log(\tan(c+dx))}{a^4d}+\frac{3b\log(a+b\tan(c+dx))}{a^4d}-\frac{\cot(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(Cot[c + d*x]/(a^3*d)) - (3*b*Log[Tan[c + d*x]])/(a^4*d) + (3*b*Log[a + b*
Tan[c + d*x]])/(a^4*d) - b/(2*a^2*d*(a + b*Tan[c + d*x])^2) - (2*b)/(a^3*d*
(a + b*Tan[c + d*x]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{1}{x^2(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{b\,\mathrm{Subst}\left(\int \left(\frac{1}{a^3x^2} - \frac{3}{a^4x} + \frac{1}{a^2(a+x)^3} + \frac{2}{a^3(a+x)^2} + \frac{3}{a^4(a+x)}\right) dx, x, b\tan(c+dx)\right)}{d}$$
$$= -\frac{\cot(c+dx)}{a^3d} - \frac{3b\log(\tan(c+dx))}{a^4d} + \frac{3b\log(a+b\tan(c+dx))}{a^4d} - \frac{b}{2a^2d(a+b\tan(c+dx)}$$

Mathematica [B] time = 2.58893, size = 241, normalized size = 2.54

$$\frac{b\left(a^2\left(-b^2\right)\sec^2(c+dx) - 2a^2\left(a^2+b^2\right)(-3\log(a\cos(c+dx)+b\sin(c+dx)) + 3\log(\sin(c+dx)) + 2) - 2b^2\tan^2(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-2*a^3*(a^2 + b^2)*Cot[c + d*x] + b*(-2*a^2*(a^2 + b^2)*(2 + 3*Log[Sin[c +
 d*x]] - 3*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]) - a^2*b^2*Sec[c + d*x]^2 +
 2*a*b*(2*a^2 + b^2 - 6*(a^2 + b^2)*Log[Sin[c + d*x]] + 6*(a^2 + b^2)*Log[a
*Cos[c + d*x] + b*Sin[c + d*x]])*Tan[c + d*x] - 2*b^2*(-3*a^2 - 2*b^2 + 3*(
a^2 + b^2)*Log[Sin[c + d*x]] - 3*(a^2 + b^2)*Log[a*Cos[c + d*x] + b*Sin[c +
 d*x]])*Tan[c + d*x]^2))/(2*a^4*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
```

Maple [A] time = 0.119, size = 96, normalized size = 1.

$$-\frac{1}{da^3\tan(dx+c)} - 3\frac{b\ln(\tan(dx+c))}{a^4d} - \frac{b}{2a^2d(a+b\tan(dx+c))^2} + 3\frac{b\ln(a+b\tan(dx+c))}{a^4d} - 2\frac{b}{da^3(a+b\tan(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x)
```

```
[Out] -1/d/a^3/tan(d*x+c)-3*b*ln(tan(d*x+c))/a^4/d-1/2*b/a^2/d/(a+b*tan(d*x+c))^2
+3*b*ln(a+b*tan(d*x+c))/a^4/d-2*b/a^3/d/(a+b*tan(d*x+c))
```

Maxima [A] time = 1.14572, size = 146, normalized size = 1.54

$$-\frac{\frac{6\,b^2\tan(dx+c)^2+9\,ab\tan(dx+c)+2\,a^2}{a^3b^2\tan(dx+c)^3+2\,a^4b\tan(dx+c)^2+a^5\tan(dx+c)}-\frac{6\,b\log(b\tan(dx+c)+a)}{a^4}+\frac{6\,b\log(\tan(dx+c))}{a^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((6*b^2*tan(d*x + c)^2 + 9*a*b*tan(d*x + c) + 2*a^2)/(a^3*b^2*tan(d*x
+ c)^3 + 2*a^4*b*tan(d*x + c)^2 + a^5*tan(d*x + c)) - 6*b*log(b*tan(d*x + c
) + a)/a^4 + 6*b*log(tan(d*x + c))/a^4)/d
```

Fricas [B] time = 2.4596, size = 1219, normalized size = 12.83

$$2\left(a^7+4\,a^5b^2-2\,a^3b^4-3\,ab^6\right)\cos\left(dx+c\right)^3-2\left(2\,a^5b^2-3\,a^3b^4-3\,ab^6\right)\cos\left(dx+c\right)+3\left(2\left(a^5b^2+2\,a^3b^4+ab^6\right)\cos\left(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a^7 + 4*a^5*b^2 - 2*a^3*b^4 - 3*a*b^6)*cos(d*x + c)^3 - 2*(2*a^5*b^
2 - 3*a^3*b^4 - 3*a*b^6)*cos(d*x + c) + 3*(2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*
cos(d*x + c)^3 - 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c) - (a^4*b^3 +
2*a^2*b^5 + b^7 + (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*cos(d*x + c)^2)*sin(d*x
 + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b
^2) - 3*(2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 2*(a^5*b^2 + 2*a^
3*b^4 + a*b^6)*cos(d*x + c) - (a^4*b^3 + 2*a^2*b^5 + b^7 + (a^6*b + a^4*b^3
 - a^2*b^5 - b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/4*cos(d*x + c)^2 + 1
/4) - (5*a^4*b^3 + 3*a^2*b^5 - 4*(a^6*b + 5*a^4*b^3 + 3*a^2*b^5)*cos(d*x +
c)^2)*sin(d*x + c))/(2*(a^9*b + 2*a^7*b^3 + a^5*b^5)*d*cos(d*x + c)^3 - 2*(
a^9*b + 2*a^7*b^3 + a^5*b^5)*d*cos(d*x + c) - ((a^10 + a^8*b^2 - a^6*b^4 -
a^4*b^6)*d*cos(d*x + c)^2 + (a^8*b^2 + 2*a^6*b^4 + a^4*b^6)*d)*sin(d*x + c)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26987, size = 153, normalized size = 1.61

$$\frac{\frac{6\,b\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^4} - \frac{6\,b\log\left(|\tan\left(dx+c\right)|\right)}{a^4} + \frac{2\,\left(3\,b\tan\left(dx+c\right)-a\right)}{a^4\tan\left(dx+c\right)} - \frac{9\,b^3\tan\left(dx+c\right)^2+22\,ab^2\tan\left(dx+c\right)+14\,a^2b}{\left(b\tan\left(dx+c\right)+a\right)^2a^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*b*log(abs(b*tan(d*x + c) + a))/a^4 - 6*b*log(abs(tan(d*x + c)))/a^4
+ 2*(3*b*tan(d*x + c) - a)/(a^4*tan(d*x + c)) - (9*b^3*tan(d*x + c)^2 + 22*
a*b^2*tan(d*x + c) + 14*a^2*b)/((b*tan(d*x + c) + a)^2*a^4))/d
```

3.71 $\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=178

$$-\frac{2b\left(a^2+2b^2\right)}{a^5d(a+b\tan(c+dx))}-\frac{b\left(a^2+b^2\right)}{2a^4d(a+b\tan(c+dx))^2}-\frac{\left(a^2+6b^2\right)\cot(c+dx)}{a^5d}-\frac{b\left(3a^2+10b^2\right)\log(\tan(c+dx))}{a^6d}+\frac{b\left(3a^2\right.}{}$$

```
[Out] -(((a^2 + 6*b^2)*Cot[c + d*x])/(a^5*d)) + (3*b*Cot[c + d*x]^2)/(2*a^4*d) -
Cot[c + d*x]^3/(3*a^3*d) - (b*(3*a^2 + 10*b^2)*Log[Tan[c + d*x]])/(a^6*d) +
 (b*(3*a^2 + 10*b^2)*Log[a + b*Tan[c + d*x]])/(a^6*d) - (b*(a^2 + b^2))/(2*
a^4*d*(a + b*Tan[c + d*x])^2) - (2*b*(a^2 + 2*b^2))/(a^5*d*(a + b*Tan[c + d
*x]))
```

Rubi [A] time = 0.152575, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 894}

$$-\frac{2b\left(a^2+2b^2\right)}{a^5d(a+b\tan(c+dx))}-\frac{b\left(a^2+b^2\right)}{2a^4d(a+b\tan(c+dx))^2}-\frac{\left(a^2+6b^2\right)\cot(c+dx)}{a^5d}-\frac{b\left(3a^2+10b^2\right)\log(\tan(c+dx))}{a^6d}+\frac{b\left(3a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```


```
[Out] -(((a^2 + 6*b^2)*Cot[c + d*x])/(a^5*d)) + (3*b*Cot[c + d*x]^2)/(2*a^4*d) -
Cot[c + d*x]^3/(3*a^3*d) - (b*(3*a^2 + 10*b^2)*Log[Tan[c + d*x]])/(a^6*d) +
 (b*(3*a^2 + 10*b^2)*Log[a + b*Tan[c + d*x]])/(a^6*d) - (b*(a^2 + b^2))/(2*
a^4*d*(a + b*Tan[c + d*x])^2) - (2*b*(a^2 + 2*b^2))/(a^5*d*(a + b*Tan[c + d
*x]))
```


Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
```

```
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{b\,\text{Subst}\left(\int \frac{b^2+x^2}{x^4(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\,\text{Subst}\left(\int \left(\frac{b^2}{a^3x^4} - \frac{3b^2}{a^4x^3} + \frac{a^2+6b^2}{a^5x^2} + \frac{-3a^2-10b^2}{a^6x} + \frac{a^2+b^2}{a^4(a+x)^3} + \frac{2(a^2+2b^2)}{a^5(a+x)^2} + \frac{3a^2+10b^2}{a^6(a+x)}\right) dx, x, b\tan(\right.}{d}$$

$$= -\frac{(a^2+6b^2)\cot(c+dx)}{a^5d} + \frac{3b\cot^2(c+dx)}{2a^4d} - \frac{\cot^3(c+dx)}{3a^3d} - \frac{b(3a^2+10b^2)\log(\tan(c+d}{a^6d}$$

Mathematica [B] time = 3.22664, size = 456, normalized size = 2.56

$$-\frac{b^3\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))}{2a^4d(a+b\tan(c+dx))^3} + \frac{\sec^3(c+dx)\left(3a^2b^2\sin(c+dx)+4b^4\sin(c+dx)\right)(a\cos(c+dx)+b\,\text{s}}{a^6d(a+b\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(b^3*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(2*a^4*d*(a + b*Tan
[c + d*x])^3) - (Csc[c + d*x]^3*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c +
d*x])^3)/(3*a^3*d*(a + b*Tan[c + d*x])^3) - (2*(a^2*Cos[c + d*x] + 9*b^2*Co
s[c + d*x])*Csc[c + d*x]*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3
)/(3*a^5*d*(a + b*Tan[c + d*x])^3) + (3*b*Csc[c + d*x]^2*Sec[c + d*x]^3*(a*
Cos[c + d*x] + b*Sin[c + d*x])^3)/(2*a^4*d*(a + b*Tan[c + d*x])^3) + ((-3*a
^2*b - 10*b^3)*Log[Sin[c + d*x]]*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c +
 d*x])^3)/(a^6*d*(a + b*Tan[c + d*x])^3) + ((3*a^2*b + 10*b^3)*Log[a*Cos[c
+ d*x] + b*Sin[c + d*x]]*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3
)/(a^6*d*(a + b*Tan[c + d*x])^3) + (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2*(3*a^2*b^2*Sin[c + d*x] + 4*b^4*Sin[c + d*x]))/(a^6*d*(a + b*Ta
n[c + d*x])^3)
```

Maple [A] time = 0.13, size = 234, normalized size = 1.3

$$-\frac{1}{3\,da^3(\tan(dx+c))^3} - \frac{1}{da^3\tan(dx+c)} - 6\,\frac{b^2}{da^5\tan(dx+c)} + \frac{3\,b}{2\,da^4(\tan(dx+c))^2} - 3\,\frac{b\ln(\tan(dx+c))}{da^4} - 10\,\frac{b^3\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x)
```

```
[Out] -1/3/d/a^3/tan(d*x+c)^3-1/d/a^3/tan(d*x+c)-6/d/a^5/tan(d*x+c)*b^2+3/2/d/a^4
*b/tan(d*x+c)^2-3*b*ln(tan(d*x+c))/a^4/d-10/d*b^3/a^6*ln(tan(d*x+c))+3*b*ln
(a+b*tan(d*x+c))/a^4/d+10/d*b^3/a^6*ln(a+b*tan(d*x+c))-1/2*b/a^2/d/(a+b*tan
(d*x+c))^2-1/2/d*b^3/a^4/(a+b*tan(d*x+c))^2-2*b/a^3/d/(a+b*tan(d*x+c))-4/d*
b^3/a^5/(a+b*tan(d*x+c))
```

Maxima [A] time = 1.20175, size = 259, normalized size = 1.46

$$\frac{\frac{5\,a^3b\tan(dx+c)-6\left(3\,a^2b^2+10\,b^4\right)\tan(dx+c)^4-2\,a^4-9\left(3\,a^3b+10\,ab^3\right)\tan(dx+c)^3-2\left(3\,a^4+10\,a^2b^2\right)\tan(dx+c)^2}{a^5b^2\tan(dx+c)^5+2\,a^6b\tan(dx+c)^4+a^7\tan(dx+c)^3}+\frac{6\left(3\,a^2b+10\,b^3\right)\log(b\tan(dx+c)+a)}{a^6}-\frac{6\left(3\,a\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*((5*a^3*b*tan(d*x + c) - 6*(3*a^2*b^2 + 10*b^4)*tan(d*x + c)^4 - 2*a^4
- 9*(3*a^3*b + 10*a*b^3)*tan(d*x + c)^3 - 2*(3*a^4 + 10*a^2*b^2)*tan(d*x +
c)^2)/(a^5*b^2*tan(d*x + c)^5 + 2*a^6*b*tan(d*x + c)^4 + a^7*tan(d*x + c)^3
) + 6*(3*a^2*b + 10*b^3)*log(b*tan(d*x + c) + a)/a^6 - 6*(3*a^2*b + 10*b^3)
*log(tan(d*x + c))/a^6)/d
```

Fricas [B] time = 2.59892, size = 1818, normalized size = 10.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(2*a^7 + 27*a^5*b^2 + a^3*b^4 - 30*a*b^6)*cos(d*x + c)^5 - 2*(3*a^7
+ 43*a^5*b^2 - 8*a^3*b^4 - 60*a*b^6)*cos(d*x + c)^3 + 6*(5*a^5*b^2 - 3*a^3*
b^4 - 10*a*b^6)*cos(d*x + c) + 3*(2*(3*a^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos
(d*x + c)^5 - 4*(3*a^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^3 + 2*(3*a
^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos(d*x + c) + (3*a^4*b^3 + 13*a^2*b^5 + 10
*b^7 - (3*a^6*b + 10*a^4*b^3 - 3*a^2*b^5 - 10*b^7)*cos(d*x + c)^4 + (3*a^6*
```

```
b + 7*a^4*b^3 - 16*a^2*b^5 - 20*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*
b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 3*(2*(3*a
^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^5 - 4*(3*a^5*b^2 + 13*a^3*b^4
+ 10*a*b^6)*cos(d*x + c)^3 + 2*(3*a^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos(d*x
+ c) + (3*a^4*b^3 + 13*a^2*b^5 + 10*b^7 - (3*a^6*b + 10*a^4*b^3 - 3*a^2*b^5
 - 10*b^7)*cos(d*x + c)^4 + (3*a^6*b + 7*a^4*b^3 - 16*a^2*b^5 - 20*b^7)*cos
(d*x + c)^2)*sin(d*x + c))*log(-1/4*cos(d*x + c)^2 + 1/4) + (24*a^4*b^3 + 3
0*a^2*b^5 + 4*(2*a^6*b + 29*a^4*b^3 + 30*a^2*b^5)*cos(d*x + c)^4 - 3*(a^6*b
 + 45*a^4*b^3 + 50*a^2*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(2*(a^9*b + a^7*b
^3)*d*cos(d*x + c)^5 - 4*(a^9*b + a^7*b^3)*d*cos(d*x + c)^3 + 2*(a^9*b + a^
7*b^3)*d*cos(d*x + c) - ((a^10 - a^6*b^4)*d*cos(d*x + c)^4 - (a^10 - a^8*b^
2 - 2*a^6*b^4)*d*cos(d*x + c)^2 - (a^8*b^2 + a^6*b^4)*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34649, size = 320, normalized size = 1.8

$$-\frac{\frac{6\left(3a^2b+10b^3\right)\log\left(|\tan(dx+c)|\right)}{a^6}-\frac{6\left(3a^2b^2+10b^4\right)\log\left(|b\tan(dx+c)+a|\right)}{a^6b}+\frac{3\left(9a^2b^3\tan(dx+c)^2+30b^5\tan(dx+c)^2+22a^3b^2\tan(dx+c)+68ab^4\tan(dx+c)\right.}{(b\tan(dx+c)+a)^2a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(6*(3*a^2*b + 10*b^3)*log(abs(tan(d*x + c)))/a^6 - 6*(3*a^2*b^2 + 10*b
^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b) + 3*(9*a^2*b^3*tan(d*x + c)^2 + 30
*b^5*tan(d*x + c)^2 + 22*a^3*b^2*tan(d*x + c) + 68*a*b^4*tan(d*x + c) + 14*
a^4*b + 39*a^2*b^3)/((b*tan(d*x + c) + a)^2*a^6) - (33*a^2*b*tan(d*x + c)^3
 + 110*b^3*tan(d*x + c)^3 - 6*a^3*tan(d*x + c)^2 - 36*a*b^2*tan(d*x + c)^2
+ 9*a^2*b*tan(d*x + c) - 2*a^3)/(a^6*tan(d*x + c)^3))/d
```

3.72 $\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=265

$$-\frac{2b\left(a^2+b^2\right)\left(a^2+3b^2\right)}{a^7d(a+b\tan(c+dx))}-\frac{b\left(a^2+b^2\right)^2}{2a^6d(a+b\tan(c+dx))^2}-\frac{2\left(a^2+3b^2\right)\cot^3(c+dx)}{3a^5d}+\frac{b\left(3a^2+5b^2\right)\cot^2(c+dx)}{a^6d}-\frac{\left(12a^2b^2\right.}{}$$

```
[Out] -(((a^4 + 12*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(a^7*d)) + (b*(3*a^2 + 5*b^2)*
Cot[c + d*x]^2)/(a^6*d) - (2*(a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*a^5*d) + (3*b
*Cot[c + d*x]^4)/(4*a^4*d) - Cot[c + d*x]^5/(5*a^3*d) - (b*(3*a^4 + 20*a^2*
b^2 + 21*b^4)*Log[Tan[c + d*x]])/(a^8*d) + (b*(3*a^4 + 20*a^2*b^2 + 21*b^4)
*Log[a + b*Tan[c + d*x]])/(a^8*d) - (b*(a^2 + b^2)^2)/(2*a^6*d*(a + b*Tan[c
 + d*x])^2) - (2*b*(a^2 + b^2)*(a^2 + 3*b^2))/(a^7*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.239401, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 894}

$$-\frac{2b\left(a^2+b^2\right)\left(a^2+3b^2\right)}{a^7d(a+b\tan(c+dx))}-\frac{b\left(a^2+b^2\right)^2}{2a^6d(a+b\tan(c+dx))^2}-\frac{2\left(a^2+3b^2\right)\cot^3(c+dx)}{3a^5d}+\frac{b\left(3a^2+5b^2\right)\cot^2(c+dx)}{a^6d}-\frac{\left(12a^2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```


```
[Out] -(((a^4 + 12*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(a^7*d)) + (b*(3*a^2 + 5*b^2)*
Cot[c + d*x]^2)/(a^6*d) - (2*(a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*a^5*d) + (3*b
*Cot[c + d*x]^4)/(4*a^4*d) - Cot[c + d*x]^5/(5*a^3*d) - (b*(3*a^4 + 20*a^2*
b^2 + 21*b^4)*Log[Tan[c + d*x]])/(a^8*d) + (b*(3*a^4 + 20*a^2*b^2 + 21*b^4)
*Log[a + b*Tan[c + d*x]])/(a^8*d) - (b*(a^2 + b^2)^2)/(2*a^6*d*(a + b*Tan[c
 + d*x])^2) - (2*b*(a^2 + b^2)*(a^2 + 3*b^2))/(a^7*d*(a + b*Tan[c + d*x]))
```


Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{\left(b^2+x^2\right)^2}{x^6(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\,\mathrm{Subst}\left(\int \left(\frac{b^4}{a^3x^6} - \frac{3b^4}{a^4x^5} + \frac{2b^2\left(a^2+3b^2\right)}{a^5x^4} - \frac{2\left(3a^2b^2+5b^4\right)}{a^6x^3} + \frac{a^4+12a^2b^2+15b^4}{a^7x^2} + \frac{-3a^4-20a^2b^2-21b^4}{a^8x} + \frac{\left(a^2+\right.}{a^6(a}\right.\right.}{d}$$

$$= -\frac{\left(a^4+12a^2b^2+15b^4\right)\cot(c+dx)}{a^7d} + \frac{b\left(3a^2+5b^2\right)\cot^2(c+dx)}{a^6d} - \frac{2\left(a^2+3b^2\right)\cot^3(c+d}{3a^5d}$$

Mathematica [A] time = 4.67453, size = 494, normalized size = 1.86

$$-\frac{\csc^5(c+dx)\left(5\sec(c+dx)\left(-3b\left(89a^4b^2+345a^2b^4+8a^6+210b^6\right)\tan(c+dx)-27a^5b^2-42a^3b^4+40a^7+135ab^6\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(Csc[c + d*x]^5*(Sec[c + d*x]^2*((8*a^7 + 567*a^5*b^2 + 630*a^3*b^4 - 1215
*a*b^6)*Cos[3*(c + d*x)] - (24*a^7 + 619*a^5*b^2 + 630*a^3*b^4 - 675*a*b^6)
*Cos[5*(c + d*x)] + 8*a^7*Cos[7*(c + d*x)] + 187*a^5*b^2*Cos[7*(c + d*x)] +
 210*a^3*b^4*Cos[7*(c + d*x)] - 135*a*b^6*Cos[7*(c + d*x)] - 126*a^6*b*Sin[
3*(c + d*x)] + 1665*a^4*b^3*Sin[3*(c + d*x)] + 4635*a^2*b^5*Sin[3*(c + d*x)
] + 1890*b^7*Sin[3*(c + d*x)] + 10*a^6*b*Sin[5*(c + d*x)] - 1215*a^4*b^3*Si
n[5*(c + d*x)] - 2565*a^2*b^5*Sin[5*(c + d*x)] - 630*b^7*Sin[5*(c + d*x)] +
 16*a^6*b*Sin[7*(c + d*x)] + 345*a^4*b^3*Sin[7*(c + d*x)] + 585*a^2*b^5*Sin
[7*(c + d*x)] + 90*b^7*Sin[7*(c + d*x)]) + 960*b*(3*a^4 + 20*a^2*b^2 + 21*b
^4)*(Log[Sin[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin[c + d*x]])*Sin[c + d*x]
^5*(a + b*Tan[c + d*x])^2 + 5*Sec[c + d*x]*(40*a^7 - 27*a^5*b^2 - 42*a^3*b^
4 + 135*a*b^6 - 3*b*(8*a^6 + 89*a^4*b^2 + 345*a^2*b^4 + 210*b^6)*Tan[c + d*
x])))/(960*a^8*d*(a + b*Tan[c + d*x])^2)
```

Maple [A] time = 0.137, size = 410, normalized size = 1.6

$$-\frac{1}{5\,da^{3}\left(\tan\left(dx+c\right)\right)^{5}}-\frac{2}{3\,da^{3}\left(\tan\left(dx+c\right)\right)^{3}}-2\,\frac{b^{2}}{da^{5}\left(\tan\left(dx+c\right)\right)^{3}}-\frac{1}{da^{3}\tan\left(dx+c\right)}-12\,\frac{b^{2}}{da^{5}\tan\left(dx+c\right)}-15\,\frac{}{da^{7}\,\mathrm{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x)
```

```
[Out] -1/5/d/a^3/tan(d*x+c)^5-2/3/d/a^3/tan(d*x+c)^3-2/d/a^5/tan(d*x+c)^3*b^2-1/d
/a^3/tan(d*x+c)-12/d/a^5/tan(d*x+c)*b^2-15/d/a^7/tan(d*x+c)*b^4+3/4/d/a^4*b
/tan(d*x+c)^4+3/d/a^4*b/tan(d*x+c)^2+5/d*b^3/a^6/tan(d*x+c)^2-3*b*ln(tan(d*
x+c))/a^4/d-20/d*b^3/a^6*ln(tan(d*x+c))-21/d*b^5/a^8*ln(tan(d*x+c))+3*b*ln(
a+b*tan(d*x+c))/a^4/d+20/d*b^3/a^6*ln(a+b*tan(d*x+c))+21/d*b^5/a^8*ln(a+b*t
an(d*x+c))-1/2*b/a^2/d/(a+b*tan(d*x+c))^2-1/d*b^3/a^4/(a+b*tan(d*x+c))^2-1/
2/d*b^5/a^6/(a+b*tan(d*x+c))^2-2*b/a^3/d/(a+b*tan(d*x+c))-8/d*b^3/a^5/(a+b*
tan(d*x+c))-6/d*b^5/a^7/(a+b*tan(d*x+c))
```

Maxima [A] time = 1.21861, size = 379, normalized size = 1.43

$$\frac{\frac{21\,a^{5}b\tan(dx+c)-60\left(3\,a^{4}b^{2}+20\,a^{2}b^{4}+21\,b^{6}\right)\tan(dx+c)^{6}-12\,a^{6}-90\left(3\,a^{5}b+20\,a^{3}b^{3}+21\,ab^{5}\right)\tan(dx+c)^{5}-20\left(3\,a^{6}+20\,a^{4}b^{2}+21\,a^{2}b^{4}\right)\tan(dx+c)^{4}+5\left(20\,a^{5}b+21\,a^{3}\right.}{a^{7}b^{2}\tan(dx+c)^{7}+2\,a^{8}b\tan(dx+c)^{6}+a^{9}\tan(dx+c)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*((21*a^5*b*tan(d*x + c) - 60*(3*a^4*b^2 + 20*a^2*b^4 + 21*b^6)*tan(d*x
 + c)^6 - 12*a^6 - 90*(3*a^5*b + 20*a^3*b^3 + 21*a*b^5)*tan(d*x + c)^5 - 20
*(3*a^6 + 20*a^4*b^2 + 21*a^2*b^4)*tan(d*x + c)^4 + 5*(20*a^5*b + 21*a^3*b^
3)*tan(d*x + c)^3 - 2*(20*a^6 + 21*a^4*b^2)*tan(d*x + c)^2)/(a^7*b^2*tan(d*
x + c)^7 + 2*a^8*b*tan(d*x + c)^6 + a^9*tan(d*x + c)^5) + 60*(3*a^4*b + 20*
a^2*b^3 + 21*b^5)*log(b*tan(d*x + c) + a)/a^8 - 60*(3*a^4*b + 20*a^2*b^3 +
21*b^5)*log(tan(d*x + c))/a^8)/d
```

Fricas [B] time = 3.01524, size = 2344, normalized size = 8.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(4*(8*a^7 + 187*a^5*b^2 + 120*a^3*b^4 - 315*a*b^6)*cos(d*x + c)^7 - 4*
(20*a^7 + 482*a^5*b^2 + 255*a^3*b^4 - 945*a*b^6)*cos(d*x + c)^5 + 10*(6*a^7
 + 157*a^5*b^2 + 60*a^3*b^4 - 378*a*b^6)*cos(d*x + c)^3 - 30*(13*a^5*b^2 +
2*a^3*b^4 - 42*a*b^6)*cos(d*x + c) + 30*(2*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b
^6)*cos(d*x + c)^7 - 6*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c)^5 +
 6*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c)^3 - 2*(3*a^5*b^2 + 20*a
^3*b^4 + 21*a*b^6)*cos(d*x + c) - (3*a^4*b^3 + 20*a^2*b^5 + 21*b^7 + (3*a^6
*b + 17*a^4*b^3 + a^2*b^5 - 21*b^7)*cos(d*x + c)^6 - (6*a^6*b + 31*a^4*b^3
- 18*a^2*b^5 - 63*b^7)*cos(d*x + c)^4 + (3*a^6*b + 11*a^4*b^3 - 39*a^2*b^5
- 63*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c)
 + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 30*(2*(3*a^5*b^2 + 20*a^3*b^4 + 21*a
*b^6)*cos(d*x + c)^7 - 6*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c)^5
 + 6*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c)^3 - 2*(3*a^5*b^2 + 20
*a^3*b^4 + 21*a*b^6)*cos(d*x + c) - (3*a^4*b^3 + 20*a^2*b^5 + 21*b^7 + (3*a
^6*b + 17*a^4*b^3 + a^2*b^5 - 21*b^7)*cos(d*x + c)^6 - (6*a^6*b + 31*a^4*b^
3 - 18*a^2*b^5 - 63*b^7)*cos(d*x + c)^4 + (3*a^6*b + 11*a^4*b^3 - 39*a^2*b^
5 - 63*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/4*cos(d*x + c)^2 + 1/4) -
(285*a^4*b^3 + 630*a^2*b^5 - 8*(8*a^6*b + 195*a^4*b^3 + 315*a^2*b^5)*cos(d*
x + c)^6 + 10*(7*a^6*b + 330*a^4*b^3 + 567*a^2*b^5)*cos(d*x + c)^4 + 15*(a^
6*b - 135*a^4*b^3 - 252*a^2*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(2*a^9*b*d*c
os(d*x + c)^7 - 6*a^9*b*d*cos(d*x + c)^5 + 6*a^9*b*d*cos(d*x + c)^3 - 2*a^9
*b*d*cos(d*x + c) - (a^8*b^2*d + (a^10 - a^8*b^2)*d*cos(d*x + c)^6 - (2*a^1
0 - 3*a^8*b^2)*d*cos(d*x + c)^4 + (a^10 - 3*a^8*b^2)*d*cos(d*x + c)^2)*sin(
d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23827, size = 516, normalized size = 1.95

$$-\frac{\frac{60\left(3a^4b+20a^2b^3+21b^5\right)\log(|\tan(dx+c)|)}{a^8}-\frac{60\left(3a^4b^2+20a^2b^4+21b^6\right)\log(|b\tan(dx+c)+a|)}{a^8b}+\frac{30\left(9a^4b^3\tan(dx+c)^2+60a^2b^5\tan(dx+c)^2+63b^7\tan(dx+c\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*(3*a^4*b + 20*a^2*b^3 + 21*b^5)*log(abs(tan(d*x + c)))/a^8 - 60*(
3*a^4*b^2 + 20*a^2*b^4 + 21*b^6)*log(abs(b*tan(d*x + c) + a))/(a^8*b) + 30*
(9*a^4*b^3*tan(d*x + c)^2 + 60*a^2*b^5*tan(d*x + c)^2 + 63*b^7*tan(d*x + c)
^2 + 22*a^5*b^2*tan(d*x + c) + 136*a^3*b^4*tan(d*x + c) + 138*a*b^6*tan(d*x
 + c) + 14*a^6*b + 78*a^4*b^3 + 76*a^2*b^5)/((b*tan(d*x + c) + a)^2*a^8) -
(411*a^4*b*tan(d*x + c)^5 + 2740*a^2*b^3*tan(d*x + c)^5 + 2877*b^5*tan(d*x
+ c)^5 - 60*a^5*tan(d*x + c)^4 - 720*a^3*b^2*tan(d*x + c)^4 - 900*a*b^4*tan
(d*x + c)^4 + 180*a^4*b*tan(d*x + c)^3 + 300*a^2*b^3*tan(d*x + c)^3 - 40*a^
5*tan(d*x + c)^2 - 120*a^3*b^2*tan(d*x + c)^2 + 45*a^4*b*tan(d*x + c) - 12*
a^5)/(a^8*tan(d*x + c)^5))/d
```

3.73 $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal. Leaf size=366

$$-\frac{a^4b}{3d\left(a^2+b^2\right)^3(a+b\tan(c+dx))^3}-\frac{a^3b\left(a^2-2b^2\right)}{d\left(a^2+b^2\right)^4(a+b\tan(c+dx))^2}-\frac{3a^2b\left(-5a^2b^2+a^4+2b^4\right)}{d\left(a^2+b^2\right)^5(a+b\tan(c+dx))}+\frac{\cos^4(c+dx}{}$$

```
[Out] ((3*a^8 - 132*a^6*b^2 + 370*a^4*b^4 - 132*a^2*b^6 + 3*b^8)*x)/(8*(a^2 + b^2
)^6) + (4*a*b*(a^2 - b^2)*(a^4 - 8*a^2*b^2 + b^4)*Log[a*Cos[c + d*x] + b*Si
n[c + d*x]])/((a^2 + b^2)^6*d) - (a^4*b)/(3*(a^2 + b^2)^3*d*(a + b*Tan[c +
d*x])^3) - (a^3*b*(a^2 - 2*b^2))/((a^2 + b^2)^4*d*(a + b*Tan[c + d*x])^2) -
 (3*a^2*b*(a^4 - 5*a^2*b^2 + 2*b^4))/((a^2 + b^2)^5*d*(a + b*Tan[c + d*x]))
 + (Cos[c + d*x]^4*(4*a*b*(a^2 - b^2) + (a^4 - 6*a^2*b^2 + b^4)*Tan[c + d*x
]))/(4*(a^2 + b^2)^4*d) - (Cos[c + d*x]^2*(16*a*b*(2*a^4 - 5*a^2*b^2 + b^4)
 + (5*a^6 - 65*a^4*b^2 + 55*a^2*b^4 - 3*b^6)*Tan[c + d*x]))/(8*(a^2 + b^2)^
5*d)
```

Rubi [A] time = 1.37025, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$-\frac{a^4b}{3d\left(a^2+b^2\right)^3(a+b\tan(c+dx))^3}-\frac{a^3b\left(a^2-2b^2\right)}{d\left(a^2+b^2\right)^4(a+b\tan(c+dx))^2}-\frac{3a^2b\left(-5a^2b^2+a^4+2b^4\right)}{d\left(a^2+b^2\right)^5(a+b\tan(c+dx))}+\frac{\cos^4(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```


```
[Out] ((3*a^8 - 132*a^6*b^2 + 370*a^4*b^4 - 132*a^2*b^6 + 3*b^8)*x)/(8*(a^2 + b^2
)^6) + (4*a*b*(a^2 - b^2)*(a^4 - 8*a^2*b^2 + b^4)*Log[a*Cos[c + d*x] + b*Si
n[c + d*x]])/((a^2 + b^2)^6*d) - (a^4*b)/(3*(a^2 + b^2)^3*d*(a + b*Tan[c +
d*x])^3) - (a^3*b*(a^2 - 2*b^2))/((a^2 + b^2)^4*d*(a + b*Tan[c + d*x])^2) -
 (3*a^2*b*(a^4 - 5*a^2*b^2 + 2*b^4))/((a^2 + b^2)^5*d*(a + b*Tan[c + d*x]))
 + (Cos[c + d*x]^4*(4*a*b*(a^2 - b^2) + (a^4 - 6*a^2*b^2 + b^4)*Tan[c + d*x
]))/(4*(a^2 + b^2)^4*d) - (Cos[c + d*x]^2*(16*a*b*(2*a^4 - 5*a^2*b^2 + b^4)
 + (5*a^6 - 65*a^4*b^2 + 55*a^2*b^4 - 3*b^6)*Tan[c + d*x]))/(8*(a^2 + b^2)^
5*d)
```


Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^4}{(a+x)^4\left(b^2+x^2\right)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^4(c+dx)\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+b^4\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^4 d} - \frac{\text{Subst}\left(\int \frac{\frac{a^4b^4\left(a^4-6a^2b^2+b^4\right)}{\left(a^2+b^2\right)^4} - \frac{4a}{}}{}\right.}{}$$

$$= \frac{\cos^4(c+dx)\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+b^4\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^4 d} - \frac{\cos^2(c+dx)\left(16ab\left(2a^4-\right.\right.}{}$$

$$= \frac{\cos^4(c+dx)\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+b^4\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^4 d} - \frac{\cos^2(c+dx)\left(16ab\left(2a^4-\right.\right.}{}$$

$$= \frac{4ab\left(a^2-b^2\right)\left(a^4-8a^2b^2+b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^6 d} - \frac{a^4b}{3\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^3}$$

$$= \frac{4ab\left(a^2-b^2\right)\left(a^4-8a^2b^2+b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^6 d} - \frac{a^4b}{3\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^3}$$

$$= \frac{\left(3a^8-132a^6b^2+370a^4b^4-132a^2b^6+3b^8\right)x}{8\left(a^2+b^2\right)^6} + \frac{4ab\left(a^2-b^2\right)\left(a^4-8a^2b^2+b^4\right)\log(\cos(c}{\left(a^2+b^2\right)^6 d}$$

Mathematica [A] time = 5.41854, size = 564, normalized size = 1.54

$$-\frac{b\left(\frac{12a^2\left(a^2+b^2\right)\left(-10a^2b^2+a^4+5b^4\right)\sin(2(c+dx))}{b} - 24a(a-b)(a+b)\left(a^2+b^2\right)^2\cos^4(c+dx) + 48a\left(a^2+b^2\right)\left(-5a^2b^2+2a^4+b^4\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(b*((24*a^2*(a^2 + b^2)*(a^4 - 10*a^2*b^2 + 5*b^4)*ArcTan[Tan[c + d*x]])/b
 + 48*a*(a^2 + b^2)*(2*a^4 - 5*a^2*b^2 + b^4)*Cos[c + d*x]^2 - 24*a*(a - b)
```

```
*(a + b)*(a^2 + b^2)^2*Cos[c + d*x]^4 + 12*a*(4*a^6 - 36*a^4*b^2 + 36*a^2*b
^4 - 4*b^6 + (-a^7 + 24*a^5*b^2 - 45*a^3*b^4 + 10*a*b^6)/Sqrt[-b^2])*Log[Sq
rt[-b^2] - b*Tan[c + d*x]] - 96*a*(a - b)*(a + b)*(a^4 - 8*a^2*b^2 + b^4)*L
og[a + b*Tan[c + d*x]] + 12*a*(4*a^6 - 36*a^4*b^2 + 36*a^2*b^4 - 4*b^6 + (a
^7 - 24*a^5*b^2 + 45*a^3*b^4 - 10*a*b^6)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan
[c + d*x]] - (6*(a^2 + b^2)^2*(a^4 - 6*a^2*b^2 + b^4)*Cos[c + d*x]^3*Sin[c
+ d*x])/b + (12*a^2*(a^2 + b^2)*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[2*(c + d*x)]
)/b - (9*(a^2 + b^2)^2*(a^4 - 6*a^2*b^2 + b^4)*(2*ArcTan[Tan[c + d*x]] + Si
n[2*(c + d*x)]))/(2*b) + (8*a^4*(a^2 + b^2)^3)/(a + b*Tan[c + d*x])^3 + (24
*a^3*(a^2 - 2*b^2)*(a^2 + b^2)^2)/(a + b*Tan[c + d*x])^2 + (72*a^2*(a^2 + b
^2)*(a^4 - 5*a^2*b^2 + 2*b^4))/(a + b*Tan[c + d*x])))/(24*(a^2 + b^2)^6*d)
```

Maple [B] time = 0.123, size = 1215, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x)
```

```
[Out] -3/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*a*b^7+4/d*b*a^7/(a^2+b^2)^6*ln(a+b*tan(
d*x+c))-36/d*b^3*a^5/(a^2+b^2)^6*ln(a+b*tan(d*x+c))+36/d*b^5*a^3/(a^2+b^2)^
6*ln(a+b*tan(d*x+c))-4/d*b^7*a/(a^2+b^2)^6*ln(a+b*tan(d*x+c))-3/d*b*a^6/(a^
2+b^2)^5/(a+b*tan(d*x+c))-1/3*a^4*b/(a^2+b^2)^3/d/(a+b*tan(d*x+c))^3-5/4/d/
(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^4*b^4-3/8/d/(a^2+b^2)^6/(1+tan(
d*x+c)^2)^2*tan(d*x+c)*a^8+15/d*b^3*a^4/(a^2+b^2)^5/(a+b*tan(d*x+c))-6/d*b^
5*a^2/(a^2+b^2)^5/(a+b*tan(d*x+c))-1/d*a^5*b/(a^2+b^2)^4/(a+b*tan(d*x+c))^2
+2/d*a^3*b^3/(a^2+b^2)^4/(a+b*tan(d*x+c))^2-2/d/(a^2+b^2)^6*ln(1+tan(d*x+c)
^2)*a^7*b+18/d/(a^2+b^2)^6*ln(1+tan(d*x+c)^2)*a^5*b^3-18/d/(a^2+b^2)^6*ln(1
+tan(d*x+c)^2)*a^3*b^5+2/d/(a^2+b^2)^6*ln(1+tan(d*x+c)^2)*a*b^7-33/2/d/(a^2
+b^2)^6*arctan(tan(d*x+c))*a^6*b^2+185/4/d/(a^2+b^2)^6*arctan(tan(d*x+c))*a
^4*b^4-33/2/d/(a^2+b^2)^6*arctan(tan(d*x+c))*a^2*b^6-5/8/d/(a^2+b^2)^6/(1+t
an(d*x+c)^2)^2*tan(d*x+c)^3*a^8+3/8/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*tan(d*
x+c)^3*b^8+5/8/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*tan(d*x+c)*b^8-3/d/(a^2+b^2
)^6/(1+tan(d*x+c)^2)^2*a^7*b+7/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*b^3*a^5+7/d
/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*a^3*b^5+3/8/d/(a^2+b^2)^6*arctan(tan(d*x+c)
)*b^8+3/8/d/(a^2+b^2)^6*arctan(tan(d*x+c))*a^8+15/2/d/(a^2+b^2)^6/(1+tan(d*
x+c)^2)^2*tan(d*x+c)^3*a^6*b^2+5/4/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*tan(d*x
+c)^3*a^4*b^4-13/2/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*a^2*b^6-4/
d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*tan(d*x+c)^2*a^7*b+6/d/(a^2+b^2)^6/(1+tan(
d*x+c)^2)^2*tan(d*x+c)^2*a^5*b^3+8/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*tan(d*x
+c)^2*a^3*b^5-2/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*tan(d*x+c)^2*a*b^7+13/2/d/
```

```
(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^6*b^2-15/2/d/(a^2+b^2)^6/(1+tan
(d*x+c)^2)^2*tan(d*x+c)*a^2*b^6
```

Maxima [B] time = 1.89595, size = 1346, normalized size = 3.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/24*(3*(3*a^8 - 132*a^6*b^2 + 370*a^4*b^4 - 132*a^2*b^6 + 3*b^8)*(d*x + c)
/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b
^12) + 96*(a^7*b - 9*a^5*b^3 + 9*a^3*b^5 - a*b^7)*log(b*tan(d*x + c) + a)/(
a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^1
2) - 48*(a^7*b - 9*a^5*b^3 + 9*a^3*b^5 - a*b^7)*log(tan(d*x + c)^2 + 1)/(a^
12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)
 - (176*a^8*b - 608*a^6*b^3 + 176*a^4*b^5 + 3*(29*a^6*b^3 - 185*a^4*b^5 + 1
03*a^2*b^7 - 3*b^9)*tan(d*x + c)^6 + 3*(71*a^7*b^2 - 411*a^5*b^4 + 165*a^3*
b^6 + 7*a*b^8)*tan(d*x + c)^5 + (149*a^8*b - 512*a^6*b^3 - 1006*a^4*b^5 + 6
00*a^2*b^7 - 15*b^9)*tan(d*x + c)^4 + 3*(5*a^9 + 152*a^7*b^2 - 822*a^5*b^4
+ 320*a^3*b^6 + 9*a*b^8)*tan(d*x + c)^3 + (331*a^8*b - 1183*a^6*b^3 - 239*a
^4*b^5 + 315*a^2*b^7)*tan(d*x + c)^2 + 3*(3*a^9 + 73*a^7*b^2 - 423*a^5*b^4
+ 147*a^3*b^6)*tan(d*x + c))/(a^13 + 5*a^11*b^2 + 10*a^9*b^4 + 10*a^7*b^6 +
 5*a^5*b^8 + a^3*b^10 + (a^10*b^3 + 5*a^8*b^5 + 10*a^6*b^7 + 10*a^4*b^9 + 5
*a^2*b^11 + b^13)*tan(d*x + c)^7 + 3*(a^11*b^2 + 5*a^9*b^4 + 10*a^7*b^6 + 1
0*a^5*b^8 + 5*a^3*b^10 + a*b^12)*tan(d*x + c)^6 + (3*a^12*b + 17*a^10*b^3 +
 40*a^8*b^5 + 50*a^6*b^7 + 35*a^4*b^9 + 13*a^2*b^11 + 2*b^13)*tan(d*x + c)^
5 + (a^13 + 11*a^11*b^2 + 40*a^9*b^4 + 70*a^7*b^6 + 65*a^5*b^8 + 31*a^3*b^1
0 + 6*a*b^12)*tan(d*x + c)^4 + (6*a^12*b + 31*a^10*b^3 + 65*a^8*b^5 + 70*a^
6*b^7 + 40*a^4*b^9 + 11*a^2*b^11 + b^13)*tan(d*x + c)^3 + (2*a^13 + 13*a^11
*b^2 + 35*a^9*b^4 + 50*a^7*b^6 + 40*a^5*b^8 + 17*a^3*b^10 + 3*a*b^12)*tan(d
*x + c)^2 + 3*(a^12*b + 5*a^10*b^3 + 10*a^8*b^5 + 10*a^6*b^7 + 5*a^4*b^9 +
a^2*b^11)*tan(d*x + c)))/d
```

Fricas [B] time = 4.00481, size = 2412, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(6*(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11)*c
os(d*x + c)^7 - 3*(11*a^10*b + 45*a^8*b^3 + 70*a^6*b^5 + 50*a^4*b^7 + 15*a^
2*b^9 + b^11)*cos(d*x + c)^5 - (6*a^10*b + 342*a^8*b^3 - 1830*a^6*b^5 + 614
*a^4*b^7 - 216*a^2*b^9 + 12*b^11 - 3*(3*a^11 - 141*a^9*b^2 + 766*a^7*b^4 -
1242*a^5*b^6 + 399*a^3*b^8 - 9*a*b^10)*d*x)*cos(d*x + c)^3 + 3*(114*a^8*b^3
 - 381*a^6*b^5 + 187*a^4*b^7 - 67*a^2*b^9 + 3*b^11 + 3*(3*a^9*b^2 - 132*a^7
*b^4 + 370*a^5*b^6 - 132*a^3*b^8 + 3*a*b^10)*d*x)*cos(d*x + c) + 48*((a^10*
b - 12*a^8*b^3 + 36*a^6*b^5 - 28*a^4*b^7 + 3*a^2*b^9)*cos(d*x + c)^3 + 3*(a
^8*b^3 - 9*a^6*b^5 + 9*a^4*b^7 - a^2*b^9)*cos(d*x + c) + (a^7*b^4 - 9*a^5*b
^6 + 9*a^3*b^8 - a*b^10 + (3*a^9*b^2 - 28*a^7*b^4 + 36*a^5*b^6 - 12*a^3*b^8
 + a*b^10)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c
) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) + (143*a^7*b^4 - 537*a^5*b^6 + 105*a^
3*b^8 + 33*a*b^10 + 6*(a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*b^6 + 5*a^3*b
^8 + a*b^10)*cos(d*x + c)^6 - 15*(a^11 + 3*a^9*b^2 + 2*a^7*b^4 - 2*a^5*b^6
- 3*a^3*b^8 - a*b^10)*cos(d*x + c)^4 + 3*(3*a^8*b^3 - 132*a^6*b^5 + 370*a^4
*b^7 - 132*a^2*b^9 + 3*b^11)*d*x + (216*a^9*b^2 - 734*a^7*b^4 + 1590*a^5*b^
6 - 522*a^3*b^8 - 54*a*b^10 + 3*(9*a^10*b - 399*a^8*b^3 + 1242*a^6*b^5 - 76
6*a^4*b^7 + 141*a^2*b^9 - 3*b^11)*d*x)*cos(d*x + c)^2)*sin(d*x + c))/((a^15
 + 3*a^13*b^2 - 3*a^11*b^4 - 25*a^9*b^6 - 45*a^7*b^8 - 39*a^5*b^10 - 17*a^3
*b^12 - 3*a*b^14)*d*cos(d*x + c)^3 + 3*(a^13*b^2 + 6*a^11*b^4 + 15*a^9*b^6
+ 20*a^7*b^8 + 15*a^5*b^10 + 6*a^3*b^12 + a*b^14)*d*cos(d*x + c) + ((3*a^14
*b + 17*a^12*b^3 + 39*a^10*b^5 + 45*a^8*b^7 + 25*a^6*b^9 + 3*a^4*b^11 - 3*a
^2*b^13 - b^15)*d*cos(d*x + c)^2 + (a^12*b^3 + 6*a^10*b^5 + 15*a^8*b^7 + 20
*a^6*b^9 + 15*a^4*b^11 + 6*a^2*b^13 + b^15)*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34998, size = 1218, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^8 - 132*a^6*b^2 + 370*a^4*b^4 - 132*a^2*b^6 + 3*b^8)*(d*x + c)
/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b
^12) - 48*(a^7*b - 9*a^5*b^3 + 9*a^3*b^5 - a*b^7)*log(tan(d*x + c)^2 + 1)/(
a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^1
2) + 96*(a^7*b^2 - 9*a^5*b^4 + 9*a^3*b^6 - a*b^8)*log(abs(b*tan(d*x + c) +
a))/(a^12*b + 6*a^10*b^3 + 15*a^8*b^5 + 20*a^6*b^7 + 15*a^4*b^9 + 6*a^2*b^1
1 + b^13) + 3*(24*a^7*b*tan(d*x + c)^4 - 216*a^5*b^3*tan(d*x + c)^4 + 216*a
^3*b^5*tan(d*x + c)^4 - 24*a*b^7*tan(d*x + c)^4 - 5*a^8*tan(d*x + c)^3 + 60
*a^6*b^2*tan(d*x + c)^3 + 10*a^4*b^4*tan(d*x + c)^3 - 52*a^2*b^6*tan(d*x +
c)^3 + 3*b^8*tan(d*x + c)^3 + 16*a^7*b*tan(d*x + c)^2 - 384*a^5*b^3*tan(d*x
 + c)^2 + 496*a^3*b^5*tan(d*x + c)^2 - 64*a*b^7*tan(d*x + c)^2 - 3*a^8*tan(
d*x + c) + 52*a^6*b^2*tan(d*x + c) - 10*a^4*b^4*tan(d*x + c) - 60*a^2*b^6*t
an(d*x + c) + 5*b^8*tan(d*x + c) - 160*a^5*b^3 + 272*a^3*b^5 - 48*a*b^7)/((
a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^1
2)*(tan(d*x + c)^2 + 1)^2) - 8*(22*a^7*b^4*tan(d*x + c)^3 - 198*a^5*b^6*tan
(d*x + c)^3 + 198*a^3*b^8*tan(d*x + c)^3 - 22*a*b^10*tan(d*x + c)^3 + 75*a^
8*b^3*tan(d*x + c)^2 - 630*a^6*b^5*tan(d*x + c)^2 + 567*a^4*b^7*tan(d*x + c
)^2 - 48*a^2*b^9*tan(d*x + c)^2 + 87*a^9*b^2*tan(d*x + c) - 666*a^7*b^4*tan
(d*x + c) + 531*a^5*b^6*tan(d*x + c) - 36*a^3*b^8*tan(d*x + c) + 35*a^10*b
- 231*a^8*b^3 + 165*a^6*b^5 - 9*a^4*b^7)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 +
 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*(b*tan(d*x + c) + a)^3))/d
```

3.74 $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4} dx$

Optimal. Leaf size=264

$$-\frac{a^2b}{3d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^3}-\frac{ab\left(a^2-b^2\right)}{d\left(a^2+b^2\right)^3(a+b\tan(c+dx))^2}-\frac{b\left(-8a^2b^2+3a^4+b^4\right)}{d\left(a^2+b^2\right)^4(a+b\tan(c+dx))}-\frac{\cos^2(c+dx)}{}$$

```
[Out] ((a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*x)/(2*(a^2 + b^2)^5) + (4*a*b*(a^4
 - 5*a^2*b^2 + 2*b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^5*
d) - (a^2*b)/(3*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^3) - (a*b*(a^2 - b^2))
/((a^2 + b^2)^3*d*(a + b*Tan[c + d*x])^2) - (b*(3*a^4 - 8*a^2*b^2 + b^4))/(
(a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^2*(4*a*b*(a^2 - b^2)
+ (a^4 - 6*a^2*b^2 + b^4)*Tan[c + d*x]))/(2*(a^2 + b^2)^4*d)
```

Rubi [A] time = 0.570884, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$-\frac{a^2b}{3d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^3}-\frac{ab\left(a^2-b^2\right)}{d\left(a^2+b^2\right)^3(a+b\tan(c+dx))^2}-\frac{b\left(-8a^2b^2+3a^4+b^4\right)}{d\left(a^2+b^2\right)^4(a+b\tan(c+dx))}-\frac{\cos^2(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```


```
[Out] ((a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*x)/(2*(a^2 + b^2)^5) + (4*a*b*(a^4
 - 5*a^2*b^2 + 2*b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^5*
d) - (a^2*b)/(3*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^3) - (a*b*(a^2 - b^2))
/((a^2 + b^2)^3*d*(a + b*Tan[c + d*x])^2) - (b*(3*a^4 - 8*a^2*b^2 + b^4))/(
(a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^2*(4*a*b*(a^2 - b^2)
+ (a^4 - 6*a^2*b^2 + b^4)*Tan[c + d*x]))/(2*(a^2 + b^2)^4*d)
```


Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^2}{(a+x)^4\left(b^2+x^2\right)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^2(c+dx)\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+b^4\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^4 d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^4b^2\left(a^4-6a^2b^2+b^4\right)}{\left(a^2+b^2\right)^4}+ \cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{\cos^2(c+dx)\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+b^4\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^4 d} - \frac{\text{Subst}\left(\int\left(-\frac{2a^2b^2}{\left(a^2+b^2\right)^2(a+x)^4} + \cdots\right.\right.}{\cdots}$$

$$= \frac{4ab\left(a^4-5a^2b^2+2b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^2b}{3\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^3} - \frac{\cdots}{\left(a^2+\cdots\right.}$$

$$= \frac{4ab\left(a^4-5a^2b^2+2b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^2b}{3\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^3} - \frac{\cdots}{\left(a^2+\cdots\right.}$$

$$= \frac{\left(a^6-25a^4b^2+35a^2b^4-3b^6\right)x}{2\left(a^2+b^2\right)^5} + \frac{4ab\left(a^4-5a^2b^2+2b^4\right)\log(\cos(c+dx))}{\left(a^2+b^2\right)^5 d} + \frac{4ab\left(a^4-5a^2b^{\cdots}\right.}{\cdots}$$

Mathematica [A] time = 3.59427, size = 395, normalized size = 1.5

$$-\frac{b\left(\frac{3\left(-6a^2b^2+a^4+b^4\right)\left(a^2+b^2\right)\sin(2(c+dx))}{2b}+12a(a-b)(a+b)\left(a^2+b^2\right)\cos^2(c+dx)+\frac{3\left(-6a^2b^2+a^4+b^4\right)\left(a^2+b^2\right)\tan^{-1}(\tan(c+dx))}{b}+\frac{2a^2\cdots}{(a+b\,\text{t}\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(b*((3*(a^2 + b^2)*(a^4 - 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]])/b + 12*a*
(a - b)*(a + b)*(a^2 + b^2)*Cos[c + d*x]^2 + 3*(4*a^5 - 20*a^3*b^2 + 8*a*b^
4 + (-a^6 + 15*a^4*b^2 - 15*a^2*b^4 + b^6)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*T
an[c + d*x]] - 24*a*(a^4 - 5*a^2*b^2 + 2*b^4)*Log[a + b*Tan[c + d*x]] + 3*(
4*a^5 - 20*a^3*b^2 + 8*a*b^4 + (a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)/Sqrt[-
b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (3*(a^2 + b^2)*(a^4 - 6*a^2*b^2 +
b^4)*Sin[2*(c + d*x)])/(2*b) + (2*a^2*(a^2 + b^2)^3)/(a + b*Tan[c + d*x])^3
```

```
 + (6*a*(a - b)*(a + b)*(a^2 + b^2)^2)/(a + b*Tan[c + d*x])^2 + (6*(a^2 + b
^2)*(3*a^4 - 8*a^2*b^2 + b^4))/(a + b*Tan[c + d*x])))/(6*(a^2 + b^2)^5*d)
```

Maple [B] time = 0.12, size = 668, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x)
```

```
[Out] -1/2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)*tan(d*x+c)*a^6+5/2/d/(a^2+b^2)^5/(1+tan
(d*x+c)^2)*tan(d*x+c)*a^4*b^2+5/2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)*tan(d*x+c)
*a^2*b^4-1/2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)*tan(d*x+c)*b^6-2/d/(a^2+b^2)^5/
(1+tan(d*x+c)^2)*a^5*b+2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)*a*b^5-2/d/(a^2+b^2)
^5*ln(1+tan(d*x+c)^2)*a^5*b+10/d/(a^2+b^2)^5*ln(1+tan(d*x+c)^2)*a^3*b^3-4/d
/(a^2+b^2)^5*ln(1+tan(d*x+c)^2)*a*b^5-25/2/d/(a^2+b^2)^5*arctan(tan(d*x+c))
*a^4*b^2+35/2/d/(a^2+b^2)^5*arctan(tan(d*x+c))*a^2*b^4-3/2/d/(a^2+b^2)^5*ar
ctan(tan(d*x+c))*b^6+1/2/d/(a^2+b^2)^5*arctan(tan(d*x+c))*a^6-1/3*a^2*b/(a^
2+b^2)^2/d/(a+b*tan(d*x+c))^3-3/d*b/(a^2+b^2)^4/(a+b*tan(d*x+c))*a^4+8/d*b^
3/(a^2+b^2)^4/(a+b*tan(d*x+c))*a^2-1/d*b^5/(a^2+b^2)^4/(a+b*tan(d*x+c))+4/d
*a^5*b/(a^2+b^2)^5*ln(a+b*tan(d*x+c))-20/d*a^3*b^3/(a^2+b^2)^5*ln(a+b*tan(d
*x+c))+8/d*a*b^5/(a^2+b^2)^5*ln(a+b*tan(d*x+c))-1/d*b*a^3/(a^2+b^2)^3/(a+b*
tan(d*x+c))^2+1/d*b^3*a/(a^2+b^2)^3/(a+b*tan(d*x+c))^2
```

Maxima [B] time = 1.71198, size = 894, normalized size = 3.39

$$\frac{\frac{3\left(a^{6}-25\,a^{4}b^{2}+35\,a^{2}b^{4}-3\,b^{6}\right)(dx+c)}{a^{10}+5\,a^{8}b^{2}+10\,a^{6}b^{4}+10\,a^{4}b^{6}+5\,a^{2}b^{8}+b^{10}}+\frac{24\left(a^{5}b-5\,a^{3}b^{3}+2\,ab^{5}\right)\log\left(b\tan\left(dx+c\right)+a\right)}{a^{10}+5\,a^{8}b^{2}+10\,a^{6}b^{4}+10\,a^{4}b^{6}+5\,a^{2}b^{8}+b^{10}}-\frac{12\left(a^{5}b-5\,a^{3}b^{3}+2\,ab^{5}\right)\log\left(\tan\left(dx+c\right)^{2}+1\right)}{a^{10}+5\,a^{8}b^{2}+10\,a^{6}b^{4}+10\,a^{4}b^{6}+5\,a^{2}b^{8}+b^{10}}-\frac{\cdots}{a^{11}+4\,a^{9}b^{2}+6\,a^{7}b^{4}\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*(d*x + c)/(a^10 + 5*a^8*b^2
+ 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 24*(a^5*b - 5*a^3*b^3 + 2*a
*b^5)*log(b*tan(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 +
 5*a^2*b^8 + b^10) - 12*(a^5*b - 5*a^3*b^3 + 2*a*b^5)*log(tan(d*x + c)^2 +
1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - (38*a^
```

```
6*b - 56*a^4*b^3 + 2*a^2*b^5 + 3*(7*a^4*b^3 - 22*a^2*b^5 + 3*b^7)*tan(d*x +
 c)^4 + 3*(17*a^5*b^2 - 46*a^3*b^4 + a*b^6)*tan(d*x + c)^3 + (35*a^6*b - 44
*a^4*b^3 - 73*a^2*b^5 + 6*b^7)*tan(d*x + c)^2 + 3*(a^7 + 20*a^5*b^2 - 43*a^
3*b^4 + 2*a*b^6)*tan(d*x + c))/(a^11 + 4*a^9*b^2 + 6*a^7*b^4 + 4*a^5*b^6 +
a^3*b^8 + (a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*tan(d*x + c)
^5 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*tan(d*x + c)^
4 + (3*a^10*b + 13*a^8*b^3 + 22*a^6*b^5 + 18*a^4*b^7 + 7*a^2*b^9 + b^11)*ta
n(d*x + c)^3 + (a^11 + 7*a^9*b^2 + 18*a^7*b^4 + 22*a^5*b^6 + 13*a^3*b^8 + 3
*a*b^10)*tan(d*x + c)^2 + 3*(a^10*b + 4*a^8*b^3 + 6*a^6*b^5 + 4*a^4*b^7 + a
^2*b^9)*tan(d*x + c)))/d
```

Fricas [B] time = 3.00205, size = 1789, normalized size = 6.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^5 +
(3*a^8*b + 111*a^6*b^3 - 231*a^4*b^5 + 65*a^2*b^7 - 12*b^9 - 3*(a^9 - 28*a^
7*b^2 + 110*a^5*b^4 - 108*a^3*b^6 + 9*a*b^8)*d*x)*cos(d*x + c)^3 - 3*(25*a^
6*b^3 - 51*a^4*b^5 + 25*a^2*b^7 - 3*b^9 + 3*(a^7*b^2 - 25*a^5*b^4 + 35*a^3*
b^6 - 3*a*b^8)*d*x)*cos(d*x + c) - 12*((a^8*b - 8*a^6*b^3 + 17*a^4*b^5 - 6*
a^2*b^7)*cos(d*x + c)^3 + 3*(a^6*b^3 - 5*a^4*b^5 + 2*a^2*b^7)*cos(d*x + c)
+ (a^5*b^4 - 5*a^3*b^6 + 2*a*b^8 + (3*a^7*b^2 - 16*a^5*b^4 + 11*a^3*b^6 - 2
*a*b^8)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (32*a^5*b^4 - 66*a^3*b^6 + 6*a*b^8 - 3
*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cos(d*x + c)^4 + 3*(a^6*
b^3 - 25*a^4*b^5 + 35*a^2*b^7 - 3*b^9)*d*x + (45*a^7*b^2 - 143*a^5*b^4 + 21
9*a^3*b^6 - 9*a*b^8 + 3*(3*a^8*b - 76*a^6*b^3 + 130*a^4*b^5 - 44*a^2*b^7 +
3*b^9)*d*x)*cos(d*x + c)^2)*sin(d*x + c))/((a^13 + 2*a^11*b^2 - 5*a^9*b^4 -
 20*a^7*b^6 - 25*a^5*b^8 - 14*a^3*b^10 - 3*a*b^12)*d*cos(d*x + c)^3 + 3*(a^
11*b^2 + 5*a^9*b^4 + 10*a^7*b^6 + 10*a^5*b^8 + 5*a^3*b^10 + a*b^12)*d*cos(d
*x + c) + ((3*a^12*b + 14*a^10*b^3 + 25*a^8*b^5 + 20*a^6*b^7 + 5*a^4*b^9 -
2*a^2*b^11 - b^13)*d*cos(d*x + c)^2 + (a^10*b^3 + 5*a^8*b^5 + 10*a^6*b^7 +
10*a^4*b^9 + 5*a^2*b^11 + b^13)*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32387, size = 867, normalized size = 3.28

$$\frac{3\left(a^6-25\,a^4b^2+35\,a^2b^4-3\,b^6\right)(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}}-\frac{12\left(a^5b-5\,a^3b^3+2\,ab^5\right)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}}+\frac{24\left(a^5b^2-5\,a^3b^4+2\,ab^6\right)\log(|b\tan(dx+c)+a|)}{a^{10}b+5\,a^8b^3+10\,a^6b^5+10\,a^4b^7+5\,a^2b^9+b^{11}}+\frac{3\left(4\,a^5b\tan(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*(a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*(d*x + c)/(a^10 + 5*a^8*b^2
+ 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - 12*(a^5*b - 5*a^3*b^3 + 2*a
*b^5)*log(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 +
 5*a^2*b^8 + b^10) + 24*(a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*log(abs(b*tan(d*x +
 c) + a))/(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11)
 + 3*(4*a^5*b*tan(d*x + c)^2 - 20*a^3*b^3*tan(d*x + c)^2 + 8*a*b^5*tan(d*x
+ c)^2 - a^6*tan(d*x + c) + 5*a^4*b^2*tan(d*x + c) + 5*a^2*b^4*tan(d*x + c)
 - b^6*tan(d*x + c) - 20*a^3*b^3 + 12*a*b^5)/((a^10 + 5*a^8*b^2 + 10*a^6*b^
4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*(tan(d*x + c)^2 + 1)) - 2*(22*a^5*b^4*ta
n(d*x + c)^3 - 110*a^3*b^6*tan(d*x + c)^3 + 44*a*b^8*tan(d*x + c)^3 + 75*a^
6*b^3*tan(d*x + c)^2 - 345*a^4*b^5*tan(d*x + c)^2 + 111*a^2*b^7*tan(d*x + c
)^2 + 3*b^9*tan(d*x + c)^2 + 87*a^7*b^2*tan(d*x + c) - 357*a^5*b^4*tan(d*x
+ c) + 87*a^3*b^6*tan(d*x + c) + 3*a*b^8*tan(d*x + c) + 35*a^8*b - 119*a^6*
b^3 + 23*a^4*b^5 + a^2*b^7)/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 +
5*a^2*b^8 + b^10)*(b*tan(d*x + c) + a)^3))/d
```

3.75 $\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal. Leaf size=116

$$-\frac{3b}{a^4d(a+b\tan(c+dx))}-\frac{b}{a^3d(a+b\tan(c+dx))^2}-\frac{b}{3a^2d(a+b\tan(c+dx))^3}-\frac{4b\log(\tan(c+dx))}{a^5d}+\frac{4b\log(a+b\tan}{a^5d}$$

```
[Out] -(Cot[c + d*x]/(a^4*d)) - (4*b*Log[Tan[c + d*x]])/(a^5*d) + (4*b*Log[a + b*
Tan[c + d*x]])/(a^5*d) - b/(3*a^2*d*(a + b*Tan[c + d*x])^3) - b/(a^3*d*(a +
 b*Tan[c + d*x])^2) - (3*b)/(a^4*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.0884231, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 44}

$$-\frac{3b}{a^4d(a+b\tan(c+dx))}-\frac{b}{a^3d(a+b\tan(c+dx))^2}-\frac{b}{3a^2d(a+b\tan(c+dx))^3}-\frac{4b\log(\tan(c+dx))}{a^5d}+\frac{4b\log(a+b\tan}{a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```


```
[Out] -(Cot[c + d*x]/(a^4*d)) - (4*b*Log[Tan[c + d*x]])/(a^5*d) + (4*b*Log[a + b*
Tan[c + d*x]])/(a^5*d) - b/(3*a^2*d*(a + b*Tan[c + d*x])^3) - b/(a^3*d*(a +
 b*Tan[c + d*x])^2) - (3*b)/(a^4*d*(a + b*Tan[c + d*x]))
```


Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{b\,\text{Subst}\left(\int \frac{1}{x^2(a+x)^4}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \left(\frac{1}{a^4x^2} - \frac{4}{a^5x} + \frac{1}{a^2(a+x)^4} + \frac{2}{a^3(a+x)^3} + \frac{3}{a^4(a+x)^2} + \frac{4}{a^5(a+x)}\right)dx, x, b\tan(c+dx)\right)}{d}$$
$$= -\frac{\cot(c+dx)}{a^4d} - \frac{4b\log(\tan(c+dx))}{a^5d} + \frac{4b\log(a+b\tan(c+dx))}{a^5d} - \frac{b}{3a^2d(a+b\tan(c+dx)}$$

Mathematica [B] time = 2.06357, size = 259, normalized size = 2.23

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{a^2b^4\tan(c+dx)}{a^2+b^2} - \frac{2a^2b^3\left(3a^2+2b^2\right)(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2} + \frac{b^2\left(23a^2b^2+18a^4+9b^4\right)\tan(c+dx)(a\cos(c+}{\left(a^2+b^2\right)^2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*(-3*a*(b + a*Cot[c + d*x]
)^3*Sin[c + d*x]^2 + (a^2*b^4*Tan[c + d*x])/(a^2 + b^2) + (b^2*(18*a^4 + 23
*a^2*b^2 + 9*b^4)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2*Tan[c + d*x])/(a^2 +
b^2)^2 - (2*a^2*b^3*(3*a^2 + 2*b^2)*(a + b*Tan[c + d*x]))/(a^2 + b^2)^2 - 1
2*b*Cos[c + d*x]^2*Log[Sin[c + d*x]]*(a + b*Tan[c + d*x])^3 + 12*b*Cos[c +
d*x]^2*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]*(a + b*Tan[c + d*x])^3))/(3*a^5
*d*(a + b*Tan[c + d*x])^4)
```

Maple [A] time = 0.131, size = 117, normalized size = 1.

$$-\frac{1}{da^4\tan(dx+c)} - 4\,\frac{b\ln(\tan(dx+c))}{a^5d} - \frac{b}{3\,a^2d\,(a+b\tan(dx+c))^3} + 4\,\frac{b\ln(a+b\tan(dx+c))}{a^5d} - 3\,\frac{b}{da^4\,(a+b\tan(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x)
```

```
[Out] -1/d/a^4/tan(d*x+c)-4*b*ln(tan(d*x+c))/a^5/d-1/3*b/a^2/d/(a+b*tan(d*x+c))^3
+4*b*ln(a+b*tan(d*x+c))/a^5/d-3*b/a^4/d/(a+b*tan(d*x+c))-b/a^3/d/(a+b*tan(d
```

```
*x+c))^2
```

Maxima [A] time = 1.14022, size = 189, normalized size = 1.63

$$-\frac{\frac{12\,b^3\tan(dx+c)^3+30\,ab^2\tan(dx+c)^2+22\,a^2b\tan(dx+c)+3\,a^3}{a^4b^3\tan(dx+c)^4+3\,a^5b^2\tan(dx+c)^3+3\,a^6b\tan(dx+c)^2+a^7\tan(dx+c)}-\frac{12\,b\log(b\tan(dx+c)+a)}{a^5}+\frac{12\,b\log(\tan(dx+c))}{a^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*((12*b^3*tan(d*x + c)^3 + 30*a*b^2*tan(d*x + c)^2 + 22*a^2*b*tan(d*x +
 c) + 3*a^3)/(a^4*b^3*tan(d*x + c)^4 + 3*a^5*b^2*tan(d*x + c)^3 + 3*a^6*b*t
an(d*x + c)^2 + a^7*tan(d*x + c)) - 12*b*log(b*tan(d*x + c) + a)/a^5 + 12*b
*log(tan(d*x + c))/a^5)/d
```

Fricas [B] time = 2.84577, size = 1901, normalized size = 16.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(13*a^6*b^4 + 15*a^4*b^6 + 6*a^2*b^8 - (3*a^10 + 18*a^8*b^2 - 49*a^6*b
^4 - 84*a^4*b^6 - 36*a^2*b^8)*cos(d*x + c)^4 + (9*a^8*b^2 - 71*a^6*b^4 - 10
2*a^4*b^6 - 42*a^2*b^8)*cos(d*x + c)^2 + 6*(a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8
 + b^10 - (3*a^8*b^2 + 8*a^6*b^4 + 6*a^4*b^6 - b^10)*cos(d*x + c)^4 + (3*a^
8*b^2 + 7*a^6*b^4 + 3*a^4*b^6 - 3*a^2*b^8 - 2*b^10)*cos(d*x + c)^2 + ((a^9*
b - 6*a^5*b^5 - 8*a^3*b^7 - 3*a*b^9)*cos(d*x + c)^3 + 3*(a^7*b^3 + 3*a^5*b^
5 + 3*a^3*b^7 + a*b^9)*cos(d*x + c))*sin(d*x + c))*log(2*a*b*cos(d*x + c)*s
in(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 6*(a^6*b^4 + 3*a^4*b^6 +
3*a^2*b^8 + b^10 - (3*a^8*b^2 + 8*a^6*b^4 + 6*a^4*b^6 - b^10)*cos(d*x + c)^
4 + (3*a^8*b^2 + 7*a^6*b^4 + 3*a^4*b^6 - 3*a^2*b^8 - 2*b^10)*cos(d*x + c)^2
 + ((a^9*b - 6*a^5*b^5 - 8*a^3*b^7 - 3*a*b^9)*cos(d*x + c)^3 + 3*(a^7*b^3 +
 3*a^5*b^5 + 3*a^3*b^7 + a*b^9)*cos(d*x + c))*sin(d*x + c))*log(-1/4*cos(d*
x + c)^2 + 1/4) - ((9*a^9*b + 78*a^7*b^3 + 69*a^5*b^5 + 4*a^3*b^7 - 12*a*b^
9)*cos(d*x + c)^3 - 3*(9*a^7*b^3 + 3*a^5*b^5 - 6*a^3*b^7 - 4*a*b^9)*cos(d*x
 + c))*sin(d*x + c))/((3*a^13*b + 8*a^11*b^3 + 6*a^9*b^5 - a^5*b^9)*d*cos(d
```

```
*x + c)^4 - (3*a^13*b + 7*a^11*b^3 + 3*a^9*b^5 - 3*a^7*b^7 - 2*a^5*b^9)*d*c
os(d*x + c)^2 - (a^11*b^3 + 3*a^9*b^5 + 3*a^7*b^7 + a^5*b^9)*d - ((a^14 - 6
*a^10*b^4 - 8*a^8*b^6 - 3*a^6*b^8)*d*cos(d*x + c)^3 + 3*(a^12*b^2 + 3*a^10*
b^4 + 3*a^8*b^6 + a^6*b^8)*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2252, size = 174, normalized size = 1.5

$$\frac{\frac{12\,b\log(|b\tan(dx+c)+a|)}{a^5}-\frac{12\,b\log(|\tan(dx+c)|)}{a^5}+\frac{3\,(4\,b\tan(dx+c)-a)}{a^5\tan(dx+c)}-\frac{22\,b^4\tan(dx+c)^3+75\,ab^3\tan(dx+c)^2+87\,a^2b^2\tan(dx+c)+35\,a^3b}{(b\tan(dx+c)+a)^3a^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(12*b*log(abs(b*tan(d*x + c) + a))/a^5 - 12*b*log(abs(tan(d*x + c)))/a^
5 + 3*(4*b*tan(d*x + c) - a)/(a^5*tan(d*x + c)) - (22*b^4*tan(d*x + c)^3 +
75*a*b^3*tan(d*x + c)^2 + 87*a^2*b^2*tan(d*x + c) + 35*a^3*b)/((b*tan(d*x +
 c) + a)^3*a^5))/d
```

3.76 $\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal. Leaf size=205

$$-\frac{b\left(3a^2+10b^2\right)}{a^6d(a+b\tan(c+dx))}-\frac{b\left(a^2+2b^2\right)}{a^5d(a+b\tan(c+dx))^2}-\frac{b\left(a^2+b^2\right)}{3a^4d(a+b\tan(c+dx))^3}-\frac{\left(a^2+10b^2\right)\cot(c+dx)}{a^6d}-\frac{4b\left(a^2+5b^2\right)}{}$$

```
[Out] -(((a^2 + 10*b^2)*Cot[c + d*x])/(a^6*d)) + (2*b*Cot[c + d*x]^2)/(a^5*d) - C
ot[c + d*x]^3/(3*a^4*d) - (4*b*(a^2 + 5*b^2)*Log[Tan[c + d*x]])/(a^7*d) + (
4*b*(a^2 + 5*b^2)*Log[a + b*Tan[c + d*x]])/(a^7*d) - (b*(a^2 + b^2))/(3*a^4
*d*(a + b*Tan[c + d*x])^3) - (b*(a^2 + 2*b^2))/(a^5*d*(a + b*Tan[c + d*x])^
2) - (b*(3*a^2 + 10*b^2))/(a^6*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.173254, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 894}

$$-\frac{b\left(3a^2+10b^2\right)}{a^6d(a+b\tan(c+dx))}-\frac{b\left(a^2+2b^2\right)}{a^5d(a+b\tan(c+dx))^2}-\frac{b\left(a^2+b^2\right)}{3a^4d(a+b\tan(c+dx))^3}-\frac{\left(a^2+10b^2\right)\cot(c+dx)}{a^6d}-\frac{4b\left(a^2+5b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(((a^2 + 10*b^2)*Cot[c + d*x])/(a^6*d)) + (2*b*Cot[c + d*x]^2)/(a^5*d) - C
ot[c + d*x]^3/(3*a^4*d) - (4*b*(a^2 + 5*b^2)*Log[Tan[c + d*x]])/(a^7*d) + (
4*b*(a^2 + 5*b^2)*Log[a + b*Tan[c + d*x]])/(a^7*d) - (b*(a^2 + b^2))/(3*a^4
*d*(a + b*Tan[c + d*x])^3) - (b*(a^2 + 2*b^2))/(a^5*d*(a + b*Tan[c + d*x])^
2) - (b*(3*a^2 + 10*b^2))/(a^6*d*(a + b*Tan[c + d*x]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
```

```
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{b\,\text{Subst}\left(\int \frac{b^2+x^2}{x^4(a+x)^4}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\,\text{Subst}\left(\int \left(\frac{b^2}{a^4x^4} - \frac{4b^2}{a^5x^3} + \frac{a^2+10b^2}{a^6x^2} - \frac{4(a^2+5b^2)}{a^7x} + \frac{a^2+b^2}{a^4(a+x)^4} + \frac{2(a^2+2b^2)}{a^5(a+x)^3} + \frac{3a^2+10b^2}{a^6(a+x)^2} + \frac{4(a^2+5b^2)}{a^7(a+x)}\right) d\right.}{d}$$

$$= -\frac{(a^2+10b^2)\cot(c+dx)}{a^6d} + \frac{2b\cot^2(c+dx)}{a^5d} - \frac{\cot^3(c+dx)}{3a^4d} - \frac{4b(a^2+5b^2)\log(\tan(c+d}{a^7d}$$

Mathematica [B] time = 2.03439, size = 528, normalized size = 2.58

$$\frac{\sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(-192b\left(a^2+5b^2\right)\log(\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3+192b\left(a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])*(-192*b*(a^2 + 5*b^2)*Log
[Sin[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3 + 192*b*(a^2 + 5*b^2)*Lo
g[a*Cos[c + d*x] + b*Sin[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3 - (C
sc[c + d*x]^3*(8*a^8 - 4*a^6*b^2 - 50*a^4*b^4 - 190*a^2*b^6 - 150*b^8 + 3*(
3*a^8 + 10*a^6*b^2 + 45*a^4*b^4 + 115*a^2*b^6 + 75*b^8)*Cos[2*(c + d*x)] +
6*(2*a^6*b^2 - 17*a^4*b^4 - 35*a^2*b^6 - 15*b^8)*Cos[4*(c + d*x)] - a^8*Cos
[6*(c + d*x)] - 22*a^6*b^2*Cos[6*(c + d*x)] + 17*a^4*b^4*Cos[6*(c + d*x)] +
 55*a^2*b^6*Cos[6*(c + d*x)] + 15*b^8*Cos[6*(c + d*x)] - 3*a^7*b*Sin[2*(c +
 d*x)] + 3*a^5*b^3*Sin[2*(c + d*x)] - 75*a^3*b^5*Sin[2*(c + d*x)] - 75*a*b^
7*Sin[2*(c + d*x)] - 6*a^7*b*Sin[4*(c + d*x)] + 84*a^5*b^3*Sin[4*(c + d*x)]
 + 156*a^3*b^5*Sin[4*(c + d*x)] + 60*a*b^7*Sin[4*(c + d*x)] - 3*a^7*b*Sin[6
*(c + d*x)] - 65*a^5*b^3*Sin[6*(c + d*x)] - 79*a^3*b^5*Sin[6*(c + d*x)] - 1
5*a*b^7*Sin[6*(c + d*x)]))/(a^2 + b^2)))/(48*a^7*d*(a + b*Tan[c + d*x])^4)
```

Maple [A] time = 0.198, size = 278, normalized size = 1.4

$$-\frac{1}{3\,da^{4}\left(\tan\left(dx+c\right)\right)^{3}}-\frac{1}{da^{4}\tan\left(dx+c\right)}-10\,\frac{b^{2}}{da^{6}\tan\left(dx+c\right)}+2\,\frac{b}{da^{5}\left(\tan\left(dx+c\right)\right)^{2}}-4\,\frac{b\ln\left(\tan\left(dx+c\right)\right)}{da^{5}}-20\,\frac{b^{3}\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x)
```

```
[Out] -1/3/d/a^4/tan(d*x+c)^3-1/d/a^4/tan(d*x+c)-10/d/a^6/tan(d*x+c)*b^2+2/d/a^5*
b/tan(d*x+c)^2-4*b*ln(tan(d*x+c))/a^5/d-20/d*b^3/a^7*ln(tan(d*x+c))-3*b/a^4
/d/(a+b*tan(d*x+c))-10/d*b^3/a^6/(a+b*tan(d*x+c))-1/3*b/a^2/d/(a+b*tan(d*x+
c))^3-1/3/d*b^3/a^4/(a+b*tan(d*x+c))^3-b/a^3/d/(a+b*tan(d*x+c))^2-2/d*b^3/a
^5/(a+b*tan(d*x+c))^2+4*b*ln(a+b*tan(d*x+c))/a^5/d+20/d*b^3/a^7*ln(a+b*tan(
d*x+c))
```

Maxima [A] time = 1.12349, size = 308, normalized size = 1.5

$$\frac{\frac{3\,a^{4}b\tan(dx+c)-12\left(a^{2}b^{3}+5\,b^{5}\right)\tan(dx+c)^{5}-a^{5}-30\left(a^{3}b^{2}+5\,ab^{4}\right)\tan(dx+c)^{4}-22\left(a^{4}b+5\,a^{2}b^{3}\right)\tan(dx+c)^{3}-3\left(a^{5}+5\,a^{3}b^{2}\right)\tan(dx+c)^{2}}{a^{6}b^{3}\tan(dx+c)^{6}+3\,a^{7}b^{2}\tan(dx+c)^{5}+3\,a^{8}b\tan(dx+c)^{4}+a^{9}\tan(dx+c)^{3}}+\frac{12\left(a^{2}b+5\,b^{3}\right)\log(b\tan}{a^{7}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*((3*a^4*b*tan(d*x + c) - 12*(a^2*b^3 + 5*b^5)*tan(d*x + c)^5 - a^5 - 30
*(a^3*b^2 + 5*a*b^4)*tan(d*x + c)^4 - 22*(a^4*b + 5*a^2*b^3)*tan(d*x + c)^3
 - 3*(a^5 + 5*a^3*b^2)*tan(d*x + c)^2)/(a^6*b^3*tan(d*x + c)^6 + 3*a^7*b^2*
tan(d*x + c)^5 + 3*a^8*b*tan(d*x + c)^4 + a^9*tan(d*x + c)^3) + 12*(a^2*b +
 5*b^3)*log(b*tan(d*x + c) + a)/a^7 - 12*(a^2*b + 5*b^3)*log(tan(d*x + c))/
a^7)/d
```

Fricas [B] time = 3.25469, size = 2732, normalized size = 13.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(19*a^6*b^4 + 51*a^4*b^6 + 30*a^2*b^8 + 2*(a^10 + 23*a^8*b^2 - 22*a^6*b
^4 - 138*a^4*b^6 - 90*a^2*b^8)*cos(d*x + c)^6 - 3*(a^10 + 25*a^8*b^2 - 46*a
^6*b^4 - 206*a^4*b^6 - 130*a^2*b^8)*cos(d*x + c)^4 + 3*(9*a^8*b^2 - 38*a^6*
b^4 - 131*a^4*b^6 - 80*a^2*b^8)*cos(d*x + c)^2 + 6*(a^6*b^4 + 7*a^4*b^6 + 1
1*a^2*b^8 + 5*b^10 + (3*a^8*b^2 + 20*a^6*b^4 + 26*a^4*b^6 + 4*a^2*b^8 - 5*b
^10)*cos(d*x + c)^6 - 3*(2*a^8*b^2 + 13*a^6*b^4 + 15*a^4*b^6 - a^2*b^8 - 5*
b^10)*cos(d*x + c)^4 + 3*(a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 - 6*a^2*b^8 - 5*b
^10)*cos(d*x + c)^2 - ((a^9*b + 4*a^7*b^3 - 10*a^5*b^5 - 28*a^3*b^7 - 15*a*
b^9)*cos(d*x + c)^5 - (a^9*b + a^7*b^3 - 31*a^5*b^5 - 61*a^3*b^7 - 30*a*b^9
)*cos(d*x + c)^3 - 3*(a^7*b^3 + 7*a^5*b^5 + 11*a^3*b^7 + 5*a*b^9)*cos(d*x +
 c))*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*
x + c)^2 + b^2) - 6*(a^6*b^4 + 7*a^4*b^6 + 11*a^2*b^8 + 5*b^10 + (3*a^8*b^2
 + 20*a^6*b^4 + 26*a^4*b^6 + 4*a^2*b^8 - 5*b^10)*cos(d*x + c)^6 - 3*(2*a^8*
b^2 + 13*a^6*b^4 + 15*a^4*b^6 - a^2*b^8 - 5*b^10)*cos(d*x + c)^4 + 3*(a^8*b
^2 + 6*a^6*b^4 + 4*a^4*b^6 - 6*a^2*b^8 - 5*b^10)*cos(d*x + c)^2 - ((a^9*b +
 4*a^7*b^3 - 10*a^5*b^5 - 28*a^3*b^7 - 15*a*b^9)*cos(d*x + c)^5 - (a^9*b +
a^7*b^3 - 31*a^5*b^5 - 61*a^3*b^7 - 30*a*b^9)*cos(d*x + c)^3 - 3*(a^7*b^3 +
 7*a^5*b^5 + 11*a^3*b^7 + 5*a*b^9)*cos(d*x + c))*sin(d*x + c))*log(-1/4*cos
(d*x + c)^2 + 1/4) + (2*(3*a^9*b + 77*a^7*b^3 + 142*a^5*b^5 + 34*a^3*b^7 -
30*a*b^9)*cos(d*x + c)^5 - (3*a^9*b + 193*a^7*b^3 + 350*a^5*b^5 + 26*a^3*b^
7 - 120*a*b^9)*cos(d*x + c)^3 + 3*(15*a^7*b^3 + 23*a^5*b^5 - 14*a^3*b^7 - 2
0*a*b^9)*cos(d*x + c))*sin(d*x + c))/((3*a^13*b + 5*a^11*b^3 + a^9*b^5 - a^
7*b^7)*d*cos(d*x + c)^6 - 3*(2*a^13*b + 3*a^11*b^3 - a^7*b^7)*d*cos(d*x + c
)^4 + 3*(a^13*b + a^11*b^3 - a^9*b^5 - a^7*b^7)*d*cos(d*x + c)^2 + (a^11*b^
3 + 2*a^9*b^5 + a^7*b^7)*d - ((a^14 - a^12*b^2 - 5*a^10*b^4 - 3*a^8*b^6)*d*
cos(d*x + c)^5 - (a^14 - 4*a^12*b^2 - 11*a^10*b^4 - 6*a^8*b^6)*d*cos(d*x +
c)^3 - 3*(a^12*b^2 + 2*a^10*b^4 + a^8*b^6)*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33433, size = 300, normalized size = 1.46

$$-\frac{\frac{12\left(a^2b+5\,b^3\right)\log(|\tan(dx+c)|)}{a^7}-\frac{12\left(a^2b^2+5\,b^4\right)\log(|b\tan(dx+c)+a|)}{a^7b}+\frac{12\,a^2b^3\tan(dx+c)^5+60\,b^5\tan(dx+c)^5+30\,a^3b^2\tan(dx+c)^4+150\,ab^4\tan(dx+c)^4+22}{\left(b\tan(d\right.}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(12*(a^2*b + 5*b^3)*log(abs(tan(d*x + c)))/a^7 - 12*(a^2*b^2 + 5*b^4)*
log(abs(b*tan(d*x + c) + a))/(a^7*b) + (12*a^2*b^3*tan(d*x + c)^5 + 60*b^5*
tan(d*x + c)^5 + 30*a^3*b^2*tan(d*x + c)^4 + 150*a*b^4*tan(d*x + c)^4 + 22*
a^4*b*tan(d*x + c)^3 + 110*a^2*b^3*tan(d*x + c)^3 + 3*a^5*tan(d*x + c)^2 +
15*a^3*b^2*tan(d*x + c)^2 - 3*a^4*b*tan(d*x + c) + a^5)/((b*tan(d*x + c)^2
+ a*tan(d*x + c))^3*a^6))/d
```

3.77 $\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal. Leaf size=300

$$-\frac{b\left(20a^2b^2+3a^4+21b^4\right)}{a^8d(a+b\tan(c+dx))}-\frac{b\left(a^2+b^2\right)\left(a^2+3b^2\right)}{a^7d(a+b\tan(c+dx))^2}-\frac{b\left(a^2+b^2\right)^2}{3a^6d(a+b\tan(c+dx))^3}-\frac{2\left(a^2+5b^2\right)\cot^3(c+dx)}{3a^6d}+\frac{2b\left(2a^2\right.}{}$$

```
[Out] -(((a^4 + 20*a^2*b^2 + 35*b^4)*Cot[c + d*x])/(a^8*d)) + (2*b*(2*a^2 + 5*b^2
)*Cot[c + d*x]^2)/(a^7*d) - (2*(a^2 + 5*b^2)*Cot[c + d*x]^3)/(3*a^6*d) + (b
*Cot[c + d*x]^4)/(a^5*d) - Cot[c + d*x]^5/(5*a^4*d) - (4*b*(a^4 + 10*a^2*b^
2 + 14*b^4)*Log[Tan[c + d*x]])/(a^9*d) + (4*b*(a^4 + 10*a^2*b^2 + 14*b^4)*L
og[a + b*Tan[c + d*x]])/(a^9*d) - (b*(a^2 + b^2)^2)/(3*a^6*d*(a + b*Tan[c +
 d*x])^3) - (b*(a^2 + b^2)*(a^2 + 3*b^2))/(a^7*d*(a + b*Tan[c + d*x])^2) -
(b*(3*a^4 + 20*a^2*b^2 + 21*b^4))/(a^8*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.271243, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 894}

$$-\frac{b\left(20a^2b^2+3a^4+21b^4\right)}{a^8d(a+b\tan(c+dx))}-\frac{b\left(a^2+b^2\right)\left(a^2+3b^2\right)}{a^7d(a+b\tan(c+dx))^2}-\frac{b\left(a^2+b^2\right)^2}{3a^6d(a+b\tan(c+dx))^3}-\frac{2\left(a^2+5b^2\right)\cot^3(c+dx)}{3a^6d}+\frac{2b\left(2a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^4,x]
```


```
[Out] -(((a^4 + 20*a^2*b^2 + 35*b^4)*Cot[c + d*x])/(a^8*d)) + (2*b*(2*a^2 + 5*b^2
)*Cot[c + d*x]^2)/(a^7*d) - (2*(a^2 + 5*b^2)*Cot[c + d*x]^3)/(3*a^6*d) + (b
*Cot[c + d*x]^4)/(a^5*d) - Cot[c + d*x]^5/(5*a^4*d) - (4*b*(a^4 + 10*a^2*b^
2 + 14*b^4)*Log[Tan[c + d*x]])/(a^9*d) + (4*b*(a^4 + 10*a^2*b^2 + 14*b^4)*L
og[a + b*Tan[c + d*x]])/(a^9*d) - (b*(a^2 + b^2)^2)/(3*a^6*d*(a + b*Tan[c +
 d*x])^3) - (b*(a^2 + b^2)*(a^2 + 3*b^2))/(a^7*d*(a + b*Tan[c + d*x])^2) -
(b*(3*a^4 + 20*a^2*b^2 + 21*b^4))/(a^8*d*(a + b*Tan[c + d*x]))
```


Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{\left(b^2+x^2\right)^2}{x^6(a+x)^4}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\,\mathrm{Subst}\left(\int \left(\frac{b^4}{a^4x^6} - \frac{4b^4}{a^5x^5} + \frac{2b^2\left(a^2+5b^2\right)}{a^6x^4} - \frac{4\left(2a^2b^2+5b^4\right)}{a^7x^3} + \frac{a^4+20a^2b^2+35b^4}{a^8x^2} - \frac{4\left(a^4+10a^2b^2+14b^4\right)}{a^9x} + \frac{\left(a^2+b\right.}{a^6(a+}\right.\right.}{d}$$

$$= -\frac{\left(a^4+20a^2b^2+35b^4\right)\cot(c+dx)}{a^8d} + \frac{2b\left(2a^2+5b^2\right)\cot^2(c+dx)}{a^7d} - \frac{2\left(a^2+5b^2\right)\cot^3(c+d}{3a^6d}$$

Mathematica [B] time = 1.63369, size = 673, normalized size = 2.24

$$\frac{\sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(-7680b\left(10a^2b^2+a^4+14b^4\right)\log(\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])*(-7680*b*(a^4 + 10*a^2*b^
2 + 14*b^4)*Log[Sin[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3 + 7680*b*
(a^4 + 10*a^2*b^2 + 14*b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]*(a*Cos[c +
 d*x] + b*Sin[c + d*x])^3 + Csc[c + d*x]^5*(-200*a^8 + 380*a^6*b^2 + 3070*a
^4*b^4 + 11375*a^2*b^6 + 11025*b^8 - 4*(52*a^8 + 194*a^6*b^2 + 1510*a^4*b^4
 + 5705*a^2*b^6 + 4410*b^8)*Cos[2*(c + d*x)] + 4*(4*a^8 - 16*a^6*b^2 + 1010
*a^4*b^4 + 4585*a^2*b^6 + 2205*b^8)*Cos[4*(c + d*x)] + 16*a^8*Cos[6*(c + d*
x)] + 776*a^6*b^2*Cos[6*(c + d*x)] - 1000*a^4*b^4*Cos[6*(c + d*x)] - 8540*a
^2*b^6*Cos[6*(c + d*x)] - 2520*b^8*Cos[6*(c + d*x)] - 8*a^8*Cos[8*(c + d*x)
] - 316*a^6*b^2*Cos[8*(c + d*x)] - 70*a^4*b^4*Cos[8*(c + d*x)] + 1645*a^2*b
^6*Cos[8*(c + d*x)] + 315*b^8*Cos[8*(c + d*x)] + 264*a^7*b*Sin[2*(c + d*x)]
 + 372*a^5*b^3*Sin[2*(c + d*x)] + 4830*a^3*b^5*Sin[2*(c + d*x)] + 1470*a*b^
```

```
7*Sin[2*(c + d*x)] + 144*a^7*b*Sin[4*(c + d*x)] - 2476*a^5*b^3*Sin[4*(c + d
*x)] - 9730*a^3*b^5*Sin[4*(c + d*x)] - 1470*a*b^7*Sin[4*(c + d*x)] - 24*a^7
*b*Sin[6*(c + d*x)] + 2756*a^5*b^3*Sin[6*(c + d*x)] + 7670*a^3*b^5*Sin[6*(c
 + d*x)] + 630*a*b^7*Sin[6*(c + d*x)] - 24*a^7*b*Sin[8*(c + d*x)] - 922*a^5
*b^3*Sin[8*(c + d*x)] - 2095*a^3*b^5*Sin[8*(c + d*x)] - 105*a*b^7*Sin[8*(c
+ d*x)])))/(1920*a^9*d*(a + b*Tan[c + d*x])^4)
```

Maple [A] time = 0.165, size = 476, normalized size = 1.6

$$-\frac{1}{5\,da^4\,(\tan(dx+c))^5}-\frac{2}{3\,da^4\,(\tan(dx+c))^3}-\frac{10\,b^2}{3\,da^6\,(\tan(dx+c))^3}-\frac{1}{da^4\tan(dx+c)}-20\,\frac{b^2}{da^6\tan(dx+c)}-35\,\frac{}{da^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x)
```

```
[Out] -1/5/d/a^4/tan(d*x+c)^5-2/3/d/a^4/tan(d*x+c)^3-10/3/d/a^6/tan(d*x+c)^3*b^2-
1/d/a^4/tan(d*x+c)-20/d/a^6/tan(d*x+c)*b^2-35/d/a^8/tan(d*x+c)*b^4+1/d/a^5*
b/tan(d*x+c)^4+4/d/a^5*b/tan(d*x+c)^2+10/d*b^3/a^7/tan(d*x+c)^2-4*b*ln(tan(
d*x+c))/a^5/d-40/d*b^3/a^7*ln(tan(d*x+c))-56/d*b^5/a^9*ln(tan(d*x+c))-3*b/a
^4/d/(a+b*tan(d*x+c))-20/d*b^3/a^6/(a+b*tan(d*x+c))-21/d*b^5/a^8/(a+b*tan(d
*x+c))-1/3*b/a^2/d/(a+b*tan(d*x+c))^3-2/3/d*b^3/a^4/(a+b*tan(d*x+c))^3-1/3/
d*b^5/a^6/(a+b*tan(d*x+c))^3-b/a^3/d/(a+b*tan(d*x+c))^2-4/d*b^3/a^5/(a+b*ta
n(d*x+c))^2-3/d*b^5/a^7/(a+b*tan(d*x+c))^2+4*b*ln(a+b*tan(d*x+c))/a^5/d+40/
d*b^3/a^7*ln(a+b*tan(d*x+c))+56/d*b^5/a^9*ln(a+b*tan(d*x+c))
```

Maxima [A] time = 1.23988, size = 439, normalized size = 1.46

$$\frac{\frac{6\,a^6 b\tan(dx+c)-60\left(a^4b^3+10\,a^2b^5+14\,b^7\right)\tan(dx+c)^7-3\,a^7-150\left(a^5b^2+10\,a^3b^4+14\,ab^6\right)\tan(dx+c)^6-110\left(a^6b+10\,a^4b^3+14\,a^2b^5\right)\tan(dx+c)^5-15\left(a^7+10\,a^5b^2+\right.}{a^8b^3\tan(dx+c)^8+3\,a^9b^2\tan(dx+c)^7+3\,a^{10}b\tan(dx+c)^6+a^{11}\tan(dx+c)^5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/15*((6*a^6*b*tan(d*x + c) - 60*(a^4*b^3 + 10*a^2*b^5 + 14*b^7)*tan(d*x +
c)^7 - 3*a^7 - 150*(a^5*b^2 + 10*a^3*b^4 + 14*a*b^6)*tan(d*x + c)^6 - 110*(
a^6*b + 10*a^4*b^3 + 14*a^2*b^5)*tan(d*x + c)^5 - 15*(a^7 + 10*a^5*b^2 + 14
*a^3*b^4)*tan(d*x + c)^4 + 6*(5*a^6*b + 7*a^4*b^3)*tan(d*x + c)^3 - 2*(5*a^
```

```
7 + 7*a^5*b^2)*tan(d*x + c)^2)/(a^8*b^3*tan(d*x + c)^8 + 3*a^9*b^2*tan(d*x
+ c)^7 + 3*a^10*b*tan(d*x + c)^6 + a^11*tan(d*x + c)^5) + 60*(a^4*b + 10*a^
2*b^3 + 14*b^5)*log(b*tan(d*x + c) + a)/a^9 - 60*(a^4*b + 10*a^2*b^3 + 14*b
^5)*log(tan(d*x + c))/a^9)/d
```

Fricas [B] time = 4.43096, size = 3507, normalized size = 11.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/15*(110*a^6*b^4 + 510*a^4*b^6 + 420*a^2*b^8 - 4*(2*a^10 + 81*a^8*b^2 + 2
9*a^6*b^4 - 660*a^4*b^6 - 630*a^2*b^8)*cos(d*x + c)^8 + 2*(10*a^10 + 423*a^
8*b^2 - 47*a^6*b^4 - 4320*a^4*b^6 - 3990*a^2*b^8)*cos(d*x + c)^6 - 15*(a^10
 + 47*a^8*b^2 - 44*a^6*b^4 - 658*a^4*b^6 - 588*a^2*b^8)*cos(d*x + c)^4 + 20
*(9*a^8*b^2 - 28*a^6*b^4 - 219*a^4*b^6 - 189*a^2*b^8)*cos(d*x + c)^2 + 30*(
a^6*b^4 + 11*a^4*b^6 + 24*a^2*b^8 + 14*b^10 - (3*a^8*b^2 + 32*a^6*b^4 + 61*
a^4*b^6 + 18*a^2*b^8 - 14*b^10)*cos(d*x + c)^8 + (9*a^8*b^2 + 95*a^6*b^4 +
172*a^4*b^6 + 30*a^2*b^8 - 56*b^10)*cos(d*x + c)^6 - 3*(3*a^8*b^2 + 31*a^6*
b^4 + 50*a^4*b^6 - 6*a^2*b^8 - 28*b^10)*cos(d*x + c)^4 + (3*a^8*b^2 + 29*a^
6*b^4 + 28*a^4*b^6 - 54*a^2*b^8 - 56*b^10)*cos(d*x + c)^2 + ((a^9*b + 8*a^7
*b^3 - 9*a^5*b^5 - 58*a^3*b^7 - 42*a*b^9)*cos(d*x + c)^7 - (2*a^9*b + 13*a^
7*b^3 - 51*a^5*b^5 - 188*a^3*b^7 - 126*a*b^9)*cos(d*x + c)^5 + (a^9*b + 2*a
^7*b^3 - 75*a^5*b^5 - 202*a^3*b^7 - 126*a*b^9)*cos(d*x + c)^3 + 3*(a^7*b^3
+ 11*a^5*b^5 + 24*a^3*b^7 + 14*a*b^9)*cos(d*x + c))*sin(d*x + c))*log(2*a*b
*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 30*(a^6*b^
4 + 11*a^4*b^6 + 24*a^2*b^8 + 14*b^10 - (3*a^8*b^2 + 32*a^6*b^4 + 61*a^4*b^
6 + 18*a^2*b^8 - 14*b^10)*cos(d*x + c)^8 + (9*a^8*b^2 + 95*a^6*b^4 + 172*a^
4*b^6 + 30*a^2*b^8 - 56*b^10)*cos(d*x + c)^6 - 3*(3*a^8*b^2 + 31*a^6*b^4 +
50*a^4*b^6 - 6*a^2*b^8 - 28*b^10)*cos(d*x + c)^4 + (3*a^8*b^2 + 29*a^6*b^4
+ 28*a^4*b^6 - 54*a^2*b^8 - 56*b^10)*cos(d*x + c)^2 + ((a^9*b + 8*a^7*b^3 -
 9*a^5*b^5 - 58*a^3*b^7 - 42*a*b^9)*cos(d*x + c)^7 - (2*a^9*b + 13*a^7*b^3
- 51*a^5*b^5 - 188*a^3*b^7 - 126*a*b^9)*cos(d*x + c)^5 + (a^9*b + 2*a^7*b^3
 - 75*a^5*b^5 - 202*a^3*b^7 - 126*a*b^9)*cos(d*x + c)^3 + 3*(a^7*b^3 + 11*a
^5*b^5 + 24*a^3*b^7 + 14*a*b^9)*cos(d*x + c))*sin(d*x + c))*log(-1/4*cos(d*
x + c)^2 + 1/4) - 2*(2*(6*a^9*b + 259*a^7*b^3 + 783*a^5*b^5 + 340*a^3*b^7 -
 210*a*b^9)*cos(d*x + c)^7 - (15*a^9*b + 1141*a^7*b^3 + 3546*a^5*b^5 + 1270
*a^3*b^7 - 1260*a*b^9)*cos(d*x + c)^5 + 5*(151*a^7*b^3 + 483*a^5*b^5 + 100*
a^3*b^7 - 252*a*b^9)*cos(d*x + c)^3 - 15*(9*a^7*b^3 + 29*a^5*b^5 - 6*a^3*b^
7 - 28*a*b^9)*cos(d*x + c))*sin(d*x + c))/((3*a^13*b + 2*a^11*b^3 - a^9*b^5
```

```
)*d*cos(d*x + c)^8 - (9*a^13*b + 5*a^11*b^3 - 4*a^9*b^5)*d*cos(d*x + c)^6 +
 3*(3*a^13*b + a^11*b^3 - 2*a^9*b^5)*d*cos(d*x + c)^4 - (3*a^13*b - a^11*b^
3 - 4*a^9*b^5)*d*cos(d*x + c)^2 - (a^11*b^3 + a^9*b^5)*d - ((a^14 - 2*a^12*
b^2 - 3*a^10*b^4)*d*cos(d*x + c)^7 - (2*a^14 - 7*a^12*b^2 - 9*a^10*b^4)*d*c
os(d*x + c)^5 + (a^14 - 8*a^12*b^2 - 9*a^10*b^4)*d*cos(d*x + c)^3 + 3*(a^12
*b^2 + a^10*b^4)*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36148, size = 578, normalized size = 1.93

$$-\frac{\frac{60\left(a^4b+10\,a^2b^3+14\,b^5\right)\log(|\tan(dx+c)|)}{a^9}-\frac{60\left(a^4b^2+10\,a^2b^4+14\,b^6\right)\log(|b\tan(dx+c)+a|)}{a^9b}+\frac{5\left(22\,a^4b^4\tan(dx+c)^3+220\,a^2b^6\tan(dx+c)^3+308\,b^8\tan(dx+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/15*(60*(a^4*b + 10*a^2*b^3 + 14*b^5)*log(abs(tan(d*x + c)))/a^9 - 60*(a^
4*b^2 + 10*a^2*b^4 + 14*b^6)*log(abs(b*tan(d*x + c) + a))/(a^9*b) + 5*(22*a
^4*b^4*tan(d*x + c)^3 + 220*a^2*b^6*tan(d*x + c)^3 + 308*b^8*tan(d*x + c)^3
 + 75*a^5*b^3*tan(d*x + c)^2 + 720*a^3*b^5*tan(d*x + c)^2 + 987*a*b^7*tan(d
*x + c)^2 + 87*a^6*b^2*tan(d*x + c) + 792*a^4*b^4*tan(d*x + c) + 1059*a^2*b
^6*tan(d*x + c) + 35*a^7*b + 294*a^5*b^3 + 381*a^3*b^5)/((b*tan(d*x + c) +
a)^3*a^9) - (137*a^4*b*tan(d*x + c)^5 + 1370*a^2*b^3*tan(d*x + c)^5 + 1918*
b^5*tan(d*x + c)^5 - 15*a^5*tan(d*x + c)^4 - 300*a^3*b^2*tan(d*x + c)^4 - 5
25*a*b^4*tan(d*x + c)^4 + 60*a^4*b*tan(d*x + c)^3 + 150*a^2*b^3*tan(d*x + c
)^3 - 10*a^5*tan(d*x + c)^2 - 50*a^3*b^2*tan(d*x + c)^2 + 15*a^4*b*tan(d*x
+ c) - 3*a^5)/(a^9*tan(d*x + c)^5))/d
```

3.78 $\int \frac{\csc(x)}{1+\tan(x)}\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}-\tanh^{-1}(\cos(x))$$

```
[Out] -ArcTanh[Cos[x]] + ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2]
```

Rubi [A] time = 0.0744782, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3518, 3110, 3770, 3074, 206}

$$\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(1 + Tan[x]),x]
```

```
[Out] -ArcTanh[Cos[x]] + ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(x)}{1+\tan(x)}\,dx &= \int \frac{\cot(x)}{\cos(x)+\sin(x)}\,dx \\
&= \int \left(\csc(x)+\frac{1}{-\cos(x)-\sin(x)}\right)dx \\
&= \int \csc(x)\,dx + \int \frac{1}{-\cos(x)-\sin(x)}\,dx \\
&= -\tanh^{-1}(\cos(x)) - \text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, -\cos(x)+\sin(x)\right) \\
&= -\tanh^{-1}(\cos(x)) - \frac{\tanh^{-1}\left(\frac{-\cos(x)+\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.0413579, size = 41, normalized size = 1.58

$$
\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)+(1+i)(-1)^{3/4}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(1 + Tan[x]),x]
```

```
[Out] (1 + I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] - Log[Cos[x/2]] + Log[S
in[x/2]]
```

Maple [A] time = 0.025, size = 26, normalized size = 1.

$$-\sqrt{2}\,\mathrm{Artanh}\left(\frac{\sqrt{2}}{4}\left(2\tan\left(x/2\right)-2\right)\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(1+tan(x)),x)
```

```
[Out] -2^(1/2)*arctanh(1/4*(2*tan(1/2*x)-2)*2^(1/2))+ln(tan(1/2*x))
```

Maxima [B] time = 1.85027, size = 68, normalized size = 2.62

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(x)}{\cos(x)+1}+1}{\sqrt{2}+\frac{\sin(x)}{\cos(x)+1}-1}\right)+\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(1+tan(x)),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - sin(x)/(cos(x) + 1) + 1)/(sqrt(2) + sin(x)/(cos
(x) + 1) - 1)) + log(sin(x)/(cos(x) + 1))
```

Fricas [B] time = 2.35363, size = 203, normalized size = 7.81

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}+\cos\left(x\right)\right)\sin\left(x\right)-2\sqrt{2}\cos\left(x\right)-3}{2\cos\left(x\right)\sin\left(x\right)+1}\right)-\frac{1}{2}\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+\frac{1}{2}\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(1+tan(x)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((2*(sqrt(2) + cos(x))*sin(x) - 2*sqrt(2)*cos(x) - 3)/(2*cos
(x)*sin(x) + 1)) - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(x)}{\tan(x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(1+tan(x)),x)
```

```
[Out] Integral(csc(x)/(tan(x) + 1), x)
```

Giac [A] time = 1.29647, size = 59, normalized size = 2.27

$$\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2} + 2\tan\left(\frac{1}{2}x\right) - 2\right|}{\left|2\sqrt{2} + 2\tan\left(\frac{1}{2}x\right) - 2\right|}\right) + \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(1+tan(x)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(1/2*x) - 2)/abs(2*sqrt(2) + 2*tan(1/
2*x) - 2)) + log(abs(tan(1/2*x)))
```

3.79 $\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=229

$$\frac{3a^2b\sin^{m+2}(c+dx)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(c+dx)\right)}{d(m+2)}+\frac{a^3\cos(c+dx)\sin^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{3a\ldots}{\ldots}$$

```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (3*a^2*b*Hyper
geometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))
/(d*(2 + m)) + (3*a*b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)
/2, (5 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(3 + m))/(d*(3 + m
)) + (b^3*Hypergeometric2F1[2, (4 + m)/2, (6 + m)/2, Sin[c + d*x]^2]*Sin[c
+ d*x]^(4 + m))/(d*(4 + m))
```

Rubi [A] time = 0.449863, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4401, 2643, 2564, 364, 2577}

$$\frac{3a^2b\sin^{m+2}(c+dx)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(c+dx)\right)}{d(m+2)}+\frac{a^3\cos(c+dx)\sin^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{3a\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^3,x]
```


```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (3*a^2*b*Hyper
geometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))
/(d*(2 + m)) + (3*a*b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)
/2, (5 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(3 + m))/(d*(3 + m
)) + (b^3*Hypergeometric2F1[2, (4 + m)/2, (6 + m)/2, Sin[c + d*x]^2]*Sin[c
+ d*x]^(4 + m))/(d*(4 + m))
```


Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a^3\sin^m(c+dx) + 3a^2b\sec(c+dx)\sin^{1+m}(c+dx) + 3ab^2\sec^2(c+dx)\sin^2\right.$$

$$= a^3\int \sin^m(c+dx)\,dx + \left(3a^2b\right)\int \sec(c+dx)\sin^{1+m}(c+dx)\,dx + \left(3ab^2\right)\int \mathrm{se}$$

$$= \frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}} + \frac{3ab^2\sqrt{\cos^2(c}}{}$$

$$= \frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}} + \frac{3a^2b\,{}_2F_1\left(1,\frac{2}{}\right.}{}$$

Mathematica [A] time = 2.51971, size = 205, normalized size = 0.9

$$\frac{\sin^{m+1}(c+dx)\left(b\sin(c+dx)\left(\frac{3a^2\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(c+dx)\right)}{m+2}+b\left(\frac{3a\sqrt{\cos^2(c+dx)}\tan(c+dx)\,{}_2F_1\left(\frac{3}{2},\frac{m+3}{2};\frac{m+5}{2};\sin^2(c+dx)\right)}{m+3}+\frac{b\sin^2(c+dx)\,{}_2F_1\left(2\right.}{}\right.\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sin[c + d*x]^(1 + m)*((a^3*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1
+ m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x])/(1 + m) + b*Sin[c + d*x]*(
(3*a^2*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2])/(2 + m)
+ b*((b*Hypergeometric2F1[2, (4 + m)/2, (6 + m)/2, Sin[c + d*x]^2]*Sin[c +
d*x]^2)/(4 + m) + (3*a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/
2, (5 + m)/2, Sin[c + d*x]^2]*Tan[c + d*x])/(3 + m)))))/d
```

Maple [F] time = 0.392, size = 0, normalized size = 0.

$$\int (\sin(dx+c))^m (a+b\tan(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x)
```

```
[Out] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)^3\sin(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^3*sin(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3\tan\left(dx+c\right)^3+3\,ab^2\tan\left(dx+c\right)^2+3\,a^2b\tan\left(dx+c\right)+a^3\right)\sin\left(dx+c\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*tan(d*x + c)^3 + 3*a*b^2*tan(d*x + c)^2 + 3*a^2*b*tan(d*x + c
) + a^3)*sin(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**m*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(dx+c\right)+a\right)^3\sin\left(dx+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^3*sin(d*x + c)^m, x)
```

3.80 $\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=179

$$\frac{a^2\cos(c+dx)\sin^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{2ab\sin^{m+2}(c+dx)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(c+dx)\right)}{d(m+2)}+\frac{b^2\sqrt{}}{}$$

```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (2*a*b*Hyperge
ometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(
d*(2 + m)) + (b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5
 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(3 + m))/(d*(3 + m))
```

Rubi [A] time = 0.268018, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4401, 2643, 2564, 364, 2577}

$$\frac{a^2\cos(c+dx)\sin^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{2ab\sin^{m+2}(c+dx)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(c+dx)\right)}{d(m+2)}+\frac{b^2\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^2,x]
```


```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (2*a*b*Hyperge
ometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(
d*(2 + m)) + (b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5
 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(3 + m))/(d*(3 + m))
```


Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a^2\sin^m(c+dx)+2ab\sec(c+dx)\sin^{1+m}(c+dx)+b^2\sec^2(c+dx)\sin^{2+m}(\right.$$

$$= a^2\int \sin^m(c+dx)\,dx+(2ab)\int \sec(c+dx)\sin^{1+m}(c+dx)\,dx+b^2\int \sec^2(c+$$

$$= \frac{a^2\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}}+\frac{b^2\sqrt{\cos^2(c+}}{}$$

$$= \frac{a^2\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}}+\frac{2ab\,{}_2F_1\left(1,\frac{2+}{2}\right.}{}$$

Mathematica [A] time = 1.19385, size = 166, normalized size = 0.93

$$\frac{\sin^{m+1}(c+dx)\left(\frac{a^2\sqrt{\cos^2(c+dx)}\sec(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{m+1}+\frac{b\sin(c+dx)\left(2a(m+3)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(c+dx)\right)+b(m+2)\sqrt{\cos^2(c+dx)}\tan\right.}{(m+2)(m+3)}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]^(1 + m)*((a^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1
+ m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x])/(1 + m) + (b*Sin[c + d*x]*
(2*a*(3 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2] + b
*(2 + m)*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)/2,
Sin[c + d*x]^2]*Tan[c + d*x]))/((2 + m)*(3 + m))))/d
```

Maple [F] time = 0.248, size = 0, normalized size = 0.

$$\int (\sin (dx+c))^{m} (a+b\tan (dx+c))^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x)
```

```
[Out] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan (dx+c)+a)^{2} \sin (dx+c)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^2*sin(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^{2} \tan (dx+c)^{2}+2\,ab\tan (dx+c)+a^{2}\right) \sin (dx+c)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)*sin(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**m*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)^2 \sin(dx+c)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^2*sin(d*x + c)^m, x)
```

3.81 $\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=109

$$\frac{a\cos(c+dx)\sin^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{b\sin^{m+2}(c+dx)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(c+dx)\right)}{d(m+2)}$$

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2
]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (b*Hypergeometri
c2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(d*(2 +
 m))
```

Rubi [A] time = 0.14461, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {4401, 2643, 2564, 364}

$$\frac{a\cos(c+dx)\sin^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{b\sin^{m+2}(c+dx)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(c+dx)\right)}{d(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2
]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (b*Hypergeometri
c2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(d*(2 +
 m))
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx &= \int \left(a\sin^m(c+dx)+b\sec(c+dx)\sin^{1+m}(c+dx)\right)\,dx\\ &= a\int \sin^m(c+dx)\,dx+b\int \sec(c+dx)\sin^{1+m}(c+dx)\,dx\\ &= \frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}}+\frac{b\,\text{Subst}\left(\int \frac{x^{1+m}}{1-x^2}\right.}{}\\ &= \frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}}+\frac{b\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+}{2}\right.}{}\end{aligned}$$

Mathematica [A] time = 0.257188, size = 109, normalized size = 1.

$$\frac{a\sqrt{\cos^2(c+dx)}\sec(c+dx)\sin^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)}+\frac{b\sin^{m+2}(c+dx)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(c+\right.}{d(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c
+ d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + m))/(d*(1 + m)) + (b*Hypergeometri
c2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(d*(2 +
 m))
```

Maple [F] time = 0.488, size = 0, normalized size = 0.

$$\int (\sin (dx+c))^{m} (a+b\tan (dx+c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^m*(a+b*tan(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)^m*(a+b*tan(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan (dx+c)+a)\sin (dx+c)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)*sin(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\tan (dx+c)+a)\sin (dx+c)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)*sin(d*x + c)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\tan (c+dx))\sin^{m} (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**m*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)\sin(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)*sin(d*x + c)^m, x)
```

3.82 $\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=765

result too large to display

```
[Out] (2^(1 + m)*Hypergeometric2F1[(1 + m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/2]
^2]*Tan[(c + d*x)/2]*(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m*(1 + Tan
[(c + d*x)/2]^2)^m)/(a*d*(1 + m)) + (2^(1 + m)*b*AppellF1[(2 + m)/2, 1 + m,
 1, (4 + m)/2, -Tan[(c + d*x)/2]^2, (a^2*Tan[(c + d*x)/2]^2)/(b - Sqrt[a^2
+ b^2])^2]*Tan[(c + d*x)/2]^2*(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m
*(1 + Tan[(c + d*x)/2]^2)^m)/(Sqrt[a^2 + b^2]*(b - Sqrt[a^2 + b^2])*d*(2 +
m)) - (2^(1 + m)*b*AppellF1[(2 + m)/2, 1 + m, 1, (4 + m)/2, -Tan[(c + d*x)/
2]^2, (a^2*Tan[(c + d*x)/2]^2)/(b + Sqrt[a^2 + b^2])^2]*Tan[(c + d*x)/2]^2*
(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m*(1 + Tan[(c + d*x)/2]^2)^m)/(
Sqrt[a^2 + b^2]*(b + Sqrt[a^2 + b^2])*d*(2 + m)) + (2^(1 + m)*a*b*AppellF1[
(3 + m)/2, 1 + m, 1, (5 + m)/2, -Tan[(c + d*x)/2]^2, (a^2*Tan[(c + d*x)/2]^
2)/(b - Sqrt[a^2 + b^2])^2]*Tan[(c + d*x)/2]^3*(Tan[(c + d*x)/2]/(1 + Tan[(
c + d*x)/2]^2))^m*(1 + Tan[(c + d*x)/2]^2)^m)/(Sqrt[a^2 + b^2]*(b - Sqrt[a^
2 + b^2])^2*d*(3 + m)) - (2^(1 + m)*a*b*AppellF1[(3 + m)/2, 1 + m, 1, (5 +
m)/2, -Tan[(c + d*x)/2]^2, (a^2*Tan[(c + d*x)/2]^2)/(b + Sqrt[a^2 + b^2])^2
]*Tan[(c + d*x)/2]^3*(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m*(1 + Tan
[(c + d*x)/2]^2)^m)/(Sqrt[a^2 + b^2]*(b + Sqrt[a^2 + b^2])^2*d*(3 + m))
```

Rubi [A] time = 4.17316, antiderivative size = 765, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {12, 6719, 6728, 364, 959, 510}

$$\frac{ab2^{m+1}\tan^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m F_1\left(\frac{m+3}{2};m+1,1;\frac{m+5}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{a^2\tan^2\left(\frac{1}{2}(c\right.}{\left(b-\sqrt{a^2+b\right.}}\right.}{d(m+3)\sqrt{a^2+b^2}\left(b-\sqrt{a^2+b^2}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^m/(a + b*Tan[c + d*x]),x]
```

```
[Out] (2^(1 + m)*Hypergeometric2F1[(1 + m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/2]
^2]*Tan[(c + d*x)/2]*(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m*(1 + Tan
[(c + d*x)/2]^2)^m)/(a*d*(1 + m)) + (2^(1 + m)*b*AppellF1[(2 + m)/2, 1 + m,
```

```
 1, (4 + m)/2, -Tan[(c + d*x)/2]^2, (a^2*Tan[(c + d*x)/2]^2)/(b - Sqrt[a^2
+ b^2])^2]*Tan[(c + d*x)/2]^2*(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m
*(1 + Tan[(c + d*x)/2]^2)^m)/(Sqrt[a^2 + b^2]*(b - Sqrt[a^2 + b^2])*d*(2 +
m)) - (2^(1 + m)*b*AppellF1[(2 + m)/2, 1 + m, 1, (4 + m)/2, -Tan[(c + d*x)/
2]^2, (a^2*Tan[(c + d*x)/2]^2)/(b + Sqrt[a^2 + b^2])^2]*Tan[(c + d*x)/2]^2*
(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m*(1 + Tan[(c + d*x)/2]^2)^m)/(
Sqrt[a^2 + b^2]*(b + Sqrt[a^2 + b^2])*d*(2 + m)) + (2^(1 + m)*a*b*AppellF1[
(3 + m)/2, 1 + m, 1, (5 + m)/2, -Tan[(c + d*x)/2]^2, (a^2*Tan[(c + d*x)/2]^
2)/(b - Sqrt[a^2 + b^2])^2]*Tan[(c + d*x)/2]^3*(Tan[(c + d*x)/2]/(1 + Tan[(
c + d*x)/2]^2))^m*(1 + Tan[(c + d*x)/2]^2)^m)/(Sqrt[a^2 + b^2]*(b - Sqrt[a^
2 + b^2])^2*d*(3 + m)) - (2^(1 + m)*a*b*AppellF1[(3 + m)/2, 1 + m, 1, (5 +
m)/2, -Tan[(c + d*x)/2]^2, (a^2*Tan[(c + d*x)/2]^2)/(b + Sqrt[a^2 + b^2])^2
]*Tan[(c + d*x)/2]^3*(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m*(1 + Tan
[(c + d*x)/2]^2)^m)/(Sqrt[a^2 + b^2]*(b + Sqrt[a^2 + b^2])^2*d*(3 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
```

```
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{2^m(1-x^2)\left(\frac{x}{1+x^2}\right)^{1+m}}{x(a+2bx-ax^2)}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$= \frac{2^{1+m}\,\mathrm{Subst}\left(\int \frac{(1-x^2)\left(\frac{x}{1+x^2}\right)^{1+m}}{x(a+2bx-ax^2)}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$= \frac{\left(2^{1+m}\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m\right)\mathrm{Subst}\left(\int \frac{x^m(1-x^2)(1+x^2)}{a+2bx-ax^2}\right.}{d}$$

$$= \frac{\left(2^{1+m}\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m\right)\mathrm{Subst}\left(\int\left(\frac{x^m(1+x^2)^{-1-m}}{a}\right.\right.}{d}$$

$$= \frac{\left(2^{1+m}\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m\right)\mathrm{Subst}\left(\int x^m\left(1+x^2\right)^{-1}\right.}{ad}$$

$$= \frac{2^{1+m}\,{}_2F_1\left(\frac{1+m}{2},1+m;\frac{3+m}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\right.}{ad(1+m)}$$

$$= \frac{2^{1+m}\,{}_2F_1\left(\frac{1+m}{2},1+m;\frac{3+m}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\right.}{ad(1+m)}$$

$$= \frac{2^{1+m}\,{}_2F_1\left(\frac{1+m}{2},1+m;\frac{3+m}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\right.}{ad(1+m)}$$

$$= \frac{2^{1+m}\,{}_2F_1\left(\frac{1+m}{2},1+m;\frac{3+m}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\right.}{ad(1+m)}$$

Mathematica [F] time = 12.7916, size = 0, normalized size = 0.

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^m/(a + b*Tan[c + d*x]),x]
```

```
[Out] Integrate[Sin[c + d*x]^m/(a + b*Tan[c + d*x]), x]
```

Maple [F] time = 0.164, size = 0, normalized size = 0.

$$\int \frac{(\sin(dx+c))^m}{a+b\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^m/(a+b*tan(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)^m/(a+b*tan(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^m}{b\tan(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^m/(b*tan(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sin(dx+c)^m}{b\tan(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)^m/(b*tan(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^m (c + dx)}{a + b \tan (c + dx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**m/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**m/(a + b*tan(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin (dx + c)^m}{b \tan (dx + c) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^m/(b*tan(d*x + c) + a), x)
```

3.83 $\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=23

$$\text{CannotIntegrate}\left(\sin^m(c+dx)(a+b\tan(c+dx))^n,x\right)$$

```
[Out] CannotIntegrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n, x]
```

Rubi [A] time = 2.46307, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] Defer[Int][Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n, x]
```

Rubi steps

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx=\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx$$

Mathematica [A] time = 3.18417, size = 0, normalized size = 0.

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n, x]
```

Maple [A] time = 0.644, size = 0, normalized size = 0.

$$\int (\sin (dx + c))^{m} (a + b \tan (dx + c))^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b \tan (dx + c) + a)^{n} \sin (dx + c)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \tan (dx + c) + a)^{n} \sin (dx + c)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**m*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.84 $\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=435

$$-\frac{\left(\sqrt{-b^2}\left(a^2b^2\left(-n^2+6n+6\right)+3a^4+b^4\left(n^2+4n+3\right)\right)+ab^2n\left(5a^2+b^2(2n+3)\right)\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1\right.}{16bd(n+1)\left(a^2+b^2\right)^2\left(a-\sqrt{-b^2}\right)}$$

```
[Out] -((a*b^2*n*(5*a^2 + b^2*(3 + 2*n)) + Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 + 6*n -
 n^2) + b^4*(3 + 4*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan
[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(16*b*(a^2 + b^2
)^2*(a - Sqrt[-b^2])*d*(1 + n)) - ((a*b^2*n*(5*a^2 + b^2*(3 + 2*n)) - Sqrt[
-b^2]*(3*a^4 + a^2*b^2*(6 + 6*n - n^2) + b^4*(3 + 4*n + n^2)))*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c
+ d*x])^(1 + n))/(16*b*(a^2 + b^2)^2*(a + Sqrt[-b^2])*d*(1 + n)) + (Cos[c +
 d*x]^4*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/(4*(a^2 + b^2)*d
) - (Cos[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n)*(b*(a^2*(7 - n) + b^2*(5 +
 n)) + a*(5*a^2 + b^2*(3 + 2*n))*Tan[c + d*x]))/(8*(a^2 + b^2)^2*d)
```

Rubi [A] time = 0.803032, antiderivative size = 435, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3516, 1649, 831, 68}

$$-\frac{\left(\sqrt{-b^2}\left(a^2b^2\left(-n^2+6n+6\right)+3a^4+b^4\left(n^2+4n+3\right)\right)+ab^2n\left(5a^2+b^2(2n+3)\right)\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1\right.}{16bd(n+1)\left(a^2+b^2\right)^2\left(a-\sqrt{-b^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] -((a*b^2*n*(5*a^2 + b^2*(3 + 2*n)) + Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 + 6*n -
 n^2) + b^4*(3 + 4*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan
[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(16*b*(a^2 + b^2
)^2*(a - Sqrt[-b^2])*d*(1 + n)) - ((a*b^2*n*(5*a^2 + b^2*(3 + 2*n)) - Sqrt[
-b^2]*(3*a^4 + a^2*b^2*(6 + 6*n - n^2) + b^4*(3 + 4*n + n^2)))*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c
+ d*x])^(1 + n))/(16*b*(a^2 + b^2)^2*(a + Sqrt[-b^2])*d*(1 + n)) + (Cos[c +
 d*x]^4*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/(4*(a^2 + b^2)*d
) - (Cos[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n)*(b*(a^2*(7 - n) + b^2*(5 +
 n)) + a*(5*a^2 + b^2*(3 + 2*n))*Tan[c + d*x]))/(8*(a^2 + b^2)^2*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^(p + 1)*(a*(e*f - d*g) +
(c*d*f + a*e*g)*x))/(2*a*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*(p + 1)
*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p +
 1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3))
 + e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, c, d, e, m},
 x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[m, 0]
 && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{x^4(a+x)^n}{\left(b^2+x^2\right)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} - \frac{\mathrm{Subst}\left(\int \frac{(a+x)^n\left(b^4\left(a^2+\right.\right.}{}\right.}{}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} - \frac{\cos^2(c+dx)(a+b\,\mathrm{ta}}{}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} - \frac{\cos^2(c+dx)(a+b\,\mathrm{ta}}{}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} - \frac{\cos^2(c+dx)(a+b\,\mathrm{ta}}{}$$

$$= -\frac{\left(ab^2n\left(5a^2+b^2(3+2n)\right)+\sqrt{-b^2}\left(3a^4+a^2b^2\left(6+6n-n^2\right)+b^4\left(3+4n+n\right.\right.\right.}{16b\left(a^2+b^2\right)^2\left(a-\sqrt{-b^2}\right.}$$

Mathematica [B] time = 6.5794, size = 910, normalized size = 2.09

$$b\left(\frac{{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{n+1}}{2\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)(n+1)} - \frac{{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{n+1}}{2\sqrt{-b^2}\left(a+\sqrt{-b^2}\right)(n+1)} + \frac{\cos^4(c+dx)\left(b^2+a\tan(c+dx)b\right)(a+b\tan(c+dx\right.}{4b^2\left(a^2+b^2\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*((Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2
])]*(a + b*Tan[c + d*x])^(1 + n))/(2*Sqrt[-b^2]*(a - Sqrt[-b^2])*(1 + n)) -
```

```
 (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]
*(a + b*Tan[c + d*x])^(1 + n))/(2*Sqrt[-b^2]*(a + Sqrt[-b^2])*(1 + n)) - (C
os[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n)*(b^2 + a*b*Tan[c + d*x]))/(b^2*(
a^2 + b^2)) + (Cos[c + d*x]^4*(a + b*Tan[c + d*x])^(1 + n)*(b^2 + a*b*Tan[c
 + d*x]))/(4*b^2*(a^2 + b^2)) + (((Sqrt[-b^2]*(a^2 + b^2*(1 - n)) - a*b^2*n
)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]
*(a + b*Tan[c + d*x])^(1 + n))/(b^2*(a - Sqrt[-b^2])*(1 + n)) - ((a^2*Sqrt[
-b^2] - (-b^2)^(3/2)*(1 - n) + a*b^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n,
(a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(b^2*(
a + Sqrt[-b^2])*(1 + n)))/(2*(a^2 + b^2)) - (b^2*((Cos[c + d*x]^2*(a + b*Ta
n[c + d*x])^(1 + n)*(b^2*(-3*a^2 - b^2*(3 - n)) + a^2*b^2*(2 - n) + b*(a*(-
3*a^2 - b^2*(3 - n)) - a*b^2*(2 - n))*Tan[c + d*x]))/(2*b^4*(a^2 + b^2)) -
(((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n - Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 - 2*n -
 n^2) + b^4*(3 - 4*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan
[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(2*b^2*(a - Sqrt
[-b^2])*(1 + n)) + ((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n + Sqrt[-b^2]*(3*a^4 +
a^2*b^2*(6 - 2*n - n^2) + b^4*(3 - 4*n + n^2)))*Hypergeometric2F1[1, 1 + n,
 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n)
)/(2*b^2*(a + Sqrt[-b^2])*(1 + n)))/(2*b^2*(a^2 + b^2))))/(4*(a^2 + b^2))))
/d
```

Maple [F] time = 0.556, size = 0, normalized size = 0.

$$\int (\sin(dx + c))^4 (a + b\tan(dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx + c) + a)^n \sin(dx + c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\left(b\tan\left(dx+c\right)+a\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(b*tan(d*x + c) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(dx+c\right)+a\right)^{n}\sin\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

3.85 $\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=276

$$-\frac{\left(\sqrt{-b^2}\left(a^2+b^2(n+1)\right)+ab^2n\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{4bd(n+1)\left(a^2+b^2\right)\left(a-\sqrt{-b^2}\right)}-\frac{\left(ab^2n-\sqrt{-b^2}\left(a^2+b^2(n+1\right.\right.}{4}$$

```
[Out] -((a*b^2*n + Sqrt[-b^2]*(a^2 + b^2*(1 + n)))*Hypergeometric2F1[1, 1 + n, 2
+ n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(
4*b*(a^2 + b^2)*(a - Sqrt[-b^2])*d*(1 + n)) - ((a*b^2*n - Sqrt[-b^2]*(a^2 +
 b^2*(1 + n)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a +
 Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(4*b*(a^2 + b^2)*(a + Sqrt[-b^2
])*d*(1 + n)) - (Cos[c + d*x]^2*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])^(
1 + n))/(2*(a^2 + b^2)*d)
```

Rubi [A] time = 0.367371, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3516, 1649, 831, 68}

$$-\frac{\left(\sqrt{-b^2}\left(a^2+b^2(n+1)\right)+ab^2n\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{4bd(n+1)\left(a^2+b^2\right)\left(a-\sqrt{-b^2}\right)}-\frac{\left(ab^2n-\sqrt{-b^2}\left(a^2+b^2(n+1\right.\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] -((a*b^2*n + Sqrt[-b^2]*(a^2 + b^2*(1 + n)))*Hypergeometric2F1[1, 1 + n, 2
+ n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(
4*b*(a^2 + b^2)*(a - Sqrt[-b^2])*d*(1 + n)) - ((a*b^2*n - Sqrt[-b^2]*(a^2 +
 b^2*(1 + n)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a +
 Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(4*b*(a^2 + b^2)*(a + Sqrt[-b^2
])*d*(1 + n)) - (Cos[c + d*x]^2*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])^(
1 + n))/(2*(a^2 + b^2)*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^(p + 1)*(a*(e*f - d*g) +
(c*d*f + a*e*g)*x))/(2*a*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*(p + 1)
*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p +
 1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3))
 + e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, c, d, e, m},
 x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[m, 0]
 && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^2(a+x)^n}{(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2(a^2+b^2)d} - \frac{\text{Subst}\left(\int \frac{(a+x)^n\left(-b^2\left(a^2\right.\right.}{b}\right.}{}$$

$$= -\frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2(a^2+b^2)d} - \frac{\text{Subst}\left(\int\left(\frac{\left(ab^4n-b^2\sqrt{-b}\right.}{2b}\right.\right.}{}$$

$$= -\frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2(a^2+b^2)d} - \frac{\left(ab^2n-\sqrt{-b^2}\left(a^2+b\right.\right.}{}$$

$$= -\frac{\left(ab^2n+\sqrt{-b^2}\left(a^2+b^2(1+n)\right)\right){}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c}{4b(a^2+b^2)\left(a-\sqrt{-b^2}\right)d(1+n)}$$

Mathematica [A] time = 1.08754, size = 270, normalized size = 0.98

$$\frac{(a+b\tan(c+dx))^{n+1}\left(\left(a^2b^2(n-1)+a^3\sqrt{-b^2}-a\left(-b^2\right)^{3/2}(2n+1)+b^4(-(n+1))\right){}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)-\right.}{4bd(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (((a^3*Sqrt[-b^2] + a^2*b^2*(-1 + n) - b^4*(1 + n) - a*(-b^2)^(3/2)*(1 + 2*
n))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]
)] - (a^3*Sqrt[-b^2] - a^2*b^2*(-1 + n) + b^4*(1 + n) - a*(-b^2)^(3/2)*(1 +
 2*n))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b
^2])] + 2*b*(a^2 + b^2)*(1 + n)*Cos[c + d*x]*(b*Cos[c + d*x] + a*Sin[c + d*
x]))*(a + b*Tan[c + d*x])^(1 + n))/(4*b*(a^2 + b^2)*(-a + Sqrt[-b^2])*(a +
Sqrt[-b^2])*d*(1 + n))
```

Maple [F] time = 0.473, size = 0, normalized size = 0.

$$\int (\sin(dx+c))^2 (a+b\tan(dx+c))^n \; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)^n \sin(dx+c)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos(dx+c)^2-1\right)(b\tan(dx+c)+a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*tan(d*x + c) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)^n\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

3.86 $\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=48

$$\frac{b(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{b\tan(c+dx)}{a}+1\right)}{a^2d(n+1)}$$

```
[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Tan[c
+ d*x])^(1 + n))/(a^2*d*(1 + n))
```

Rubi [A] time = 0.0548995, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 65}

$$\frac{b(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{b\tan(c+dx)}{a}+1\right)}{a^2d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```


```
[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Tan[c
+ d*x])^(1 + n))/(a^2*d*(1 + n))
```


Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^n}{x^2}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{b\,{}_2F_1\left(2,1+n;2+n;1+\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^{1+n}}{a^2d(1+n)}$$

Mathematica [A] time = 0.875393, size = 48, normalized size = 1.

$$\frac{b(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{b\tan(c+dx)}{a}+1\right)}{a^2d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Tan[c
+ d*x])^(1 + n))/(a^2*d*(1 + n))
```

Maple [F] time = 0.239, size = 0, normalized size = 0.

$$\int (\csc(dx+c))^2 (a+b\tan(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)^n \csc(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\tan(dx+c)+a)^n\csc(dx+c)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)^n\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

3.87 $\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=140

$$\frac{b\left(6a^2+b^2\left(n^2-3n+2\right)\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{b\tan(c+dx)}{a}+1\right)}{6a^4d(n+1)}+\frac{b(2-n)\cot^2(c+dx)(a+b\tan(c+dx)}{6a^2d}$$

```
[Out] (b*(2 - n)*Cot[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n))/(6*a^2*d) - (Cot[c
+ d*x]^3*(a + b*Tan[c + d*x])^(1 + n))/(3*a*d) + (b*(6*a^2 + b^2*(2 - 3*n +
 n^2))*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Ta
n[c + d*x])^(1 + n))/(6*a^4*d*(1 + n))
```

Rubi [A] time = 0.12994, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3516, 950, 78, 65}

$$\frac{b\left(6a^2+b^2\left(n^2-3n+2\right)\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{b\tan(c+dx)}{a}+1\right)}{6a^4d(n+1)}+\frac{b(2-n)\cot^2(c+dx)(a+b\tan(c+dx)}{6a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*(2 - n)*Cot[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n))/(6*a^2*d) - (Cot[c
+ d*x]^3*(a + b*Tan[c + d*x])^(1 + n))/(3*a*d) + (b*(6*a^2 + b^2*(2 - 3*n +
 n^2))*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Ta
n[c + d*x])^(1 + n))/(6*a^4*d*(1 + n))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 950

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + c*x^2)^p, d + e*x
, x], R = PolynomialRemainder[(a + c*x^2)^p, d + e*x, x]}, Simp[(R*(d + e*x
)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)), x] + Dist[1/((m + 1)*(e
*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*ExpandToSum[(m + 1)*(e*f - d*
```

```
g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^n\left(b^2+x^2\right)}{x^4}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cot^3(c+dx)(a+b\tan(c+dx))^{1+n}}{3ad} - \frac{b\,\text{Subst}\left(\int \frac{(a+x)^n\left(b^2(2-n)-3ax\right)}{x^3}\,dx, x, b\,\text{ta}\right.}{3ad}$$

$$= \frac{b(2-n)\cot^2(c+dx)(a+b\tan(c+dx))^{1+n}}{6a^2d} - \frac{\cot^3(c+dx)(a+b\tan(c+dx))}{3ad}$$

$$= \frac{b(2-n)\cot^2(c+dx)(a+b\tan(c+dx))^{1+n}}{6a^2d} - \frac{\cot^3(c+dx)(a+b\tan(c+dx))}{3ad}$$

Mathematica [A] time = 1.24389, size = 78, normalized size = 0.56

$$\frac{b(a+b\tan(c+dx))^{n+1}\left(a^2\,{}_2F_1\left(2,n+1;n+2;\frac{b\tan(c+dx)}{a}+1\right)+b^2\,{}_2F_1\left(4,n+1;n+2;\frac{b\tan(c+dx)}{a}+1\right)\right)}{a^4d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*(a^2*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a] + b^2*Hy
pergeometric2F1[4, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a])*(a + b*Tan[c + d*
x])^(1 + n))/(a^4*d*(1 + n))
```

Maple [F] time = 0.229, size = 0, normalized size = 0.

$$\int (\csc (dx+c))^4 (a+b\tan (dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan (dx+c)+a)^n \csc (dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\tan (dx+c)+a)^n \csc (dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)^n \csc(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

3.88 $\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=23

$$\text{CannotIntegrate}\left(\sin^3(c+dx)(a+b\tan(c+dx))^n,x\right)$$

```
[Out] CannotIntegrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]
```

Rubi [A] time = 1.97873, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] Defer[Int][Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]
```

Rubi steps

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx=\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

Mathematica [A] time = 3.06851, size = 0, normalized size = 0.

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]
```

Maple [A] time = 0.542, size = 0, normalized size = 0.

$$\int (\sin (dx+c))^{3} (a+b\tan (dx+c))^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\tan (dx+c)+a)^{n} \sin (dx+c)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos (dx+c)^{2}-1\right)(b\tan (dx+c)+a)^{n} \sin (dx+c)\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*tan(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)^n \sin(dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

3.89 $\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=21

$$\text{CannotIntegrate}\left(\sin(c+dx)(a+b\tan(c+dx))^n,x\right)$$

```
[Out] CannotIntegrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^n, x]
```

Rubi [A] time = 0.855052, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] Defer[Int][Sin[c + d*x]*(a + b*Tan[c + d*x])^n, x]
```

Rubi steps

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx=\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx$$

Mathematica [A] time = 2.18546, size = 0, normalized size = 0.

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^n, x]
```

Maple [A] time = 0.041, size = 0, normalized size = 0.

$$\int \sin (dx + c) (a + b \tan (dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b \tan (dx + c) + a)^n \sin (dx + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \tan (dx + c) + a)^n \sin (dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan (c + dx))^n \sin (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*sin(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)^n \sin(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c), x)
```

3.90 $\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=21

$$\text{CannotIntegrate}\left(\csc(c+dx)(a+b\tan(c+dx))^n, x\right)$$

```
[Out] CannotIntegrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^n, x]
```

Rubi [A] time = 0.486547, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] Defer[Int][Csc[c + d*x]*(a + b*Tan[c + d*x])^n, x]
```

Rubi steps

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx = \int \csc(c+dx)(a+b\tan(c+dx))^n\,dx$$

Mathematica [A] time = 1.52046, size = 0, normalized size = 0.

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^n, x]
```

Maple [A] time = 0.23, size = 0, normalized size = 0.

$$\int \csc (dx + c) (a + b \tan (dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b \tan (dx + c) + a)^n \csc (dx + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \tan (dx + c) + a)^n \csc (dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*csc(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)^n \csc(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c), x)
```

3.91 $\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=23

$$\text{CannotIntegrate}\left(\csc^3(c+dx)(a+b\tan(c+dx))^n,x\right)$$

```
[Out] CannotIntegrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]
```

Rubi [A] time = 1.69981, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]

[Out] Defer[Int][Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx=\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

Mathematica [A] time = 2.85234, size = 0, normalized size = 0.

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]

[Out] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]
```

Maple [A] time = 0.189, size = 0, normalized size = 0.

$$\int \left(\csc \left(dx+c\right)\right)^{3} \left(a+b\tan \left(dx+c\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\tan (dx+c)+a)^{n}\csc (dx+c)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\tan (dx+c)+a)^{n}\csc (dx+c)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)^n \csc(dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType, Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```